Lawton Chiles High School

AP Calculus BC
Summer Assignment

Hello calculus students!

In preparation for the coming academic year, complete the problems in this packet by the first day of school. Label your work and make sure it is organized so anyone could look at it and know which problem you are doing and how you reached your conclusion. On the free response questions on the AP exam you get points for work shown so we're going to start practicing now.

All of the concepts and skills included in these questions are prerequisites for AP Calculus BC. If there is a skill or concept you are unfamiliar with make sure you review it prior to the start of the next school year. If you need help, check the resources page at the end. The videos linked there will give you a good place to start.

I'll check my email every couple of weeks during the summer, so if you have any questions you can reach me at rayc3@leonschools.net.

Best,
Ms. Ray

Introduction: Knowledge and Skills Required for Calculus

- Familiarity with vocabulary introduced in the prerequisite math courses
- Ability to verbalize concepts and processes in mathematics
- Algebraic manipulation is second nature (e.g. fraction arithmetic, radicals, factoring, completing the square, fraction decomposition, polynomial division, substitution, exponents, factorials, etc.)
- Understanding of the difference between exact and approximate answers and how to round
- Analyze functions for their characteristics including zeros, positive/negative values, extrema, boundedness, discontinuities, etc.
- *If you need to review some of these concepts, there are video resources linked on the last page.*

Topic 1: Simplifying Expressions

Directions: Simplify each expression as much as possible without a calculator. Reduce all fractions, condense fractions into a single fraction, and eliminate negative exponents.

1. $\frac{3x+3}{2x-1} \div \frac{x^2+2x+1}{4x}$
2. $2^{x+1} \cdot 4^{2x} \cdot \frac{(-4)^x}{4^{x+3}}$
3. $e^x + e^{-x} + \ln 1$
4. $\frac{3^{2-x}}{9} \cdot \frac{1}{1-x^{-1}}$
5. $\frac{15!}{13!} \cdot \frac{6}{4!}$
6. $\frac{x^{n+1}}{(n+1)!} \div \frac{x}{n!}$

Directions: Use the strategy listed next to the expression to simplify it.

7. Long division: $\frac{x^3+4x+1}{x+2}$
8. Fraction decomposition: $\frac{x}{2x^2+7x+3}$
9. Completing the square: $3x^2 + 12x - 10$

Topic 2: Linear Equations and Slope

Directions: Complete the following problems without a calculator.

10. What is the point-slope form of a line? Write it in the form $y = f(x)$.
11. In the same form as #10, write the equation of the line that passes through the point $(-3,7)$ and is parallel to $y = -\frac{2}{3}(x+1) + 4$.
12. In the same form as #10, write the equation of the line that is perpendicular to $f(x) = \frac{1}{2}(x-3) + 5$ and has the same zero as $f(x)$.

Topic 3: Solving Equations

Directions: Solve for y.

13. $\frac{1}{5}\ln y = \frac{1}{3}x^3 + 4$
14. $\frac{1}{y} = e^{-x} + 2$
15. $\tan y + \pi = x^2$

Topic 4: Sequences and Series

Directions: Write each series in summation notation.

16. $\frac{2}{3}+\frac{5}{3}+\frac{25}{6}+\frac{125}{12}+\frac{625}{24}$

17. $1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\frac{1}{120}+\frac{1}{720}$

18. $\frac{1}{4}+\frac{1}{6}+\frac{1}{9}+\frac{2}{27}+\cdots$

Directions: Determine whether each sum converges or diverges. If it converges, find the sum without a calculator.

19. $\sum_{n=0}^{\infty}\frac{1}{2}\cdot\left(\frac{4}{7}\right)^n$

20. $\sum_{n=0}^{\infty}\frac{5\cdot 3^n}{2^n}$

21. $\sum_{n=0}^{\infty}\frac{n!}{5}$

22. $\sum_{n=0}^{\infty}\left(-\frac{e}{3}\right)^n$

Directions: Write an explicit formula for sequence and then determine the tenth term in the sequence.

23. $1, -\frac{x^2}{4!}, \frac{x^4}{8!}, -\frac{x^6}{12!}, \ldots$

24. $3,\ 5(x+1),\ 7(x+1)^2,\ 9(x+1)^3,\ \ldots$

Topic 5: Function Inequalities

Directions: Without a calculator, answer the following questions for each function:

A) What are the zeros of the function? When is the function undefined?
B) Solve $f(x)>0$
C) Solve $f(x)<0$

25. $f(x)=x^2(x+3)(x-5)^3$

26. $f(x)=x^4-9x^2+4x+12$

27. $f(x)=2^{x+3}-5$

28. $f(x)=\log_4(x+1)$

29. $f(x)=\frac{(x+1)(x-4)}{x-8}$

30. $f(x)=\frac{\sqrt{x-3}}{x+4}$

Directions: With a calculator (tutorials on last page), solve each inequality. Round to the nearest thousandth.

31. $x^3+2x^2-x-5\geq 0$

32. $\sin 2x+\cos 3x<0$ on the closed interval $[0,\pi]$

Topic 6: Describing Functions

Directions: Describe the end behavior, domain, and continuity of each function without a calculator.

33. $y=\frac{2x+5}{\sqrt{x^2+1}}$

34. $y=\frac{10}{e^x-3}$

35. $y=\begin{cases}(x-2)^2+1 & if\ x\geq 2\\ |x| & if\ x<2\end{cases}$

Directions: Using a calculator (tutorials on last page), determine the extreme values to the nearest thousandth (remember that these are y-values). Are they relative or absolute extreme values? Where is the function increasing and decreasing?

36. $f(x)=x^4+x^3-x^2-x$

37. $f(x)=x\cos x$ on the closed interval $\left[0,\frac{3\pi}{2}\right]$

Topic 7: Trigonometry

38. What are the maximum and minimum values and period of $y = 2 - 5\cos 3x$

39. Describe the graph of $y = \arctan x$. Consider domain, range, end behavior, and asymptotes.

40. Rewrite $5\cos^2\theta + \sin^2(2\theta)$ so that it has no exponents. (Hint: double angle identities)

Directions: Simplify each expression as much as possible and leave in terms of sine and/or cosine.

41. $\dfrac{\frac{2}{3}\tan\theta}{\sqrt{4\tan^2\theta + 4}} \cdot \dfrac{2}{3}\sec^2$

42. $\dfrac{5}{25\sec^2\theta\sqrt{25\sec^2\theta - 25}}$

43. $\cos\theta\sqrt{7 - 7\sin^2\theta}$

Topic 8: Polar Equations

44. Graph the polar points: $A\left(2, \frac{\pi}{6}\right)$, $B\left(-3, \frac{\pi}{2}\right)$, $C\left(5, -\frac{2\pi}{3}\right)$, $D\left(-4, -\frac{\pi}{4}\right)$

45. What is the slope of the polar line $\theta = \frac{\pi}{3}$?

Directions: Graph the polar equations over the interval $[0, 2\pi]$.

46. $r(\theta) = 2$

47. $r(\theta) = 2\cos 3\theta$

48. $r(\theta) = 4 + 4\sin\theta$

Topic 9: Parametric Equations

Directions: Graph the parametric equations with the given parameter.

49. $x(t) = -t$, $y(t) = t^2$

50. $x = \cos t$ and $y = 1 - \sin^2 t$ if $0 \le t \le 2\pi$

51. $x(t) = \frac{1}{t}$ and $y(t) = \frac{1}{t^2}$ if $t > 0$

Directions: Find the x- and y-intercepts for each set of parametric equations

52. $x(t) = t^2 + 5t + 6$, $y(t) = 2t^2 - 7$

53. $x = \sin t$ and $y = t^3 + 1$ if $0 \le t \le 2\pi$

Directions: Write each polar equation as a vector equation $r(\theta) = \langle x(\theta), y(\theta)\rangle$. *(Hint: Write equations for x and y using the conversion equations.)*

54. $r(\theta) = \sin 2\theta$

55. $r(\theta) = 4 - 3\cos\theta$

Topic 10: Applications and Verbalizing

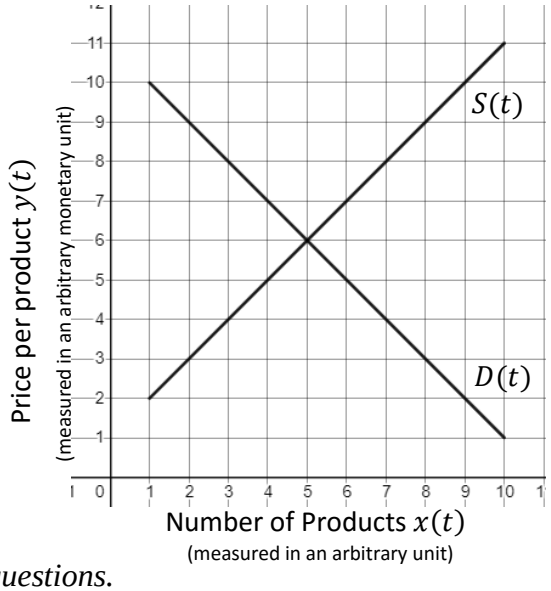
A company's profit when producing x hundred products can be determined by $P(x) = 21 - \frac{(x-6.2)^4}{17}$ *when* $0 \le x \le 13$. *Answer each question in context of the problem. A calculator may be used.*

56. Find the *x*- and *y*-intercepts for $P(x)$.

57. Find the solution for $P(x) < 0$.

58. Find the extreme values (including end points) and boundedness of $P(x)$.

The graph for the supply and demand of a product are given to the right. The parametric graph for supply, $S(t)$*, shows the number of products a company will produce* $x(t)$ *at each price point* $y(t)$*. The parametric graph for demand,* $D(t)$*, shows the number of products consumers want to buy* $x(t)$ *at each price point* $y(t)$.

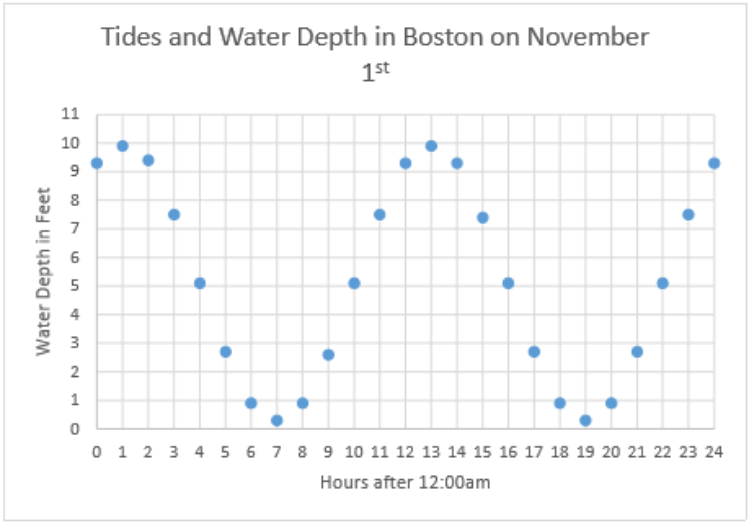


59. Draw a graph for $S(t) - D(t)$.

60. What is the *x*-intercept in context of the question?

61. What does it mean when $S(t) - D(t) < 0$?

Use the data from the graph and chart below to answer the following questions.



Time	Water depth (feet)
12:00 am	9.3
1:00 am	9.9
2:00 am	9.4
3:00 am	7.5
4:00 am	5.1
5:00 am	2.7
6:00 am	0.9
7:00 am	0.3
8:00 am	0.9
9:00 am	2.6
10:00 am	5.1
11:00 am	7.5
12:00 pm	9.3
1:00 pm	9.9

62. Write an equation $D(x)$ for the water depth in Boston, where x represents the number of hours since midnight November 1st. (Midnight on Nov. 1 is $x = 0$.)

63. Use your equation to predict the high tide times for November 2 – November 3.

Resources

Calculator tutorials for the TI-84 family:

- Graphing functions: https://youtu.be/q1OEXc_Gio4
- Finding zeros: https://youtu.be/V3MHTV_6wWw
- Finding extrema: https://youtu.be/-Ytczz1YFyc

Topic 1

- Factorials: https://youtu.be/pxh__ugRKz8
- Polynomial division: https://youtu.be/_FSXJmESFmQ
- Partial fractions and fraction decomposition: https://youtu.be/QKkdYW77xNI
- Completing the square: https://youtu.be/sh-MP-dVhD4

Topic 3

- Solving exponential equations: https://youtu.be/9tutJ5xrRwg
- Solving logarithmic equations: https://youtu.be/fnhFneOz6n8
- Solving trig equations: https://youtu.be/kEcbxiLeGTc
- Solving rational equations: https://youtu.be/1fR_9ke5-n8

Topic 4

- Geometric sequences: https://youtu.be/4NT9E-wbJMg
- Summation notation: https://youtu.be/hEPk36Yncxg
- Convergent and divergent series: https://youtu.be/oIlwWTigHsk

Topic 5

- Solving polynomial inequalities: https://youtu.be/Fd5ys4PQ-aM
- Solving rational inequalities: https://youtu.be/gfnVHwhEe6U

Topic 6

- End behavior and horizontal asymptotes (includes limit notation): https://youtu.be/80GLyehX8i8
- Domain for rational functions and roots: https://youtu.be/djT6-YamHaA
- Continuity: https://youtu.be/joewRl1CTL8
- Extrema: https://youtu.be/bZYTDst1MOo

Topic 7

- Trig graphs: https://youtu.be/U3ktjhKRdng and https://youtu.be/UPmJ9_X2BbE
- Trig Identities review: https://youtu.be/a70-dYvDJZY

Topic 8

- Graphing polar points and converting polar points: https://youtu.be/aSdaT62ndYE

Topic 9

- Graphing parametric equations: https://youtu.be/tsnHL1Lb5MU and https://youtu.be/97pe-QlSGqA