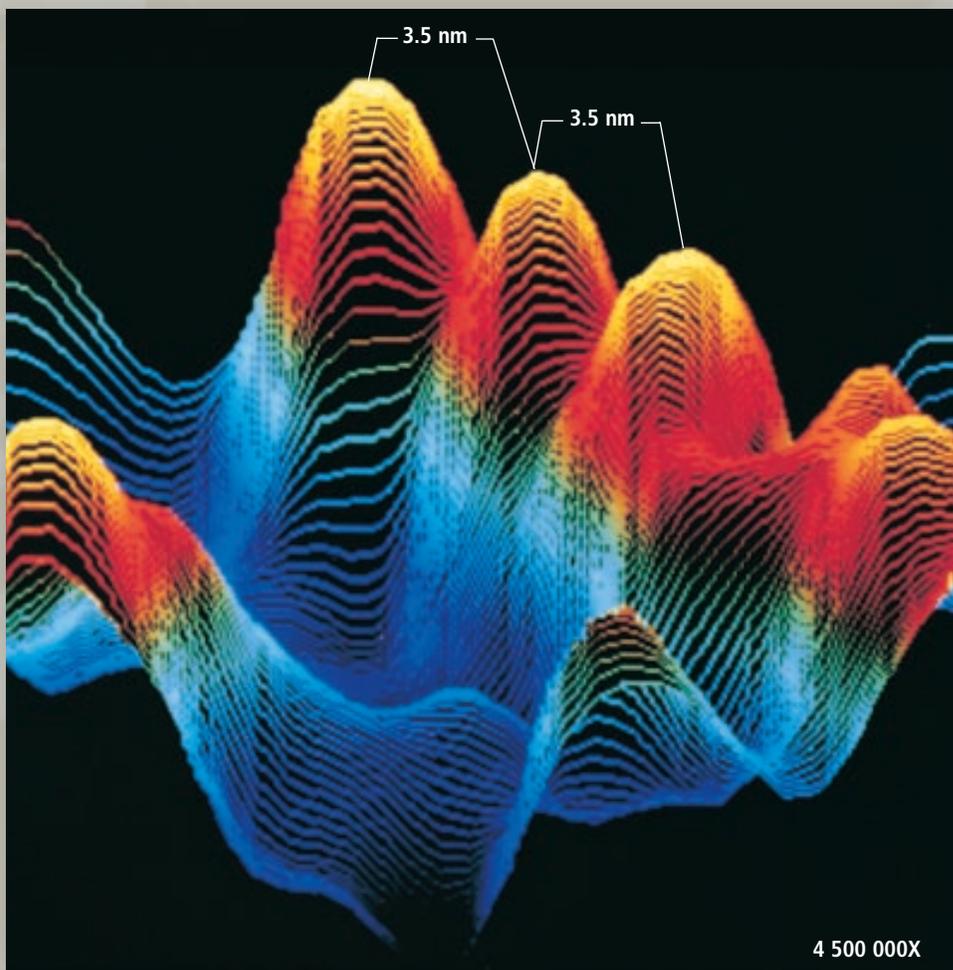


# *Measurements and Calculations*

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*Measurements provide  
quantitative information.*

# Scientific Method

## SECTION 2-1

### OBJECTIVES

- Describe the purpose of the scientific method.
- Distinguish between qualitative and quantitative observations.
- Describe the differences between hypotheses, theories, and models.

Sometimes progress in science comes about through accidental discoveries. However, most scientific advances result from carefully planned investigations. The process researchers use to carry out their investigations is often called the scientific method. *The scientific method is a logical approach to solving problems by observing and collecting data, formulating hypotheses, testing hypotheses, and formulating theories that are supported by data.*

### Observing and Collecting Data

*Observing* is the use of the senses to obtain information. Observation often involves making measurements and collecting data. The data may be descriptive (qualitative) or numerical (quantitative) in nature. Numerical information, such as the fact that a sample of copper ore has a mass of 25.7 grams, is *quantitative*. Non-numerical information, such as the fact that the sky is blue, is *qualitative*.

Experimenting involves carrying out a procedure under controlled conditions to make observations and collect data. To learn more about matter, chemists study systems. A **system** is a specific portion of matter in a given region of space that has been selected for study during an experiment or observation. When you observe a reaction in a test tube, the test tube and its contents form a system.

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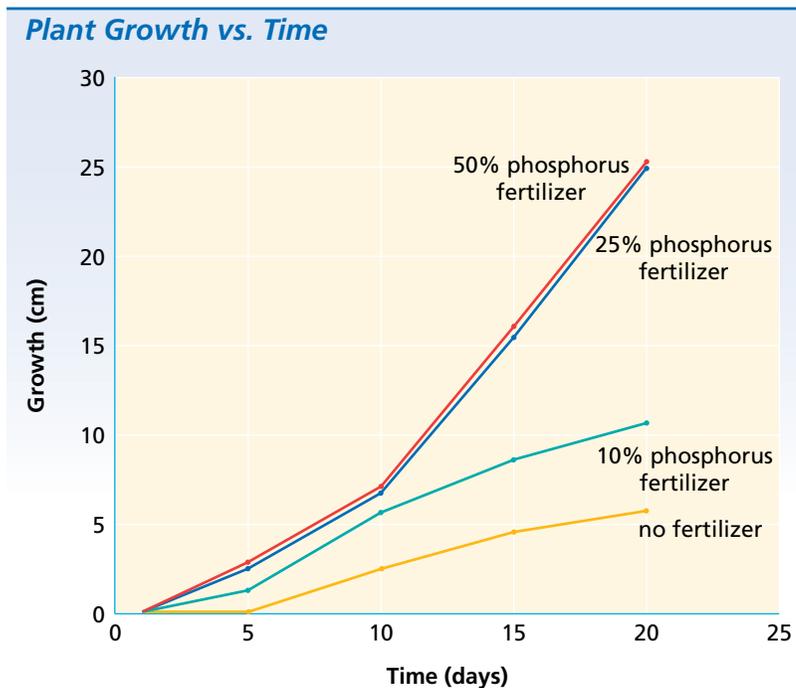
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**FIGURE 2-1** These students are designing an experiment to determine how to get the largest volume of popped corn from a fixed number of kernels. They think that the volume is likely to increase as the moisture in the kernels increases. Their experiment will involve soaking some kernels in water and observing whether the volume of the popped corn is greater than that of corn popped from kernels that have not been soaked.

**FIGURE 2-2** A graph of data can show relationships between two variables. In this case the graph shows data collected during an experiment to determine the effect of phosphorus fertilizer compounds on plant growth. The following is one possible hypothesis: *If phosphorus stimulates corn-plant growth, then corn plants treated with a soluble phosphorus compound should grow faster, under the same conditions, than corn plants that are not treated.*



## Formulating Hypotheses

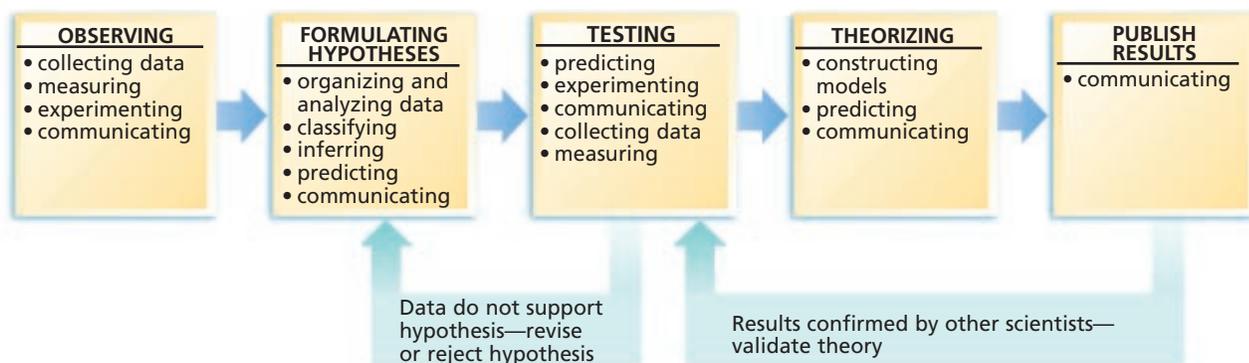
As scientists examine and compare the data from their own experiments, they attempt to find relationships and patterns—in other words, they make generalizations based on the data. Generalizations are statements that apply to a range of information. To make generalizations, data are sometimes organized in tables and analyzed using statistics or other mathematical techniques, often with the aid of graphs and a computer.

Scientists use generalizations about the data to formulate a **hypothesis**, or *testable statement*. The hypothesis serves as a basis for making predictions and for carrying out further experiments. Hypotheses are often drafted as “if-then” statements. The “then” part of the hypothesis is a prediction that is the basis for testing by experiment. Figure 2-2 shows data collected to test a hypothesis.

## Testing Hypotheses

Testing a hypothesis requires experimentation that provides data to support or refute a hypothesis or theory. Do the data in Figure 2-2 support the hypothesis? If testing reveals that the predictions were not correct, the generalizations on which the predictions were based must be discarded or modified. One of the most difficult, yet most important, aspects of science is rejecting a hypothesis that is not supported by data.

## STAGES IN THE SCIENTIFIC METHOD



**FIGURE 2-3** The scientific method is not a stepwise process. Scientists may repeat steps many times before there is sufficient evidence to formulate a theory. You can see that each stage represents a number of different activities.

## Theorizing

When the data from experiments show that the predictions of the hypothesis are successful, scientists typically try to explain the phenomena they are studying by constructing a model. *A model in science is more than a physical object; it is often an explanation of how phenomena occur and how data or events are related.* Models may be visual, verbal, or mathematical. One of the most important models in chemistry is the atomic model of matter, which states that matter is composed of tiny particles called atoms.

If a model successfully explains many phenomena, it may become part of a theory. The atomic model is a part of the atomic theory, which you will study in Chapter 3. *A theory is a broad generalization that explains a body of facts or phenomena.* Theories are considered successful if they can predict the results of many new experiments. Examples of the important theories you will study in chemistry are kinetic-molecular theory and collision theory. Figure 2-3 shows where theory fits in the scheme of the scientific method.

## SECTION REVIEW

1. What is the scientific method?
2. Which of the following are quantitative?
  - a. the liquid floats on water
  - b. the metal is malleable
  - c. the liquid has a temperature of 55.6°C
3. How do hypotheses and theories differ?
4. How are models related to theories and hypotheses?
5. What are the components of the system in the graduated cylinder shown on page 38?



# Chemistry's Holy Grail

From *Chemistry Imagined, Reflections on Science*, written by Roald Hoffmann and illustrated by Vivian Torrence

**I**n other fields the important questions seem to be breathtakingly simple: How does the brain work? Let's land a manned space craft on Mars . . . What is the "cure" for cancer? The outsider romanticizes, to be sure. But what about chemistry, where is the Holy Grail of the molecular science? . . .

To search for the Holy Grail, 150 of King Arthur's knights committed their hearts and resources. Translated to modern times, a like group effort demands some gigantic, or at least intricately expensive, machinery. Big science, in other words: a supercollider to search the innards, a space telescope to probe the outer fringes, a genome project to map human heredity. None of these is typical of chemistry, where from the beginning small groupings of people, working with relatively cheap cookware, have transformed a wondrous variety of matter.

Chemistry is an intermediate science. Its universe is defined not by reduction to a few elementary particles, or even to the hundred or so elements, but by a reaching out to the infinities of molecules that can be synthesized. A registry of new molecules contains over ten million man-made entries. A small fraction of these is of natural origin, though millions are waiting to be analyzed. And millions more are lost in species that our ecologi-

cal pressure extinguishes. Most molecules are man- and woman-made. The beauty I would claim for chemistry is that of richness and complexity, the realm of the possible. There is no end to the range of structure and function that molecules exhibit. . . .



Roald Hoffmann

There is no Holy Grail in chemistry. Yes, we would like to have a magic machine that separates the most awful mixture, purifies every component to 99.4% purity (or better if we pay more) and determines the precise arrangement of atoms in space in each molecule. Yes, we'd like to know in complete detail the resistance of a molecule to every twist, bend, stretch, rock, and roll. And, yes, we certainly must espy the secret, rapid motions molecules undergo in their most

intimate transformations. And, most of all, most fundamental to the science of transformations, we desire control—ways to synthesize to order, in a short time, using cheap materials, in one pot, any molecule in the world.

The secret of the Holy Grail is that it is to be found not in the consummation but in the search. Imagine every woman rejuvenated, every man saved, all ills, physical and mental, cured, all humanity perfect, and, of course, at peace. What a dull world! . . .

If the grand desires of chemistry were achieved—to know what one has, how things happen on the molecular scale, how to create molecules with absolute control—chemistry would simply vanish. To come to terms with complexity and the never-ending search, to find joy and beauty in the plain thing, the small step—that is the grail.

## Reading for Meaning

What does Hoffmann mean when he says that chemistry is an intermediate science?

## Read Further

Hoffmann states that, most of all, chemists desire ways to control chemical reactions. Research three ways that chemists control reactions, and write a paragraph describing each.

# Units of Measurement

## SECTION 2-2

### OBJECTIVES

- Distinguish between a quantity, a unit, and a measurement standard.
- Name SI units for length, mass, time, volume, and density.
- Distinguish between mass and weight.
- Perform density calculations.
- Transform a statement of equality to a conversion factor.

**M**easurements are quantitative information. A measurement is more than just a number, even in everyday life. Suppose a chef were to write a recipe listing quantities such as 1 salt, 3 sugar, and 2 flour. The cooks could not use the recipe without more information. They would need to know whether the number 3 represented teaspoons, tablespoons, cups, ounces, grams, or some other unit for sugar.

Measurements *represent* quantities. A **quantity** is something that has *magnitude, size, or amount*. A quantity is not the same as a measurement. For example, the quantity represented by a teaspoon is volume. The teaspoon is a unit of measurement, while volume is a quantity. A teaspoon is a measurement standard in this country. Units of measurement compare what is to be measured with a previously defined size. Nearly every measurement is a number plus a unit. The choice of unit depends on the quantity being measured.

Many centuries ago, people sometimes marked off distances in the number of foot lengths it took to cover the distance. But this system was unsatisfactory because the number of foot lengths used to express a distance varied with the size of the measurer's foot. Once there was agreement on a standard for foot length, confusion as to the real length was eliminated. It no longer mattered who made the measurement, as long as the standard measuring unit was correctly applied.

## SI Measurement

Scientists all over the world have agreed on a single measurement system called *Le Système International d'Unités*, abbreviated **SI**. This system was adopted in 1960 by the General Conference on Weights and Measures. SI has seven base units, and most other units are derived from these seven. Some non-SI units are still commonly used by chemists and are also used in this book.

SI units are defined in terms of standards of measurement. The standards are objects or natural phenomena that are of constant value, easy to preserve and reproduce, and practical in size. International organizations monitor the defining process. In the United States, the National Institute of Standards and Technology plays the main role in maintaining standards and setting style conventions. For example, numbers are written in a form that is agreed upon internationally. The number seventy-five thousand is written 75 000, not 75,000, because the comma is used in other countries to represent a decimal point.

**TABLE 2-1 SI Base Units**

Quantity	Quantity symbol	Unit name	Unit abbreviation	Defined standard
Length	<i>l</i>	meter	m	the length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second
Mass	<i>m</i>	kilogram	kg	the unit of mass equal to the mass of the international prototype of the kilogram
Time	<i>t</i>	second	s	the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom
Temperature	<i>T</i>	kelvin	K	the fraction 1/273.16 of the thermodynamic temperature of the triple point of water
Amount of substance	<i>n</i>	mole	mol	the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12
Electric current	<i>I</i>	ampere	A	the constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to $2 \times 10^{-7}$ newton per meter of length
Luminous intensity	<i>I<sub>v</sub></i>	candela	cd	the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \times 10^{12}$ hertz and that has a radiant intensity in that direction of 1/683 watt per steradian

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## SI Base Units

The seven SI base units and their standard abbreviated symbols are listed in Table 2-1. All the other SI units can be derived from the fundamental units.

Prefixes added to the names of SI base units are used to represent quantities that are larger or smaller than the base units. Table 2-2 lists SI prefixes using units of length as examples. For example, the prefix *centi-*, abbreviated c, represents an exponential factor of  $10^{-2}$ , which equals 1/100. Thus, 1 centimeter, 1 cm, equals 0.01 m, or 1/100 of a meter.

### Mass

As you learned in Chapter 1, mass is a measure of the quantity of matter. The SI standard unit for mass is the kilogram. The standard for mass defined in Table 2-1 is used to calibrate balances all over the world.

**TABLE 2-2 SI Prefixes**

Prefix	Unit abbreviation	Exponential factor	Meaning	Example
tera	T	$10^{12}$	1 000 000 000 000	1 terameter (Tm) = $1 \times 10^{12}$ m
giga	G	$10^9$	1 000 000 000	1 gigometer (Gm) = $1 \times 10^9$ m
mega	M	$10^6$	1 000 000	1 megameter (Mm) = $1 \times 10^6$ m
kilo	k	$10^3$	1000	1 kilometer (km) = 1000 m
hecto	h	$10^2$	100	1 hectometer (hm) = 100 m
deka	da	$10^1$	10	1 dekameter (dam) = 10 m
		<b><math>10^0</math></b>	<b>1</b>	<b>1 meter (m)</b>
deci	d	$10^{-1}$	1/10	1 decimeter (dm) = 0.1 m
centi	c	$10^{-2}$	1/100	1 centimeter (cm) = 0.01 m
milli	m	$10^{-3}$	1/1000	1 millimeter (mm) = 0.001 m
micro	$\mu$	$10^{-6}$	1/1 000 000	1 micrometer ( $\mu$ m) = $1 \times 10^{-6}$ m
nano	n	$10^{-9}$	1/1 000 000 000	1 nanometer (nm) = $1 \times 10^{-9}$ m
pico	p	$10^{-12}$	1/1 000 000 000 000	1 picometer (pm) = $1 \times 10^{-12}$ m
femto	f	$10^{-15}$	1/1 000 000 000 000 000	1 femtometer (fm) = $1 \times 10^{-15}$ m
atto	a	$10^{-18}$	1/1 000 000 000 000 000 000	1 attometer (am) = $1 \times 10^{-18}$ m

The mass of a typical textbook is about 1 kg. The gram, g, which is 1/1000 of a kilogram, is more useful for measuring masses of small objects, such as flasks and beakers. For even smaller objects, such as tiny quantities of chemicals, the milligram, mg, is often used. One milligram is 1/1000 of a gram, or 1/1 000 000 of a kilogram.

Mass is often confused with weight because people often express the weight of an object in grams. Mass is determined by comparing the mass of an object with a set of standard masses that are part of the balance. **Weight** is a measure of the gravitational pull on matter. Unlike weight, mass does not depend on such an attraction. Mass is measured on instruments such as a balance, and weight is typically measured on a spring scale. Taking weight measurements involves reading the amount that an object pulls down on a spring. As the force of Earth's gravity on an object increases, the object's weight increases. The weight of an object on the moon is about one-sixth of its weight on Earth.

## Length

The SI standard unit for length is the meter. A distance of 1 m is about the width of an average doorway. To express longer distances, the kilometer, km, is used. One kilometer equals 1000 m. Road signs in the United States sometimes show distances in kilometers as well as miles. The kilometer is the unit used to express highway distances in most other countries of the world. To express shorter distances, the centimeter

**FIGURE 2-4** The meter is the SI unit of length, but the centimeter is often used to measure smaller distances. What is the width in cm of the rectangular piece of aluminum foil shown?



is often used. From Table 2-2, you can see that one centimeter equals 1/100 of a meter. The width of this book is just over 20 cm.

## Derived SI Units

Many SI units are combinations of the quantities shown in Table 2-1. *Combinations of SI base units form derived units.* Some derived units are shown in Table 2-3.

Derived units are produced by multiplying or dividing standard units. For example, area, a derived unit, is length times width. If both length and width are expressed in meters, the area unit equals meters times meters, or square meters, abbreviated  $m^2$ . The last column of

**TABLE 2-3** Derived SI Units

Quantity	Quantity symbol	Unit	Unit abbreviation	Derivation
Area	$A$	square meter	$m^2$	length $\times$ width
Volume	$V$	cubic meter	$m^3$	length $\times$ width $\times$ height
Density	$D$	kilograms per cubic meter	$\frac{kg}{m^3}$	$\frac{\text{mass}}{\text{volume}}$
Molar mass	$M$	kilograms per mole	$\frac{kg}{mol}$	$\frac{\text{mass}}{\text{amount of substance}}$
Concentration	$c$	moles per liter	M	$\frac{\text{amount of substance}}{\text{volume}}$
Molar volume	$V_m$	cubic meters per mole	$\frac{m^3}{mol}$	$\frac{\text{volume}}{\text{amount of substance}}$
Energy	$E$	joule	J	force $\times$ length

Table 2-3 shows the combination of fundamental units used to obtain derived units.

Some combination units are given their own names. For example, pressure expressed in base units is the following.

$$\text{kg/m}\cdot\text{s}^2$$

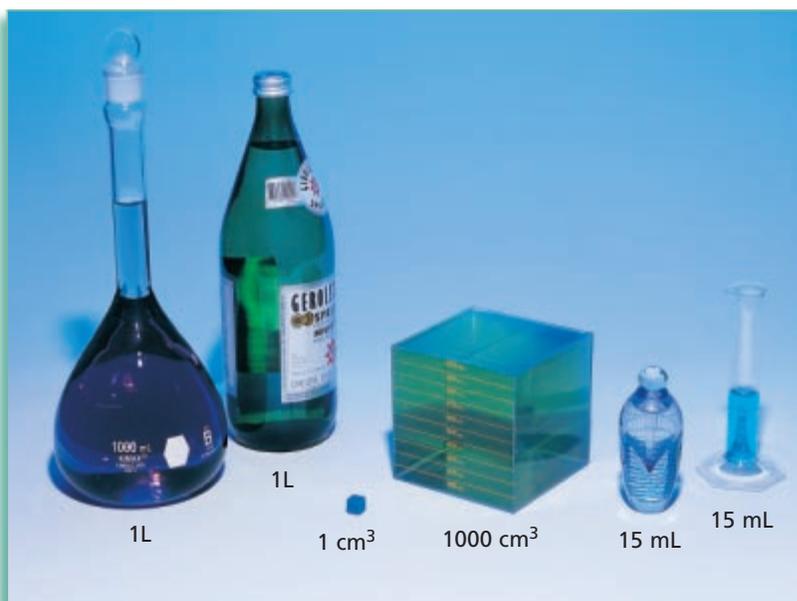
The name *pascal*, Pa, is given to this combination. You will learn more about pressure in Chapter 10. Prefixes can also be added to express derived units. Area can be expressed in  $\text{cm}^2$ , square centimeters, or  $\text{mm}^2$ , square millimeters.

## Volume

**Volume** is the amount of space occupied by an object. The derived SI unit of volume is cubic meters,  $\text{m}^3$ . One cubic meter is equal to the volume of a cube whose edges are 1 m long. Such a large unit is inconvenient for expressing the volume of materials in a chemistry laboratory. Instead, a smaller unit, the cubic centimeter,  $\text{cm}^3$ , is often used. There are 100 centimeters in a meter, so a cubic meter contains 1 000 000  $\text{cm}^3$ .

$$1 \text{ m}^3 \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{100 \text{ cm}}{1 \text{ m}} = 1\,000\,000 \text{ cm}^3$$

When chemists measure the volumes of liquids and gases, they often use a non-SI unit called the liter. The liter is equivalent to one cubic decimeter. Thus, a liter, L, is also equivalent to 1000  $\text{cm}^3$ . Another non-SI unit, the milliliter, mL, is used for smaller volumes. There are 1000 mL in 1 L. Because there are also 1000  $\text{cm}^3$  in a liter, the two units—milliliter and cubic centimeter—are interchangeable.



**FIGURE 2-6** The relationships between various volumes are shown here. One liter contains 1000 mL of liquid, and 1 mL is equivalent to 1  $\text{cm}^3$ . A small perfume bottle contains about 15 mL of liquid. The volumetric flask and graduated cylinder are used for measuring liquid volumes in the lab.



**FIGURE 2-5** The speed that registers on a speedometer represents distance traveled per hour and is expressed in the derived units kilometers per hour or miles per hour.



**FIGURE 2-7** Density is the ratio of mass to volume. Both water and copper shot float on mercury because mercury is more dense.

## Density

An object made of cork feels lighter than a lead object of the same size. What you are actually comparing in such cases is how massive objects are compared with their size. This property is called density. **Density is the ratio of mass to volume, or mass divided by volume.** Mathematically, the relationship for density can be written in the following way.

$$\text{density} = \frac{\text{mass}}{\text{volume}} \text{ or } D = \frac{m}{V}$$

The quantity  $m$  is mass,  $V$  is volume, and  $D$  is density.

The SI unit for density is derived from the base units for mass and volume—the kilogram and the cubic meter, respectively—and can be expressed as kilograms per cubic meter,  $\text{kg/m}^3$ . This unit is inconveniently large for the density measurements you will make in the laboratory. You will often see density expressed in grams per cubic centimeter,  $\text{g/cm}^3$ , or grams per milliliter,  $\text{g/mL}$ . The densities of gases are generally reported either in kilograms per cubic meter,  $\text{kg/m}^3$ , or in grams per liter,  $\text{g/L}$ .

Density is a characteristic physical property of a substance. It does not depend on the size of the sample because as the sample's mass increases, its volume increases proportionately, and the ratio of mass to volume is constant. Therefore, density can be used as one property to help identify a substance. Table 2-4 shows the densities of some common materials. As you can see, cork has a density of only  $0.24 \text{ g/cm}^3$ , which is less than the density of liquid water. Because cork is less dense than water, it floats on water. Lead, on the other hand, has a density of  $11.35 \text{ g/cm}^3$ . The density of lead is greater than that of water, so lead sinks in water.

Note that Table 2-4 specifies the temperatures at which the densities were measured. That is because density varies with temperature. Most objects expand as temperature increases, thereby increasing in volume. Because density is mass divided by volume, density usually decreases with increasing temperature.

**TABLE 2-4** Densities of Some Familiar Materials

Solids	Density at 20°C ( $\text{g/cm}^3$ )	Liquids	Density at 20°C ( $\text{g/mL}$ )
cork	0.24*	gasoline	0.67*
butter	0.86	ethyl alcohol	0.791
ice	0.92 <sup>†</sup>	kerosene	0.82
sucrose	1.59	turpentine	0.87
bone	1.85*	water	0.998
diamond	3.26*	sea water	1.025**
copper	8.92	milk	1.031*
lead	11.35	mercury	13.6

<sup>†</sup> measured at 0°C

\* typical density

\*\* measured at 15°C



## Density of Pennies

Wear Safety Goggles and an Apron.



### Materials

- balance
- 100 mL graduated cylinder
- 40 pennies dated before 1982
- 40 pennies dated after 1982
- water

### Procedure

1. Using the balance, determine the mass of the 40 pennies minted prior to 1982. Repeat this measurement two more times. Average the results of the three trials to determine the average mass of the pennies.
2. Repeat step 1 with the 40 pennies minted after 1982.
3. Pour about 50 mL of water into the 100 mL graduated cylinder. Record the exact volume of the water. Add the 40 pennies minted before 1982. Record the volume of the water and pennies. Repeat this process two more times. Determine the volume of the pennies for each trial. Average the results of those trials to determine the average volume of the pennies.
4. Repeat step 3 with the 40 pennies minted after 1982.
5. Review your data for any large differences between trials that could increase the error of your results. Repeat those measurements.

6. Use the average volume and average mass to calculate the average density for each group of pennies.
7. Compare the average calculated densities with the density of the copper listed in Table 2-4.

### Discussion

1. Why is it best to use the results of three trials rather than a single trial for determining the density?
2. How did the densities of the two groups of pennies compare? How do you account for any difference?
3. Use the results of this investigation to formulate a hypothesis about the composition of the two groups of pennies. How could you test your hypothesis?

### SAMPLE PROBLEM 2-1

A sample of aluminum metal has a mass of 8.4 g. The volume of the sample is 3.1 cm<sup>3</sup>. Calculate the density of aluminum.

#### SOLUTION

Given: mass ( $m$ ) = 8.4 g  
volume ( $V$ ) = 3.1 cm<sup>3</sup>  
Unknown: density ( $D$ )

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{8.4 \text{ g}}{3.1 \text{ cm}^3} = 2.7 \text{ g/cm}^3$$

**PRACTICE**

- |   |   |
|---|---|
| 1. What is the density of a block of marble that occupies 310 cm <sup>3</sup> and has a mass of 853 g?                          | <i>Answer</i><br>2.75 g/cm <sup>3</sup> |
| 2. Diamond has a density of 3.26 g/cm <sup>3</sup> . What is the mass of a diamond that has a volume of 0.350 cm <sup>3</sup> ? | <i>Answer</i><br>1.14 g                 |
| 3. What is the volume of a sample of liquid mercury that has a mass of 76.2 g, given that the density of mercury is 13.6 g/mL?  | <i>Answer</i><br>5.60 mL                |

## Conversion Factors

A **conversion factor** is a ratio derived from the equality between two different units that can be used to convert from one unit to the other. For example, suppose you want to know how many quarters there are in a certain number of dollars. To figure out the answer, you need to know how quarters and dollars are related. There are four quarters per dollar and one dollar for every four quarters. Those facts can be expressed as ratios in three conversion factors.

$$\frac{4 \text{ quarters}}{1 \text{ dollar}} = 1 \quad \frac{1 \text{ dollar}}{4 \text{ quarters}} = 1 \quad \frac{0.25 \text{ dollar}}{1 \text{ quarter}} = 1$$

Notice that each conversion factor equals 1. That is because the two quantities divided in any conversion factor are equivalent to each other—as in this case, where 4 quarters equal 1 dollar. Because conversion factors are equal to 1, they can be multiplied by other factors in equations without changing the validity of the equations. When you want to use a conversion factor to change a unit in a problem, you can set up the problem in the following way.

$$\text{quantity sought} = \text{quantity given} \times \text{conversion factor}$$

For example, to determine the number of quarters in 12 dollars, you would carry out the unit conversion that allows you to change from dollars to quarters.

$$\text{number of quarters} = 12 \text{ dollars} \times \text{conversion factor}$$

Next you would have to decide which conversion factor gives you an answer in the desired unit. In this case, you have dollars and you want quarters. To obtain quarters, you must divide the quantity by dollars. Therefore, the conversion factor in this case must have dollars in the denominator. That factor is 4 quarters/1 dollar. Thus, you would set up the calculation as follows.

$$\begin{aligned} ? \text{ quarters} &= 12 \text{ dollars} \times \text{conversion factor} \\ &= 12 \text{ dollars} \times \frac{4 \text{ quarters}}{1 \text{ dollar}} = 48 \text{ quarters} \end{aligned}$$

Notice that the dollars have divided out, leaving an answer in the desired unit—quarters.

Suppose you had guessed wrong and used 1 dollar/4 quarters when choosing which of the two conversion factors to use. You would have an answer with entirely inappropriate units.

$$? \text{ quarters} = 12 \text{ dollars} \times \frac{1 \text{ dollar}}{4 \text{ quarters}} = \frac{3 \text{ dollars}^2}{\text{quarter}}$$

You will work many problems in this book. It is always best to begin with an idea of the units you will need in your final answer. When working through the Sample Problems, keep track of the units needed for the unknown quantity. Check your final answer against what you've written as the unknown quantity.

## Deriving Conversion Factors

You can derive conversion factors if you know the relationship between the unit you have and the unit you want. For example, from the fact that *deci-* means “1/10,” you know that there is 1/10 of a meter per decimeter and that each meter must have 10 decimeters. Thus, from the equality

$$1 \text{ m} = 10 \text{ dm}$$

you can write the following conversion factors relating meters and decimeters.

$$\frac{1 \text{ m}}{10 \text{ dm}} \text{ and } \frac{0.1 \text{ m}}{\text{dm}^*} \text{ and } \frac{10 \text{ dm}}{\text{m}}$$

The following sample problem illustrates an example of deriving conversion factors to make a unit conversion.

### SAMPLE PROBLEM 2-2

**Express a mass of 5.712 grams in milligrams and in kilograms.**

#### SOLUTION

**Given:** 5.712 g

**Unknown:** mass in mg and kg

The expression that relates grams to milligrams is

$$1 \text{ g} = 1000 \text{ mg}$$

The possible conversion factors that can be written from this expression are

$$\frac{1000 \text{ mg}}{\text{g}} \text{ and } \frac{1 \text{ g}}{1000 \text{ mg}}$$

\*In this book, when there is no digit shown in the denominator, you can assume the value is 1.

To derive an answer in mg, you'll need to multiply 5.712 g by 1000 mg/g.

$$5.712 \text{ g} \times \frac{1000 \text{ mg}}{\text{g}} = 5712 \text{ mg}$$

This answer makes sense because milligrams is a smaller unit than grams and, therefore, there should be more of them.

The kilogram problem is solved similarly.

$$1 \text{ kg} = 1000 \text{ g}$$

Conversion factors representing this expression are

$$\frac{1 \text{ kg}}{1000 \text{ g}} \text{ and } \frac{1000 \text{ g}}{\text{kg}}$$

To derive an answer in kg, you'll need to multiply 5.712 g by 1 kg/1000 g.

$$5.712 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 0.005712 \text{ kg}$$

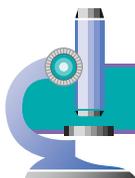
The answer makes sense because kilograms is a larger unit than grams and, therefore, there should be fewer of them.

### PRACTICE

- Express a length of 16.45 m in centimeters and in kilometers. *Answer*  
1645 cm, 0.01645 km
- Express a mass of 0.014 mg in grams. *Answer*  
0.000 014 g

## SECTION REVIEW

- Why are standards needed for measured quantities?
- Label each of the following measurements by the quantity each represents. For instance, a measurement of 10.6 kg/m<sup>3</sup> represents density.
  - 5.0 g/mL
  - 37 s
  - 47 J
  - 39.56 g
  - 25.3 cm<sup>3</sup>
  - 325 ms
  - 500 m<sup>2</sup>
  - 30.23 mL
  - 2.7 mg
  - 0.005 L
- Complete the following conversions.
  - 10.5 g = \_\_\_\_ kg
  - 1.57 km = \_\_\_\_ m
  - 3.54 μg = \_\_\_\_ g
  - 3.5 mol = \_\_\_\_ μmol
  - 1.2 L = \_\_\_\_ mL
  - 358 cm<sup>3</sup> = \_\_\_\_ m<sup>3</sup>
  - 548.6 mL = \_\_\_\_ cm<sup>3</sup>
- Write conversion factors to represent the following equalities.
  - 1 m<sup>3</sup> = 1 000 000 cm<sup>3</sup>
  - 1 in. = 2.54 cm
  - 1 μg = 0.000 001 g
  - 1 Mm = 1 000 000 m
- What is the density of an 84.7 g sample of an unknown substance if the sample occupies 49.6 cm<sup>3</sup>?
  - What volume would be occupied by 7.75 g of this same substance?



# Roadside Pollution Detector

**D**r. Donald Stedman, a chemist at the University of Denver, has developed a device that monitors exhaust emissions on highways.

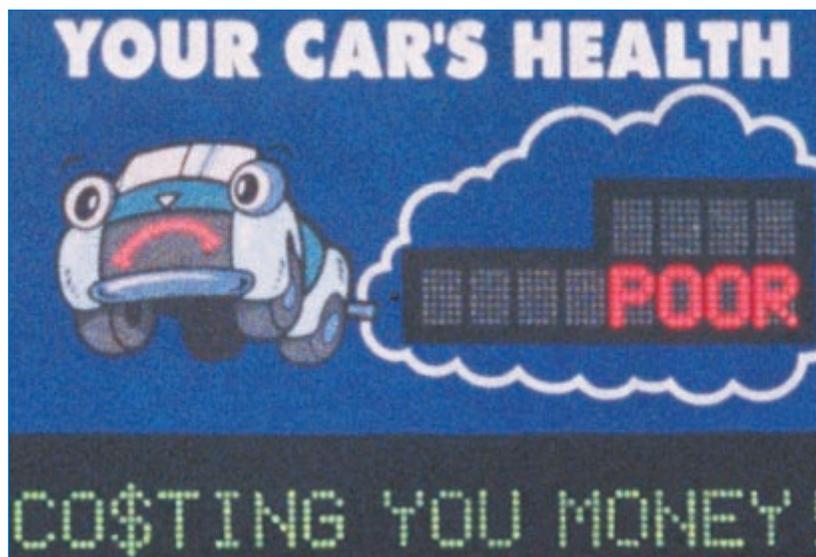
The pollution detector sits on the side of a highway and shines a beam of infrared light across the road. After the beam passes through a car's exhaust fumes, it strikes a rotating mirror on the other side of the highway, which reflects the light onto four different sensors. These sensors detect changes in the infrared beam, and then each sensor uses that information to make different measurements. One detector gauges the amount of carbon dioxide in the exhaust. The second calculates the amount of carbon monoxide. A third sensor measures the amount of hydrocarbons, which contribute to the production of smog.

A car driving down the highway will break the infrared beam, then an exhaust reading is taken after the car passes for half a second to ensure that the beam measures data from the middle of the exhaust fumes.

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A carbon monoxide rating of higher than 4.5% gets a poor rating on the display.

Stedman put the detector into action on a highway exit ramp in Denver. The device gives every car that drives by an emissions rating and automatically displays the rating on a nearby billboard. If less than 1.3% of the car's exhaust is carbon monoxide, it earns a "good" rating. A rating of less than 4.5% carbon monoxide receives a "fair" rating. A rating higher than 4.5% is a "poor" rating. Stedman has found that the billboard not only informs people that their cars are polluters but also motivates the drivers to get their cars fixed.

Stedman has determined that only a small percentage of cars are responsible for automobile pollution. In fact, half of all the

pollution from automobiles is created by about 10% of the cars on the road.

Stedman adds that the drivers will share the economic benefits of cleaning up their act. "If you have a gross-polluting car," he says, "you will save the amount of money that the repair might cost you in your fuel economy in a couple of years because you get a tremendous 10 to 15% fuel-economy improvement by fixing a gross-polluting car."

1. How does Dr. Stedman think his device will benefit society?
2. Why do you feel Dr. Stedman's research is important?

## SECTION 2-3

### OBJECTIVES

- Distinguish between accuracy and precision.
- Determine the number of significant figures in measurements.
- Perform mathematical operations involving significant figures.
- Convert measurements into scientific notation.
- Distinguish between inversely and directly proportional relationships.

# Using Scientific Measurements

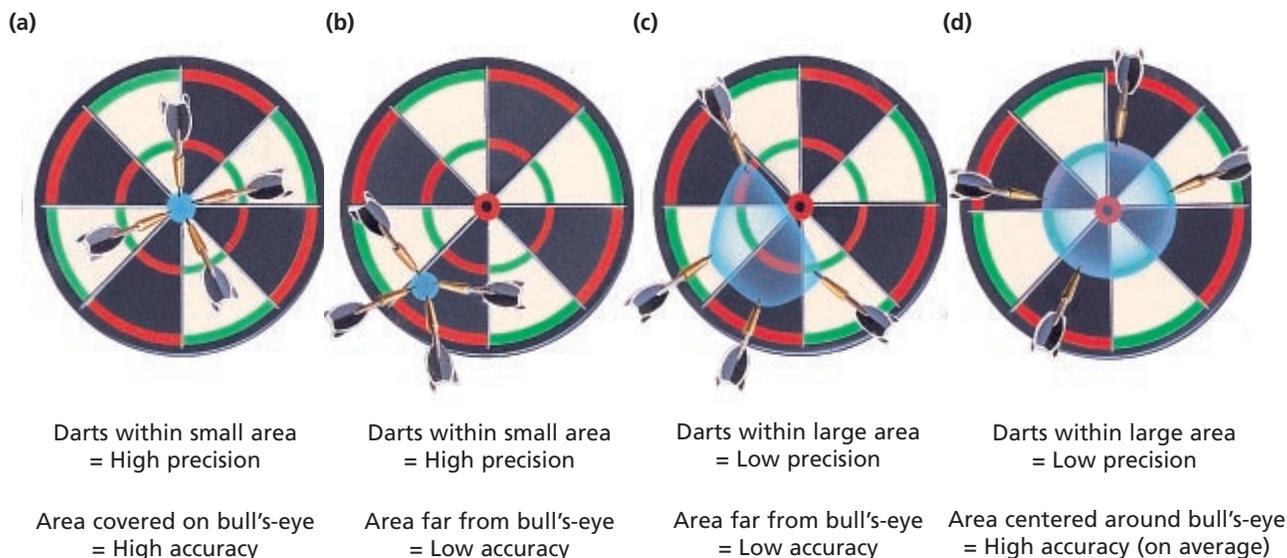
If you have ever measured something several times, you know that the results can vary. In science, for a reported measurement to be useful, there must be some indication of its reliability or uncertainty.

## Accuracy and Precision

The terms *accuracy* and *precision* mean the same thing to most people. However, in science their meanings are quite distinct. **Accuracy** refers to the closeness of measurements to the correct or accepted value of the quantity measured. **Precision** refers to the closeness of a set of measurements of the same quantity made in the same way. Thus, measured values that are accurate are close to the accepted value. Measured values that are precise are close to one another but not necessarily close to the accepted value.

Figure 2-8 should help you visualize the difference between precision and accuracy. A set of darts thrown separately at a dartboard may land in various positions, relative to the bull's-eye and to one another. The

**FIGURE 2-8** The sizes and locations of the areas covered by thrown darts illustrate the difference between precision and accuracy.



closer the darts land to the bull's-eye, the more accurately they were thrown. The closer they land to one another, the more precisely they were thrown. Thus, the set of results shown in Figure 2-8(a) is both accurate and precise because the darts are close to the bull's-eye and close to each other. In Figure 2-8(b), the set of results is inaccurate but precise because the darts are far from the bull's-eye but close to each other. In Figure 2-8(c), the set of results is both inaccurate and imprecise because the darts are far from the bull's-eye and far from each other. Notice also that the darts are not evenly distributed around the bull's-eye, so the set, even considered on average, is inaccurate. In Figure 2-8(d), the set on average is accurate compared with the third case, but it is imprecise. That is because the darts are distributed evenly around the bull's-eye but are far from each other.

## Percent Error

The accuracy of an individual value or of an average experimental value can be compared quantitatively with the correct or accepted value by calculating the percent error. **Percent error** is calculated by subtracting the experimental value from the accepted value, dividing the difference by the accepted value, and then multiplying by 100.

$$\text{Percent error} = \frac{\text{Value}_{\text{accepted}} - \text{Value}_{\text{experimental}}}{\text{Value}_{\text{accepted}}} \times 100$$

Percent error has a positive value if the accepted value is greater than the experimental value. It has a negative value if the accepted value is less than the experimental value. The following sample problem illustrates the concept of percent error.

### SAMPLE PROBLEM 2-3

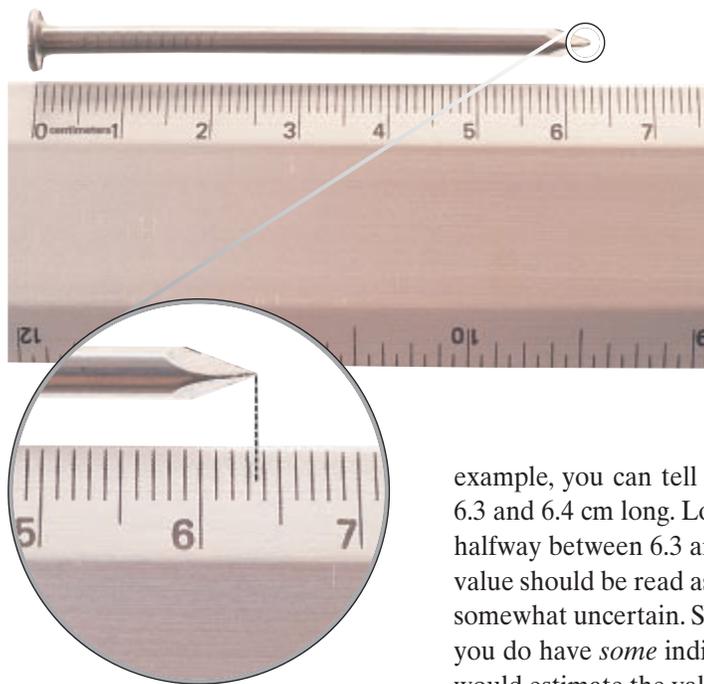
**A student measures the mass and volume of a substance and calculates its density as 1.40 g/mL. The correct, or accepted, value of the density is 1.30 g/mL. What is the percent error of the student's measurement?**

#### SOLUTION

$$\begin{aligned} \text{Percent error} &= \frac{\text{Value}_{\text{accepted}} - \text{Value}_{\text{experimental}}}{\text{Value}_{\text{accepted}}} \times 100 \\ &= \frac{1.30 \text{ g/mL} - 1.40 \text{ g/mL}}{1.30 \text{ g/mL}} \times 100 = -7.7\% \end{aligned}$$

#### PRACTICE

1. What is the percent error for a mass measurement of 17.7 g, given that the correct value is 21.2 g? *Answer* 17%
2. A volume is measured experimentally as 4.26 mL. What is the percent error, given that the correct value is 4.15 mL? *Answer* -2.7%



**FIGURE 2-9** What value should be recorded for the length of this nail?

## Error in Measurement

Some error or uncertainty always exists in any measurement. The skill of the measurer places limits on the reliability of results. The conditions of measurement also affect the outcome. The measuring instruments themselves place limitations on precision. Some balances can be read more precisely than others. The same is true of rulers, graduated cylinders, and other measuring devices.

When you use a properly calibrated measuring device, you can be almost certain of a particular number of digits in a reading.

For example, you can tell that the nail in Figure 2-9 is definitely between 6.3 and 6.4 cm long. Looking more closely, you can see that the value is halfway between 6.3 and 6.4 cm. However, it is hard to tell whether the value should be read as 6.35 cm or 6.36 cm. The hundredths place is thus somewhat uncertain. Simply leaving it out would be misleading because you do have *some* indication of the value's likely range. Therefore, you would estimate the value to the final questionable digit, perhaps reporting the length of the nail as 6.36 cm. You might include a plus-or-minus value to express the range, for example,  $6.36 \text{ cm} \pm 0.01 \text{ cm}$ .

## Significant Figures

In science, measured values are reported in terms of significant figures. **Significant figures** in a measurement consist of all the digits known with certainty plus one final digit, which is somewhat uncertain or is estimated. For example, in the reported nail length of 6.36 cm discussed above, the last digit, 6, is uncertain. All the digits, including the uncertain one, are significant, however. All contain information and are included in the reported value. Thus, the term *significant* does not mean *certain*. In any correctly reported measured value, the final digit is significant but not certain. Insignificant digits are never reported. As a chemistry student, you will need to use and recognize significant figures when you work with measured quantities and report your results, and when you evaluate measurements reported by others.

## Determining the Number of Significant Figures

When you look at a measured quantity, you need to determine which digits are significant. That process is very easy if the number has no zeros because all the digits shown are significant. For example, if you see a number reported as 3.95, all three digits are significant. The significance of zeros in a number depends on their location, however. You need to learn and follow several rules involving zeros. After you have studied the rules in Table 2-5, use them to express the answers in the sample problem that follows.

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**TABLE 2-5 Rules for Determining Significant Zeros**

Rule	Examples
1. Zeros appearing between nonzero digits are significant.	a. 40.7 L has three significant figures. b. 87 009 km has five significant figures.
2. Zeros appearing in front of all nonzero digits are not significant.	a. 0.095 897 m has five significant figures. b. 0.0009 kg has one significant figure.
3. Zeros at the end of a number and to the right of a decimal point are significant.	a. 85.00 g has four significant figures. b. 9.000 000 000 mm has 10 significant figures.
4. Zeros at the end of a number but to the left of a decimal point may or may not be significant. If a zero has not been measured or estimated but is just a placeholder, it is not significant. A decimal point placed after zeros indicates that they are significant.	a. 2000 m may contain from one to four significant figures, depending on how many zeros are placeholders. <b>For measurements given in this text, assume that 2000 m has one significant figure.</b> b. 2000. m contains four significant figures, indicated by the presence of the decimal point.

**SAMPLE PROBLEM 2-4**

How many significant figures are in each of the following measurements?

- a. 28.6 g
- b. 3440. cm
- c. 910 m
- d. 0.046 04 L
- e. 0.006 700 0 kg

**SOLUTION**

Determine the number of significant figures in each measurement using the rules listed in Table 2-5.

- a. 28.6 g  
There are no zeros, so all three digits are significant.
- b. 3440. cm  
By rule 4, the zero is significant because it is immediately followed by a decimal point; there are 4 significant figures.
- c. 910 m  
By rule 4, the zero is not significant; there are 2 significant figures.
- d. 0.046 04 L  
By rule 2, the first two zeros are not significant; by rule 1, the third zero is significant; there are 4 significant figures.
- e. 0.006 700 0 kg  
By rule 2, the first three zeros are not significant; by rule 3, the last three zeros are significant; there are 5 significant figures.

**PRACTICE**

- |  |   |
|--|---|
| <p>1. Determine the number of significant figures in each of the following.</p> <p>a. 804.05 g</p> <p>b. 0.014 403 0 km</p> <p>c. 1002 m</p> <p>d. 400 mL</p> <p>e. 30 000. cm</p> <p>f. 0.000 625 000 kg</p>  | <p><i>Answer</i></p> <p>1. a. 5</p> <p style="padding-left: 20px;">b. 6</p> <p style="padding-left: 20px;">c. 4</p> <p style="padding-left: 20px;">d. 1</p> <p style="padding-left: 20px;">e. 5</p> <p style="padding-left: 20px;">f. 6</p> |
| <p>2. Suppose the value “seven thousand centimeters” is reported to you. How should the number be expressed if it is intended to contain the following?</p> <p>a. 1 significant figure</p> <p>b. 4 significant figures</p> <p>c. 6 significant figures</p> | <p>2. a. 7000 cm</p> <p style="padding-left: 20px;">b. 7000. cm</p> <p style="padding-left: 20px;">c. 7000.00 cm</p>  |

**Rounding**

When you perform calculations involving measurements, you need to know how to handle significant figures. This is especially true when you are using a calculator to carry out mathematical operations. The answers given on a calculator can be derived results with more digits than are justified by the measurements.

Suppose you used a calculator to divide a measured value of 154 g by a measured value of 327 mL. Each of these values has three significant figures. The calculator would show a numerical answer of 0.470948012. The answer contains digits not justified by the measurements used to calculate it. Such an answer has to be rounded off to make its degree of certainty match that in the original measurements. The answer should be 0.471 g/mL.

The rules for rounding are shown in Table 2-6. The extent of rounding required in a given case depends on whether the numbers are being added, subtracted, multiplied, or divided.

**TABLE 2-6 Rules for Rounding Numbers**

If the digit following the last digit to be retained is:	then the last digit should:	Example (rounded to three significant figures)
greater than 5	be increased by 1	42.68 g → 42.7 g
less than 5	stay the same	17.32 m → 17.3 m
5, followed by nonzero digit(s)	be increased by 1	2.7851 cm → 2.79 cm
5, not followed by nonzero digit(s), and preceded by an odd digit	be increased by 1	4.635 kg → 4.64 kg (because 3 is odd)
5, not followed by nonzero digit(s), and the preceding significant digit is even	stay the same	78.65 mL → 78.6 mL (because 6 is even)

## Addition or Subtraction with Significant Figures

Consider two mass measurements, 25.1 g and 2.03 g. The first measurement, 25.1 g, has one digit to the right of the decimal point, in the tenths place. There is no information on possible values for the hundredths place. That place is simply blank and cannot be assumed to be zero. The other measurement, 2.03 g, has two digits to the right of the decimal point. It provides information up to and including the hundredths place.

Suppose you were asked to add the two measurements. Simply carrying out the addition would result in an answer of  $25.1 \text{ g} + 2.03 \text{ g} = 27.13 \text{ g}$ . That answer suggests there is certainty all the way to the hundredths place. However, that result is not justified because the hundredths place in 25.1 g is completely unknown. The answer must be adjusted to reflect the uncertainty in the numbers added.

*When adding or subtracting decimals, the answer must have the same number of digits to the right of the decimal point as there are in the measurement having the fewest digits to the right of the decimal point.* When working with whole numbers, the answer should be rounded so that the final digit is in the same place as the leftmost uncertain digit. Comparing the two values 25.1 g and 2.03 g, the measurement with the fewest digits to the right of the decimal point is 25.1 g. It has only one such digit. Following the rule, the answer must be rounded so that it has no more than one digit to the right of the decimal point. It should therefore be rounded to 27.1 g.

## Multiplication and Division with Significant Figures

Suppose you calculated the density of an object that has a mass of 3.05 g and a volume of 8.47 mL. The following division on a calculator will give a value of 0.360094451.

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{3.05 \text{ g}}{8.47 \text{ mL}} = 0.360094451 \text{ g/mL}$$

The answer must be rounded to the correct number of significant figures. The values of mass and volume used to obtain the answer have only three significant figures each. The degree of certainty in the calculated result is not justified. *For multiplication or division, the answer can have no more significant figures than are in the measurement with the fewest number of significant figures.* In the calculation just described, the answer, 0.360094451 g/mL, would be rounded to three significant figures to match the significant figures in 8.47 mL and 3.05 g. The answer would thus be 0.360 g/mL.

### SAMPLE PROBLEM 2-5

Carry out the following calculations. Express each answer to the correct number of significant figures.

a.  $5.44 \text{ m} - 2.6103 \text{ m}$

b.  $2.4 \text{ g/mL} \times 15.82 \text{ mL}$

**SOLUTION**

Carry out each mathematical operation. Follow the rules in Table 2-5 and Table 2-6 for determining significant figures and for rounding.

- The answer is rounded to 2.83 m (for subtraction there should be two digits to the right of the decimal point, to match 5.44 m).
- The answer is rounded to 38 g (for multiplication there should be two significant figures in the answer, to match 2.4 g/mL).

**PRACTICE**

- What is the sum of 2.099 g and 0.05681 g? *Answer*  
2.156 g
- Calculate the quantity 87.3 cm – 1.655 cm. *Answer*  
85.6 cm
- Calculate the area of a crystal surface that measures 1.34 μm by 0.7488 μm. (Hint: Recall that *area* = *length* × *width* and is measured in square units.) *Answer*  
1.00 μm<sup>2</sup>
- Polycarbonate plastic has a density of 1.2 g/cm<sup>3</sup>. A photo frame is constructed from two 3.0 mm sheets of polycarbonate. Each sheet measures 28 cm by 22 cm. What is the mass of the photo frame? *Answer*  
440 g

### Conversion Factors and Significant Figures

Earlier in this chapter, you learned how conversion factors are used to change one unit to another. Such conversion factors are typically exact. That is, there is no uncertainty in them. For example, there are exactly 100 cm in a meter. If you were to use the conversion factor 100 cm/m to change meters to centimeters, the 100 would not limit the degree of certainty in the answer. Thus, 4.608 m could be converted to centimeters as follows.

$$4.608 \text{ m} \times \frac{100 \text{ cm}}{\text{m}} = 460.8 \text{ cm}$$

The answer still has four significant figures. Because the conversion factor is considered exact, the answer would not be rounded. Most exact conversion factors are defined, rather than measured, quantities. Counted numbers also produce conversion factors of unlimited precision. For example, if you counted that there are 10 test tubes for every student, that would produce an exact conversion factor of 10 test tubes/student. There is no uncertainty in that factor.

### Scientific Notation

In **scientific notation**, numbers are written in the form  $M \times 10^n$ , where the factor  $M$  is a number greater than or equal to 1 but less than 10 and  $n$  is a whole number. For example, to write the quantity 65 000 km in

scientific notation and show the first two digits as significant, you would write the following.

$$6.5 \times 10^4 \text{ km}$$

Writing the  $M$  factor as 6.5 shows that there are exactly two significant figures. If, instead, you intended the first three digits in 65 000 to be significant, you would write  $6.50 \times 10^4$  km. When numbers are written in scientific notation, only the significant figures are shown.

Suppose you are expressing a very small quantity, such as the length of a flu virus. In ordinary notation this length could be 0.000 12 mm. That length can be expressed in scientific notation as follows.

$$0.000\ 12 \text{ mm} = 1.2 \times 10^{-4} \text{ mm}$$

move the decimal point four places to the right and multiply the number by  $10^{-4}$

1. Determine  $M$  by moving the decimal point in the original number to the left or the right so that only one nonzero digit remains to the left of the decimal point.
2. Determine  $n$  by counting the number of places that you moved the decimal point. If you moved it to the left,  $n$  is positive. If you moved it to the right,  $n$  is negative.

## Mathematical Operations Using Scientific Notation

1. *Addition and subtraction* These operations can be performed only if the values have the same exponent ( $n$  factor). If they do not, adjustments must be made to the values so that the exponents are equal. Once the exponents are equal, the  $M$  factors can be added or subtracted. The exponent of the answer can remain the same, or it may then require adjustment if the  $M$  factor of the answer has more than one digit to the left of the decimal point. Consider the example of the addition of  $4.2 \times 10^4$  kg and  $7.9 \times 10^3$  kg.

We can make both exponents either 3 or 4. The following solutions are possible.

$$\begin{array}{r} 4.2 \times 10^4 \text{ kg} \\ +0.79 \times 10^4 \text{ kg} \\ \hline 4.99 \times 10^4 \text{ kg rounded to } 5.0 \times 10^4 \text{ kg} \end{array}$$

or

$$\begin{array}{r} 7.9 \times 10^3 \text{ kg} \\ +42 \times 10^3 \text{ kg} \\ \hline 49.9 \times 10^3 \text{ kg} = 4.99 \times 10^4 \text{ kg rounded to } 5.0 \times 10^4 \text{ kg} \end{array}$$

Note that the units remain kg throughout.

**FIGURE 2-10** When you use a scientific calculator to work problems in scientific notation, don't forget to express the value on the display to the correct number of significant figures and show the units when you write the final answer.

$$5.44 \text{ EE } 7 \div 8.1 \text{ EE } 4 \text{ ENTER}$$

$$671.6049383$$

rounded to  $6.7 \times 10^2 \text{ g/mol}$

$$5.44 \text{ EXP } 7 \div 8.1 \text{ EXP } 4 \text{ =}$$

$$671.6049383$$

rounded to  $6.7 \times 10^2 \text{ g/mol}$



2. **Multiplication** The  $M$  factors are multiplied, and the exponents are added algebraically.

Consider the multiplication of  $5.23 \times 10^6 \mu\text{m}$  by  $7.1 \times 10^{-2} \mu\text{m}$ .

$$\begin{aligned} (5.23 \times 10^6 \mu\text{m})(7.1 \times 10^{-2} \mu\text{m}) &= (5.23 \times 7.1)(10^6 \times 10^{-2}) \\ &= 37.133 \times 10^4 \mu\text{m}^2 \text{ (adjust to two} \\ &\quad \text{significant digits)} \\ &= 3.7 \times 10^5 \mu\text{m}^2 \end{aligned}$$

Note that when length measurements are multiplied, the result is area. The unit is now  $\mu\text{m}^2$ .

3. **Division** The  $M$  factors are divided, and the exponent of the denominator is subtracted from that of the numerator. The calculator keystrokes for this problem are shown in Figure 2-10.

$$\begin{aligned} \frac{5.44 \times 10^7 \text{ g}}{8.1 \times 10^4 \text{ mol}} &= \frac{5.44}{8.1} \times 10^{7-4} \text{ g/mol} \\ &= 0.6716049383 \times 10^3 \text{ (adjust to two significant figures)} \\ &= 6.7 \times 10^2 \text{ g/mol} \end{aligned}$$

Note that the unit for the answer is the ratio of grams to moles.

## Using Sample Problems

Learning to analyze and solve such problems requires practice and a logical approach. In this section, you will review a process that can help you analyze problems effectively. Most Sample Problems in this book are organized by four basic steps to guide your thinking in how to work out the solution to a problem.

## Analyze

*The first step in solving a quantitative word problem is to read the problem carefully at least twice and to analyze the information in it.* Note any important descriptive terms that clarify or add meaning to the problem. Identify and list the data given in the problem. Also identify the unknown—the quantity you are asked to find.

## Plan

*The second step is to develop a plan for solving the problem.* The plan should show how the information given is to be used to find the unknown. In the process, reread the problem to make sure you have gathered all the necessary information. It is often helpful to draw a picture that represents the problem. For example, if you were asked to determine the volume of a crystal given its dimensions, you could draw a representation of the crystal and label the dimensions. This drawing would help you visualize the problem.

Decide which conversion factors, mathematical formulas, or chemical principles you will need to solve the problem. Your plan might suggest a single calculation or a series of them involving different conversion factors. Once you understand how you need to proceed, you may wish to sketch out the route you will take, using arrows to point the way from one stage of the solution to the next. Sometimes you will need data that are not actually part of the problem statement. For instance, you'll often use data from the periodic table.

## Compute

*The third step involves substituting the data and necessary conversion factors into the plan you have developed.* At this stage you calculate the answer, cancel units, and round the result to the correct number of significant figures. It is very important to have a plan worked out in step 2 before you start using the calculator. All too often, students start multiplying or dividing values given in the problem before they really understand what they need to do to get an answer.

## Evaluate

*Examine your answer to determine whether it is reasonable.* Use the following methods, when appropriate, to carry out the evaluation.

1. Check to see that the units are correct. If they are not, look over the setup. Are the conversion factors correct?
2. Make an estimate of the expected answer. Use simpler, rounded numbers to do so. Compare the estimate with your actual result. The two should be similar.
3. Check the order of magnitude in your answer. Does it seem reasonable compared with the values given in the problem? If you calculated the density of vegetable oil and got a value of 54.9 g/mL, you would know that something is wrong. Oil floats on water; therefore, its density is less than water, so the value obtained should be less than 1.0 g/mL.
4. Be sure that the answer given for any problem is expressed using the correct number of significant figures.

Look over the following quantitative Sample Problems. Notice how the four-step approach is used in each, and then apply the approach yourself in solving the practice problems that follow.

### SAMPLE PROBLEM 2-6

**Calculate the volume of a sample of aluminum that has a mass of 3.057 kg. The density of aluminum is 2.70 g/cm<sup>3</sup>.**

#### SOLUTION

##### 1 ANALYZE

**Given:** mass = 3.057 kg, density = 2.70 g/cm<sup>3</sup>  
**Unknown:** volume of aluminum

##### 2 PLAN

The density unit in the problem is g/cm<sup>3</sup>, and the mass given in the problem is expressed in kg. Therefore, in addition to using the density equation, you will need a conversion factor representing the relationship between grams and kilograms.

$$1000 \text{ g} = 1 \text{ kg}$$

Also, rearrange the density equation to solve for volume.

$$\text{density} = \frac{\text{mass}}{\text{volume}} \quad \text{or} \quad D = \frac{m}{V}$$

$$V = \frac{m}{D}$$

##### 3 COMPUTE

$$V = \frac{3.057 \text{ kg}}{2.70 \text{ g/cm}^3} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 1132.222 \dots \text{ cm}^3 \text{ (calculator answer)}$$

The answer should be rounded to three significant figures.

$$V = 1.13 \times 10^3 \text{ cm}^3$$

##### 4 EVALUATE

The unit of volume, cm<sup>3</sup>, is correct. An order-of-magnitude estimate would put the answer at over 1000 cm<sup>3</sup>.

$$\frac{3}{2} \times 1000$$

The correct number of significant digits is three, to match the number of significant figures in 2.70 g/cm<sup>3</sup>.

#### PRACTICE

- |   |   |
|---|---|
| 1. What is the volume of a sample of helium that has a mass of $1.73 \times 10^{-3}$ g, given that the density is 0.178 47 g/L?           | <i>Answer</i><br>9.69 mL                  |
| 2. What is the density of a piece of metal that has a mass of $6.25 \times 10^5$ g and is 92.5 cm × 47.3 cm × 85.4 cm?                    | <i>Answer</i><br>1.67 g/cm <sup>3</sup>   |
| 3. How many millimeters are there in $5.12 \times 10^5$ kilometers?   | <i>Answer</i><br>$5.12 \times 10^{11}$ mm |
| 4. A clock gains 0.020 second per minute. How many seconds will the clock gain in exactly six months, assuming exactly 30 days per month? | <i>Answer</i><br>$5.2 \times 10^3$ s      |

## Direct Proportions

Two quantities are **directly proportional** to each other if dividing one by the other gives a constant value. For example, if the masses and volumes of different samples of aluminum are measured, the masses and volumes will be directly proportional to each other. As the masses of the samples increase, their volumes increase at the same rate, as you can see from the data in Table 2-7. Doubling the mass doubles the volume. Halving the mass halves the volume.

When two variables,  $x$  and  $y$ , are directly proportional to each other, the relationship can be expressed as  $y \propto x$ , which is read as “ $y$  is *proportional* to  $x$ .” The general equation for a directly proportional relationship between the two variables can also be written as follows.

$$\frac{y}{x} = k$$

The value of  $k$  is a constant called the proportionality constant. Written in this form, the equation expresses an important fact about direct proportion: the ratio between the variables remains constant. Note that using the mass and volume values in Table 2-7 gives a mass-volume ratio that is constant (neglecting measurement error). The equation can be rearranged into the following form.

$$y = kx$$

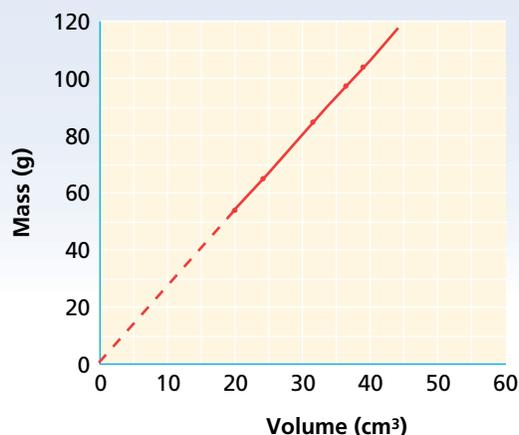
The equation  $y = kx$  may look familiar to you. It is the equation for a special case of a straight line. If two variables related in this way are graphed versus one another, a straight line, or linear plot that passes through the origin (0,0), results. The data for aluminum from Table 2-7 are graphed in Figure 2-11. The mass and volume of a pure substance are directly proportional to each other. Consider mass to be  $y$  and volume to be  $x$ . The constant ratio,  $k$ , for the two variables is density. The slope of the line reflects the constant density, or mass-volume ratio, of

**FIGURE 2-11** The graph of mass versus volume shows a relationship of direct proportion. Notice that the line is extrapolated to pass through the origin.

**TABLE 2-7** Mass-Volume Data for Aluminum at 20°C

Mass (g)	Volume (cm <sup>3</sup> )	$\frac{m}{V}$ (g/cm <sup>3</sup> )
54.4	20.1	2.70
65.7	24.15	2.72
83.5	30.9	2.70
97.2	35.8	2.71
105.7	39.1	2.70

**Mass vs. Volume of Aluminum**



aluminum, which is  $2.70 \text{ g/cm}^3$  at  $20^\circ\text{C}$ . Notice also that the plotted line passes through the origin. All directly proportional relationships produce linear graphs that pass through the origin.

## Inverse Proportions

Two quantities are **inversely proportional** to each other if their product is constant. An example of an inversely proportional relationship is that between speed of travel and the time required to cover a fixed distance. The greater the speed, the less time that is needed to go a certain fixed distance. Doubling the speed cuts the required time in half. Halving the speed doubles the required time.

When two variables,  $x$  and  $y$ , are inversely proportional to each other, the relationship can be expressed as follows.

$$y \propto \frac{1}{x}$$

This is read “ $y$  is *proportional* to 1 divided by  $x$ .” The general equation for an inversely proportional relationship between the two variables can be written in the following form.

$$xy = k$$

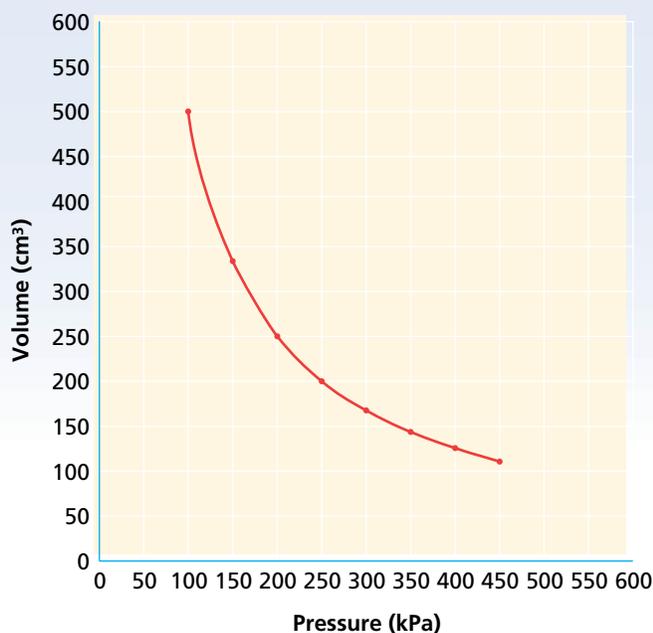
In the equation,  $k$  is the proportionality constant. If  $x$  increases,  $y$  must decrease to keep the product constant.

A graph of variables that are inversely proportional produces a curve called a hyperbola. Such a graph is illustrated in Figure 2-12. When the temperature of the gas is kept constant, the volume ( $V$ ) of the gas sample decreases as the pressure ( $P$ ) increases. Look at the data shown in Table 2-8. Note that  $P \times V$  gives a reasonably constant value. The graph of this data is shown in Figure 2-12.

**TABLE 2-8** Pressure-Volume Data for Nitrogen at Constant Temperature

Pressure (kPa)	Volume (cm <sup>3</sup> )	$P \times V$
100	500	50 000
150	333	49 500
200	250	50 000
250	200	50 000
300	166	49 800
350	143	50 500
400	125	50 000
450	110	49 500

### Volume vs. Pressure of Nitrogen



**FIGURE 2-12** The graph of volume versus pressure shows an inversely proportional relationship. Note the difference between the shape of this graph and that of the graph in Figure 2-11.

## SECTION REVIEW

- The density of copper is listed as  $8.94 \text{ g/cm}^3$ . Two students each make three density determinations of samples of the substance. Student A's results are  $7.3 \text{ g/mL}$ ,  $9.4 \text{ g/mL}$ , and  $8.3 \text{ g/mL}$ . Student B's results are  $8.4 \text{ g/cm}^3$ ,  $8.8 \text{ g/cm}^3$ , and  $8.0 \text{ g/cm}^3$ . Compare the two sets of results in terms of precision and accuracy.
- How many significant figures are there in each of the following measured values?
  - $6.002 \text{ cm}$
  - $0.0020 \text{ m}$
  - $10.0500 \text{ g}$
  - $7000 \text{ kg}$
  - $7000. \text{ kg}$
- Round  $2.6765$  to two significant figures.
- Carry out the following calculations.
  - $52.13 \text{ g} + 1.7502 \text{ g}$
  - $12 \text{ m} \times 6.41 \text{ m}$
  - $$\frac{16.25 \text{ g}}{5.1442 \text{ mL}}$$
- Perform the following operations. Express each answer in scientific notation.
  - $(1.54 \times 10^{-2} \text{ g}) + (2.86 \times 10^{-1} \text{ g})$
  - $(7.023 \times 10^9 \text{ g}) - (6.62 \times 10^7 \text{ g})$
  - $(8.99 \times 10^{-4} \text{ m}) \times (3.57 \times 10^4 \text{ m})$
  - $$\frac{2.17 \times 10^{-3} \text{ g}}{5.022 \times 10^4 \text{ mL}}$$
- Write the following numbers in scientific notation.
  - $560\,000$
  - $33\,400$
  - $0.000\,4120$
- A student measures the mass of a beaker filled with corn oil. The mass reading averages  $215.6 \text{ g}$ . The mass of the beaker is  $110.4 \text{ g}$ .
  - What is the mass of the corn oil?
  - What is the density of the corn oil if its volume is  $114 \text{ cm}^3$ ?
- Calculate the mass of a sample of gold that occupies  $5.0 \times 10^{-3} \text{ cm}^3$ . The density of gold is  $19.3 \text{ g/cm}^3$ .
- What is the difference between a graph representing data that are directly proportional and a graph of data that are inversely proportional?

# CHAPTER 2 REVIEW

## CHAPTER SUMMARY

- 2-1**
- The scientific method is a logical approach to solving problems that lend themselves to investigation.
  - The processes of observing, generalizing, theorizing, and testing are aspects of the scientific method.
  - A hypothesis is a testable statement that serves as the basis for predictions and further experiments.
  - A theory is a broad generalization that explains a body of known facts or phenomena.

### Vocabulary

hypothesis (30)                      scientific method (29)                      system (29)                      theory (31)  
model (31)

- 2-2**
- The result of nearly every measurement is a number and a unit.
  - The SI system of measurement is used in science. It has seven base units: the meter (length), kilogram (mass), second (time), kelvin (temperature), mole (quantity of substance), ampere (electric current), and candela (luminous intensity).
  - Weight is a measure of the gravitational pull on matter.
  - Derived SI units include the square meter (area) and the cubic meter (volume).
  - Density is the ratio of mass to volume.
  - Conversion factors are used to convert from one unit to another.

### Vocabulary

conversion factor (40)                      derived unit (36)                      SI (33)                      weight (35)  
density (38)                      quantity (33)                      volume (37)

- 2-3**
- Accuracy refers to the closeness of a measurement to the correct or accepted value. Precision refers to the closeness of values for a set of measurements.
  - The measurement average is the sum of a group of measurements divided by the total number of measurements.
  - Percent error is the difference between the accepted and the experimental value, divided by the accepted value, then multiplied by 100.
  - The significant figures in a number consist of all digits known with certainty plus one final digit, which is uncertain or estimated. A set of logical rules must be followed to determine the number of significant figures in numbers containing zeros.
  - After addition or subtraction, the answer should be rounded so that it has no more digits to the right of the decimal point than there are in the measurement with the smallest number of digits to the right of the decimal point. After multiplication or division, the answer should be rounded so that it has no more significant figures than there are in the measurement with the fewest number of significant figures.
  - Exact conversion factors are completely certain and do not limit the number of digits in a calculation.
  - A number written in scientific notation is of the form  $M \times 10^n$ , where  $M$  is greater than or equal to 1 but less than 10 and  $n$  is an integer.
  - Two quantities are directly proportional to each other if dividing one by the other gives a constant value. The graphs of variables related in this way are straight lines that pass through the origin.
  - Two quantities are inversely proportional to each other if their product has a constant value. The graphs of variables related in this way are hyperbolas.

### Vocabulary

accuracy (44)                      indirectly proportional (56)                      precision (44)                      significant figures (46)  
directly proportional (55)                      percent error (45)                      scientific notation (50)

## REVIEWING CONCEPTS

- How does quantitative information differ from qualitative information? (2-1)
- What is a hypothesis? (2-1)
- What is a model in the scientific sense?
  - How does a model differ from a theory? (2-1)
- Why is it important for a measurement system to have an international standard? (2-2)
- How does a quantity differ from a unit? Use two examples to explain the difference. (2-2)
- List the seven SI base units and the quantities they represent. (2-2)
- What is the numerical equivalent of each of the following SI prefixes?
 

a. kilo-	d. micro-
b. centi-	e. milli-
c. mega-	

 (2-2)
- Identify the SI unit that would be most appropriate for expressing the length of the following.
  - width of a gymnasium
  - length of a finger
  - distance between your town and the closest border of the next state
  - length of a bacterial cell (2-2)
- Identify the SI unit that would be most appropriate for measuring the mass of each of the following objects.
  - table
  - coin
  - a 250 mL beaker (2-2)
- Explain why the second is not defined by the length of the day. (2-2)
- What is a derived unit?
  - What is the SI derived unit for area? (2-2)
- List two SI derived units for volume.
  - List two non-SI units for volume, and explain how they relate to the cubic centimeter. (2-2)
- Why are the units used to express the densities of gases different from those used to express the densities of solids or liquids?
  - Name two units for density.
  - Why is the temperature at which a density is measured usually specified? (2-2)
- Which of the solids listed in Table 2-4 will float on water?
  - Which of the liquids will sink in milk?
- Define *conversion factor*.
  - Explain how conversion factors are used. (2-2)
- Contrast accuracy and precision. (2-3)
- Write the equation that is used to calculate percent error.
  - Under what condition will percent error be negative? (2-3)
- How is the average for a set of values calculated?
- What is meant by a mass measurement expressed in this form:  $4.6 \text{ g} \pm 0.2 \text{ g}$ ?
- Suppose a graduated cylinder were not correctly calibrated. How would this affect the results of a measurement? How would it affect the results of a calculation using this measurement?
- Round each of the following measurements to the number of significant figures indicated.
  - 67.029 g to three significant figures
  - 0.15 L to one significant figure
  - 52.8005 mg to five significant figures
  - 3.174 97 mol to three significant figures (2-3)
- State the rules governing the number of significant figures that result from each of the following operations.
  - addition and subtraction
  - multiplication and division (2-3)
- What is the general form for writing numbers in scientific notation? (2-3)
- State the general equation for quantities that are directly proportional.
  - For two directly proportional quantities, what happens to one variable when the other increases? (2-3)
- State the general equation for quantities that are inversely proportional.
  - For two inversely proportional quantities, what happens to one variable when the other increases? (2-3)
- Arrange in proper order the following four basic steps in working out the solution to a problem: compute, plan, evaluate, analyze. (2-3)

## PROBLEMS

### Volume and Density

27. What is the volume, in cubic meters, of a rectangular solid that is 0.25 m long, 6.1 m wide, and 4.9 m high?
28. Find the density of a material, given that a 5.03 g sample occupies 3.24 mL. (Hint: See Sample Problem 2-1.)
29. What is the mass of a sample of material that has a volume of 55.1 cm<sup>3</sup> and a density of 6.72 g/cm<sup>3</sup>?
30. A sample of a substance that has a density of 0.824 g/mL has a mass of 0.451 g. Calculate the volume of the sample.

### Conversion Factors

31. How many grams are there in 882 μg? (Hint: See Sample Problem 2-2.)
32. Calculate the number of mL in 0.603 L.
33. The density of gold is 19.3 g/cm<sup>3</sup>.
  - a. What is the volume, in cm<sup>3</sup>, of a sample of gold with mass 0.715 kg?
  - b. If this sample of gold is a cube, how long is each edge in cm?
34. a. Find the number of km in 92.25 m.  
b. Convert the answer in km to cm.

### Percent Error

35. A student measures the mass of a sample as 9.67 g. Calculate the percent error, given that the correct mass is 9.82 g. (Hint: See Sample Problem 2-3.)
36. A handbook gives the density of calcium as 1.54 g/cm<sup>3</sup>. What is the percent error of a density calculation of 1.25 g/cm<sup>3</sup> based on lab measurements?
37. What is the percent error of a length measurement of 0.229 cm if the correct value is 0.225 cm?

### Significant Figures

38. How many significant figures are there in each of the following measurements? (Hint: See Sample Problem 2-4.)
  - a. 0.4004 mL
  - b. 6000 g

- c. 1.000 30 km
- d. 400. mm

39. Calculate the sum of 6.078 g and 0.3329 g.
40. Subtract 7.11 cm from 8.2 cm. (Hint: See Sample Problem 2-5.)
41. What is the product of 0.8102 m and 3.44 m?
42. Divide 94.20 g by 3.167 22 mL.

### Scientific Notation

43. Write the following numbers in scientific notation.
  - a. 0.000 673 0
  - b. 50 000.0
  - c. 0.000 003 010
44. The following numbers are in scientific notation. Write them in ordinary notation.
  - a.  $7.050 \times 10^{-3}$  g
  - b.  $4.000\ 05 \times 10^7$  mg
  - c.  $2.350\ 0 \times 10^4$  mL
45. Perform the following operation. Express the answer in scientific notation and with the correct number of significant figures.
 
$$\frac{6.124\ 33 \times 10^6\ \text{m}^3}{7.15 \times 10^{-3}\ \text{m}}$$
46. A sample of a certain material has a mass of  $2.03 \times 10^{-3}$  g. Calculate the volume of the sample, given that the density is  $9.133 \times 10^{-1}$  g/cm<sup>3</sup>. Use the four-step method in solving the problem. (Hint: See Sample Problem 2-6.)

## MIXED REVIEW

47. A man finds that he has a mass of 100.6 kg. He goes on a diet, and several months later he finds that he has a mass of 96.4 kg. Express each number in scientific notation, and calculate the number of kilograms the man has lost by dieting.
48. A large office building is  $1.07 \times 10^2$  m long, 31 m wide, and  $4.25 \times 10^2$  m high. What is its volume?
49. An object is found to have a mass of 57.6 g. Find the object's density, given that its volume is 40.25 cm<sup>3</sup>.

50. A student measures the mass of some sucrose as 0.947 mg. Convert that quantity to grams and to kilograms.
51. A student calculates the density of iron as  $6.80 \text{ g/cm}^3$  using lab data for mass and volume. A handbook reveals that the correct value is  $7.86 \text{ g/cm}^3$ . What is the percent error?



### HANDBOOK SEARCH

52. Find the table of properties for Group 1 elements in the *Elements Handbook*, pages 726–783. Calculate the volume of a single atom of each element listed in the table using the equation for the volume of a sphere.

$$\frac{4}{3}\pi r^3$$

53. Use the radius of a sodium atom from the *Elements Handbook* to calculate the number of sodium atoms in a row 5.00 cm long. Assume that each sodium atom touches its two neighbors.
54. a. A block of sodium with measurements  $3.00 \text{ cm} \times 5.00 \text{ cm} \times 5.00 \text{ cm}$  has a measured mass of 75.5 g. Calculate the density of sodium.
- b. Compare your calculated density with the value in the properties table for Group 1 elements. Calculate the percent error for your density determination.

### RESEARCH & WRITING

55. Find out how the metric system, which was once a standard for measurement, differs from SI. Why was it necessary to change to SI?
56. Find out what ISO 9000 standards are. How do they affect industry on an international level?

### ALTERNATIVE ASSESSMENT

57. **Performance** Obtain three metal samples from your teacher. Determine the mass and volume of each sample. Calculate the density of each metal from your measurement data. (Hint:

Consider using the water displacement technique to measure the volume of your samples.)

58. Using the data from the Nutrition Facts label below, answer the following.
- Use the data given on the label for grams of fat and Calories from fat to construct a conversion factor with the units Calories per gram.
  - Calculate the mass in kilograms of 20 servings of the food.
  - Calculate the mass of protein in micrograms for one serving of the food.
  - What is the correct number of significant figures for the answer in item a? Why?

### Nutrition Facts

Serving Size  $\frac{3}{4}$  cup (30g)  
Servings Per Container About 14

Amount Per Serving	Corn Crunch	with $\frac{1}{2}$ cup skim milk
<b>Calories</b>	120	160
Calories from Fat	15	20
% Daily Value**		
<b>Total Fat</b> 2g*	<b>3%</b>	<b>3%</b>
Saturated Fat 0g	<b>0%</b>	<b>0%</b>
<b>Cholesterol</b> 0mg	<b>0%</b>	<b>1%</b>
<b>Sodium</b> 160mg	<b>7%</b>	<b>9%</b>
<b>Potassium</b> 65mg	<b>2%</b>	<b>8%</b>
<b>Total Carbohydrate</b> 25g	<b>8%</b>	<b>10%</b>
Dietary Fiber 3g		
Sugars 3g		
Other Carbohydrate 11g		
<b>Protein</b> 2g		

\*Amount in Cereal. A serving of cereal plus skim milk provides 2g fat, less 5mg cholesterol, 220mg sodium, 270mg potassium, 31g carbohydrate (19g sugars) and 6g protein.

\*\*Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs:

	Calories	2,000	2,500
Total Fat	Less than	65g	80g
Sat Fat	Less than	20g	25g
Cholesterol	Less than	300mg	300mg
Sodium	Less than	2,400mg	2,400mg
Potassium		3,500mg	3,500mg
Total Carbohydrate		300g	375g
Dietary Fiber		25g	30g