

Florida's B.E.S.T.



Go Math!™

Reteach Book

Provides Tier 1 Intervention
for Every Lesson

GRADE

4

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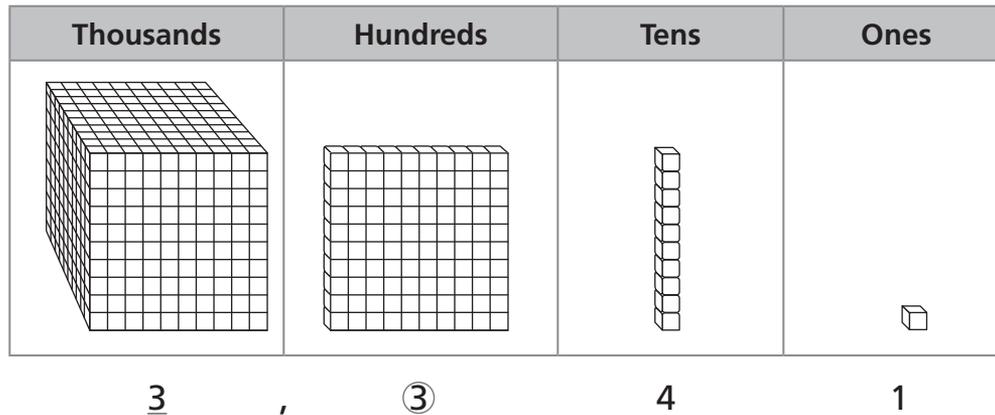
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Place Value and Patterns

Use a place-value chart and base-ten blocks to understand the relationships between digits.

The Atlantic shoreline of Florida is 3,341 miles long. Identify and compare the values of the 3s in that number.



The base-ten blocks show the value of each place.

- A.** Underline the first 3 and circle the second 3.
B. Use the place-value chart to find the values.

The underlined digit 3 has a value of 3 thousands or 3,000.

The circled digit 3 has a value of 3 hundreds or 300.

- C.** Use the base-ten blocks to compare the values.

3 thousands is 10 times the value of 3 hundreds.

Write the value of the underlined digit. Possible answers shown.

1 9,804

4 ones; 4

2 23,614

3 thousands; 3,000

3 26,550

5 tens; 50

4 178,296

1 hundred thousand; 100,000

5 43,829

4 ten thousands; 40,000

6 590,362

3 hundreds; 300

Read and Write Numbers

Look at the digit 6 in the place-value chart below. It is in the hundred thousands place. So, its value is 6 hundred thousands.

In word form, the value of this digit is six hundred thousand.

In standard form, the value of the digit 6 is 600,000.

↓			PERIOD	↓		
THOUSANDS			ONES			
Hundreds	Tens	Ones	Hundreds	Tens	Ones	
6	5	9,	0	5	8	

Read the number shown in the place-value chart.

In word form, this number is written as six hundred fifty-nine thousand, fifty-eight.

You can also write the number in expanded form:

$$600,000 + 50,000 + 9,000 + 50 + 8$$

Note that when writing a number in words, a comma separates periods.

Read and write each number in two other forms.

1 $40,000 + 1,000 + 300 + 70 + 8$

41,378; forty-one thousand, three hundred seventy-eight

2 twenty-one thousand, four hundred

21,400; $20,000 + 1,000 + 400$

3 391,032

three hundred ninety-one thousand,

thirty-two; $300,000 + 90,000 + 1,000 + 30 + 2$

Compare and Order Numbers

Compare 31,072 and 34,318. Write $<$, $>$, or $=$.

Step 1 Align the numbers by place value using grid paper.

Step 2 Compare the digits in each place value. Start at the greatest place.

Are the digits in the ten thousands place the same?

Yes. Move to the thousands place.

Are the digits in the thousands place the same?

No. 1 thousand is less than 4 thousands.

start here	↓			3	1	0	7	2		↓			3	1	0	7	2
				3	4	3	1	8					3	4	3	1	8
				$3 = 3$									$1 < 4$				

Step 3 Use the symbols $<$, $>$, or $=$ to compare the numbers.

$<$ means *is less than*. $>$ means *is greater than*. $=$ means *is equal to*.

There are two ways to write the comparison.

31,072 $<$ 34,318 or 34,318 $>$ 31,072

1 Use the grid paper to compare 21,409 and 20,891.

Write $<$, $>$, or $=$.

21,409 $>$ 20,891

		2	1	4	0	9
		2	0	8	9	1

Compare. Write $<$, $>$, or $=$.

2 \$53,621 $<$ \$53,760

3 82,550 $>$ 80,711

Order from greatest to least.

4 16,451; 16,250; 17,014

5 561,028; 582,073; 549,006

17,014; 16,451; 16,250

582,073; 561,028; 549,006

Round Numbers

When you round a number, you replace it with a number that is easier to work with but not as exact. You can round numbers to different place values.

Round 8,756 to the place value of the underlined digit.

Step 1 Identify the underlined digit.

The underlined digit, 8, is in the thousands place.

Step 2 Look at the number to the right of the underlined digit.

If that number is 0–4, the underlined digit stays the same.

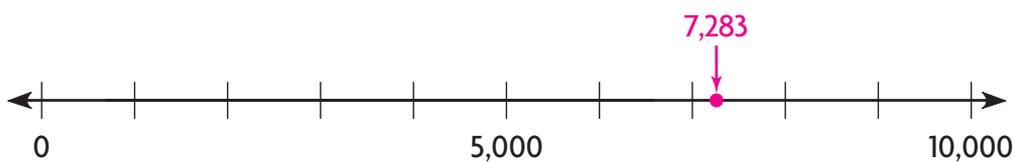
If that number is 5–9, the underlined digit is increased by 1.

The number to the right of the underlined digit is 7, so the underlined digit, 8, will be increased by one; $8 + 1 = \underline{9}$.

Step 3 Change all the digits to the right of the thousands place to zeros.

So, 8,756 rounded to the nearest thousand is 9,000.

- 1** In 2019, the population of Long Boat Key, Florida was 7,283. Use the number line to round this number to the nearest thousand.



7,283 is closer to 7,000 than 8,000,
so it rounds to 7,000.

Round to the place value of the underlined digit.

2 3,452

3,500

3 180

200

4 \$2,471

\$2,000

5 8,600

9,000

6 950

1,000

7 6,495

6,000

8 835

840

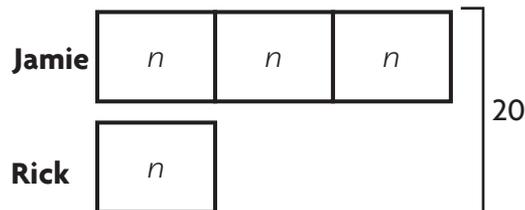
9 6,625

6,600

Comparison Problems

Jamie has 3 times as many baseball cards as Rick. Together, they have 20 baseball cards. How many cards does Jamie have?

Step 1 Draw a box with the letter n in it to show that Rick has an unknown number of cards. Jamie has 3 times as many cards as Rick, so draw three identical boxes to represent Jamie's cards.



Step 2 Use the model to write an equation.

Think: There are 4 equal boxes. The number in each box is represented by n .

There are a total of 20 cards. So, $4 \times n = 20$.

Step 3 Solve the equation to find the value of n .

Think: 4 times what number is 20?

Since $4 \times 5 = 20$, the value of n is 5.

Rick has 5 cards.

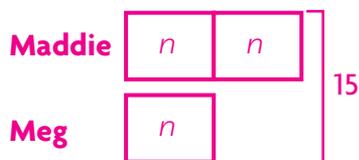
Step 4 Find how many cards Jamie has.

Think: Jamie has 3 times as many cards as Rick.

So, Jamie has $3 \times 5 = 15$ baseball cards.

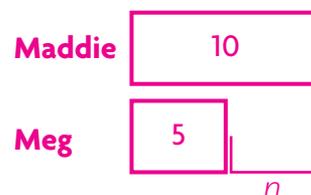
Draw a model. Write an equation and solve.

- 1** Maddie has 2 times as many stickers on her notebook as Meg. Together, they have 15 stickers. How many stickers are on Maddie's notebook?



10 stickers

- 2** How many more stickers are on Maddie's notebook than on Meg's notebook?



5 more stickers

Multiply Tens, Hundreds, and Thousands

You can use a pattern to multiply with tens, hundreds, and thousands.

Count the number of zeros in the factors.

$$4 \times 6 = 24 \quad \leftarrow \text{basic fact}$$

$$4 \times \underline{60} = \underline{240} \quad \leftarrow \text{When you multiply by tens, the last digit in the product is 0.}$$

$$4 \times \underline{600} = \underline{2,400} \quad \leftarrow \text{When you multiply by hundreds, the last two digits in the product are 0.}$$

$$4 \times \underline{6,000} = \underline{24,000} \quad \leftarrow \text{When you multiply by thousands, the last three digits in the product are 0.}$$

When the basic fact has a zero in the product, there will be an extra zero in the final product:

$$5 \times 4 = 20, \text{ so } 5 \times \underline{4,000} = \underline{20,000}$$

Complete the pattern.

1 $9 \times 2 = 18$
 $9 \times 20 = \underline{180}$
 $9 \times 200 = \underline{1,800}$
 $9 \times 2,000 = \underline{18,000}$

2 $8 \times 4 = 32$
 $8 \times 40 = \underline{320}$
 $8 \times 400 = \underline{3,200}$
 $8 \times 4,000 = \underline{32,000}$

3 $6 \times 6 = 36$
 $6 \times 60 = \underline{360}$
 $6 \times 600 = \underline{3,600}$
 $6 \times 6,000 = \underline{36,000}$

4 $4 \times 7 = 28$
 $4 \times 70 = \underline{280}$
 $4 \times 700 = \underline{2,800}$
 $4 \times 7,000 = \underline{28,000}$

Find the product.

5 $7 \times 300 = 7 \times \underline{3}$ hundreds
 $= \underline{21}$ hundreds
 $= \underline{2,100}$

6 $5 \times 8,000 = 5 \times \underline{8}$ thousands
 $= \underline{40}$ thousands
 $= \underline{40,000}$

Estimate Products by 1-Digit Numbers

You can use rounding to estimate products.

Round the greater factor. Then use mental math to estimate the product.

$$6 \times 95$$

Step 1 Round 95 to the nearest ten.

95 rounds to **100**.

Step 2 Use patterns and mental math.

$$6 \times 1 = 6$$

$$6 \times 10 = 60$$

$$6 \times 100 = 600$$

Find two numbers the exact answer is between.

$$7 \times 759$$

Step 1 Estimate by rounding to the lesser hundred.

$$7 \times 759$$

Think: $7 \times 7 = 49$

$$7 \times 70 = 490$$

$$7 \times 700 = 4,900$$

$$7 \times 700 = 4,900$$

Step 2 Estimate by rounding to the greater hundred.

$$7 \times 759$$

Think: $7 \times 8 = 56$

$$7 \times 80 = 560$$

$$7 \times 800 = 5,600$$

$$7 \times 800 = 5,600$$

So, the product is between 4,900 and 5,600.

Estimate the product by rounding. Possible estimates are given.

1 6×316

1,800

2 5×29

150

3 4×703

2,800

Estimate the product by finding two numbers the exact answer is between. Possible estimates are given.

4 3×558

1,500 and

1,800

5 7×252

1,400 and

2,100

6 8×361

2,400 and

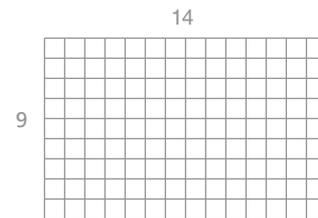
3,200

Multiply Using the Distributive Property

You can use rectangular models to multiply 2-digit numbers by 1-digit numbers.

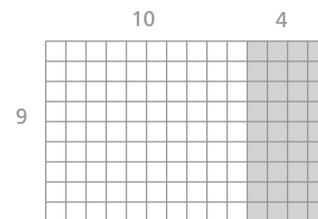
Find 9×14 .

Step 1 Draw a 9 by 14 rectangle on grid paper.



Step 2 Use the Distributive Property and products you know to break apart the model into two smaller rectangles.

Think: $14 = 10 + 4$.



Step 3 Find the product each smaller rectangle represents.

$$9 \times 10 = 90$$

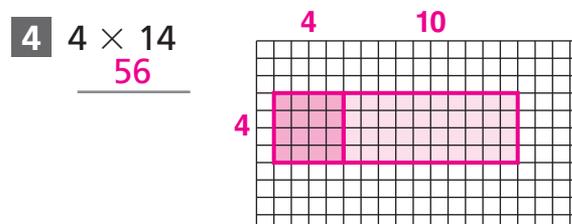
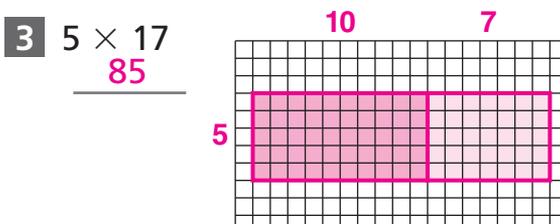
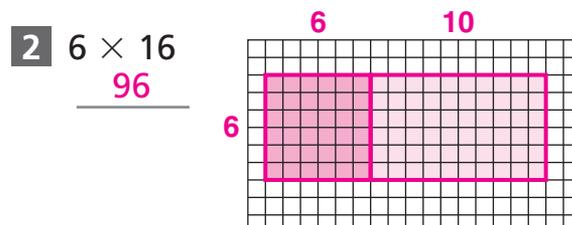
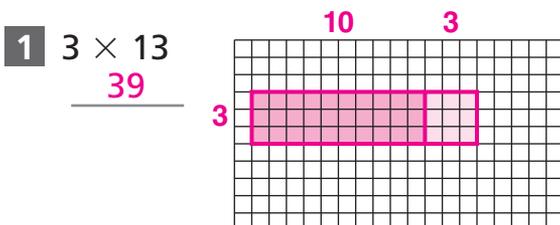
$$9 \times 4 = 36$$

Step 4 Find the sum of the products. $90 + 36 = 126$

So, $9 \times 14 = 126$.

Model the product on the grid. Possible models are shown.

Record the product.



Multiply Using Expanded Form

You can use expanded form or a model to find products.

Multiply. 3×26

Think and Write

Step 1 Write 26 in expanded form.

$$26 = 20 + 6$$

$$3 \times 26 = 3 \times (20 + 6)$$

Step 2 Use the Distributive Property.

$$3 \times 26 = (3 \times 20) + (3 \times 6)$$

Step 3 Multiply the tens. Multiply the ones.

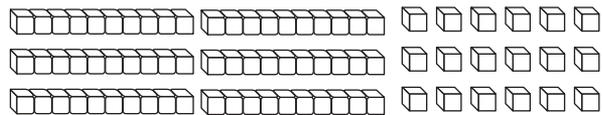
$$\begin{array}{r} 3 \times 26 = (3 \times 20) + (3 \times 6) \\ = \underline{60} + \underline{18} \end{array} \quad \begin{array}{r} 60 \\ +18 \\ \hline 78 \end{array}$$

Step 4 Add the partial products.

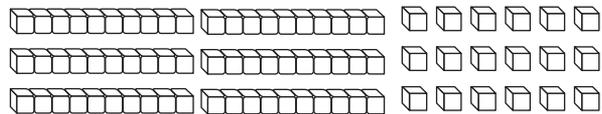
So, $3 \times 26 = \underline{78}$.

Use a Model

Step 1 Show 3 groups of 26.



Step 2 Break the model into tens and ones.



(3 × 2 tens)

(3 × 6 ones)

$$(3 \times 20)$$

$$(3 \times 6)$$

$$\underline{60}$$

$$\underline{18}$$

Step 3 Add to find the total product.

$$\underline{60} + \underline{18} = \underline{78}$$

Record the product. Use expanded form to help.

1 $6 \times 14 = \underline{84}$

$$\underline{6 \times (10 + 4)}$$

$$\underline{(6 \times 10) + (6 \times 4)}$$

	6	0
+	2	4
<hr/>		
	8	4

2 $4 \times 52 = \underline{208}$

$$\underline{4 \times (50 + 2)}$$

$$\underline{(4 \times 50) + (4 \times 2)}$$

	2	0	0
+			8
<hr/>			
	2	0	8

3 $5 \times 162 = \underline{810}$

$$\underline{5 \times (100 + 60 + 2)}$$

$$\underline{(5 \times 100) + (5 \times 60) + (5 \times 2)}$$

	5	0	0
	3	0	0
+		1	0
<hr/>			
	8	1	0

4 $3 \times 279 = \underline{837}$

$$\underline{3 \times (200 + 70 + 9)}$$

$$\underline{(3 \times 200) + (3 \times 70) + (3 \times 9)}$$

	6	0	0
	2	1	0
+		2	7
<hr/>			
	8	3	7

Multiply Using Partial Products

Use partial products to multiply.

Multiply. $7 \times \$332$

Step 1 Estimate the product.

$$332 \text{ rounds to } 300; 7 \times \$300 = \underline{\$2,100}.$$

Step 2 Multiply the 3 hundreds, or 300, by 7.

$$\begin{array}{r} \$332 \\ \times \quad 7 \\ \hline \end{array}$$

or

$$\begin{array}{r} \$300 \\ \times \quad 7 \\ \hline \$2,100 \end{array}$$

Step 3 Multiply the 3 tens, or 30, by 7.

$$\begin{array}{r} \$332 \\ \times \quad 7 \\ \hline \end{array}$$

or

$$\begin{array}{r} \$30 \\ \times \quad 7 \\ \hline \$210 \end{array}$$

Step 4 Multiply the 2 ones, or 2, by 7.

$$\begin{array}{r} \$332 \\ \times \quad 7 \\ \hline \end{array}$$

or

$$\begin{array}{r} \$2 \\ \times \quad 7 \\ \hline \$14 \end{array}$$

Step 5 Add the partial products.

$$\$2,100 + \$210 + \$14 = \underline{\$2,324}$$

So, $7 \times \$332 = \$2,324$. Since $\$2,324$ is close to the estimate of $\$2,100$, it is reasonable.

Estimate. Then record the product.

Possible estimates are given.

1 Estimate: 400

$$\begin{array}{r} 181 \\ \times \quad 2 \\ \hline 362 \end{array}$$

2 Estimate: 800

$$\begin{array}{r} 156 \\ \times \quad 4 \\ \hline 624 \end{array}$$

3 Estimate: \$1,000

$$\begin{array}{r} \$210 \\ \times \quad 5 \\ \hline \$1,050 \end{array}$$

4 Estimate: 1,800

$$\begin{array}{r} 303 \\ \times \quad 6 \\ \hline 1,818 \end{array}$$

5 Estimate: \$800

$$\begin{array}{r} \$427 \\ \times \quad 2 \\ \hline \$854 \end{array}$$

6 Estimate: \$2,000

$$\begin{array}{r} \$367 \\ \times \quad 5 \\ \hline \$1,835 \end{array}$$

Multiply Using Mental Math

Use addition to break apart the larger factor.

Find 8×214 .

Think: $214 = 200 + 14$

$$\begin{aligned} 8 \times 214 &= (8 \times 200) + (8 \times 14) \\ &= \underline{1,600} + \underline{112} \\ &= \underline{1,712} \end{aligned}$$

Use subtraction to break apart the larger factor.

Find 6×298 .

Think: $298 = 300 - 2$

$$\begin{aligned} 6 \times 298 &= (6 \times 300) - (6 \times 2) \\ &= \underline{1,800} - \underline{12} \\ &= \underline{1,788} \end{aligned}$$

Use halving and doubling.

Find 14×50 .

Think: 14 can be evenly divided by 2.

$$\begin{aligned} 14 \div 2 &= \underline{7} \\ 7 \times 50 &= \underline{350} \\ 2 \times 350 &= \underline{700} \end{aligned}$$

When multiplying more than two numbers, use the Commutative Property to change the order of the factors.

Find $2 \times 9 \times 50$.

Think: $2 \times 50 = \underline{100}$

$$\begin{aligned} 2 \times 9 \times 50 &= 2 \times \underline{50} \times 9 \\ &= \underline{100} \times 9 \\ &= \underline{900} \end{aligned}$$

Find the product. Tell which strategy you used.

Possible estimates are given.

1 $5 \times 7 \times 20$

2 6×321

700; Commutative Property

1,926; use addition

3 86×50

4 9×399

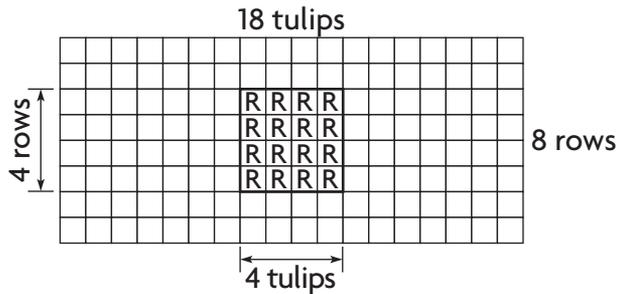
4,300; halving and doubling

3,591; use subtraction

Multi-Step Multiplication Problems

Use the strategy *draw a diagram* to solve a multistep multiplication problem.

Amy planted 8 rows with 18 tulips in each row. In each of the 4 middle rows, there are 4 red tulips. All of the other tulips are yellow. How many of the tulips are yellow tulips?

Read the Problem	Solve the Problem				
<p>What do I need to find?</p> <p>I need to find the total number of <u>yellow</u> tulips.</p>	<p>I drew a diagram for each color of tulip.</p>  <p>Next, I found the number in each section.</p> <table style="width: 100%; text-align: center;"> <tr> <td>All Tulips</td> <td>Red Tulips</td> </tr> <tr> <td>$8 \times 18 = 144$</td> <td>$4 \times 4 = 16$</td> </tr> </table> <p>Last, I subtracted the number of red tulips from the total number of tulips.</p> $\underline{144} - \underline{16} = \underline{128}$ <p>So, there are <u>128</u> yellow tulips.</p>	All Tulips	Red Tulips	$8 \times 18 = 144$	$4 \times 4 = 16$
All Tulips		Red Tulips			
$8 \times 18 = 144$		$4 \times 4 = 16$			
<p>What information do I need to use?</p> <p>There are <u>8</u> rows of tulips with <u>18</u> tulips in each row.</p> <p>There are <u>4</u> rows of tulips with <u>4</u> red tulips in each row.</p>					
<p>How will I use the information?</p> <p>I can <u>multiply</u> to find the total number of tulips and the number of red tulips.</p> <p>Then I can <u>subtract</u> to find the number of yellow tulips.</p>					

- 1** A car dealer has 8 rows of cars with 16 cars in each row. In each of the first 3 rows, 6 are used cars. The rest of the cars are new cars. How many new cars does the dealer have?

110 new cars

- 2** An orchard has 4 rows of apple trees with 12 trees in each row. There are also 6 rows of pear trees with 15 trees in each row. How many apple and pear trees are in the orchard?

138 apple and pear trees

Multiply 3-Digit and 4-Digit Numbers with Regrouping

When you multiply 3-digit and 4-digit numbers, you may need to regroup.

Estimate. Then find the product.

$$\begin{array}{r} \$1,324 \\ \times \quad 7 \\ \hline \end{array}$$

Step 1 Estimate the product.

$$\$1,324 \text{ rounds to } \$1,000; \$1,000 \times 7 = \$7,000.$$

Step 2 Multiply the 4 ones by 7.

Regroup the 28 ones as 2 tens 8 ones.

$$\begin{array}{r} \$1,324 \\ \times \quad 7 \\ \hline 8 \end{array}$$

Step 3 Multiply the 2 tens by 7.

Add the regrouped tens.

Regroup the 16 tens as 1 hundred 6 tens.

$$\begin{array}{r} \$1,324 \\ \times \quad 7 \\ \hline 68 \end{array}$$

Step 4 Multiply the 3 hundreds by 7.

Add the regrouped hundred.

Regroup the 22 hundreds as 2 thousands 2 hundreds.

$$\begin{array}{r} \$1,324 \\ \times \quad 7 \\ \hline 268 \end{array}$$

Step 5 Multiply the 1 thousand by 7.

Add the regrouped thousands.

$$\begin{array}{r} \$1,324 \\ \times \quad 7 \\ \hline \$9,268 \end{array}$$

$$\text{So, } 7 \times \$1,324 = \$9,268.$$

Since \$9,268 is close to the estimate of \$7,000, the answer is reasonable.

Estimate. Then find the product. Possible estimates are given.

1 Estimate: 6,000 **2** Estimate: \$3,200 **3** Estimate: 15,000 **4** Estimate: \$49,000

$$\begin{array}{r} 3,184 \\ \times \quad 2 \\ \hline 6,368 \end{array}$$

$$\begin{array}{r} \$828 \\ \times \quad 4 \\ \hline \$3,312 \end{array}$$

$$\begin{array}{r} 2,637 \\ \times \quad 5 \\ \hline 13,185 \end{array}$$

$$\begin{array}{r} \$6,900 \\ \times \quad 7 \\ \hline \$48,300 \end{array}$$

Solve Multi-Step Problems Using Equations

You can use single-step equations to model and solve a problem with more than one step.

Aliyah buys 4 packages of pens with 18 pens in each package. Kailyn buys 6 packages of pens with 14 pens in each package. How many more pens does Kailyn buy than Aliyah?

- A.** Write multiplication equations to find how many pens each girl buys. Let a = number of pens Aliyah buys. Let k = number of pens Kailyn buys.

Aliyah

$$4 \times 18 = a$$

$$72 = a$$

Kailyn

$$6 \times 14 = k$$

$$72 = k$$

- B.** Write an equation to find how many more pens Kailyn buys. Let p = how many more pens.

$$84 - 72 = p$$

$$12 = p$$

Kailyn buys 12 more pens than Aliyah.

Find the answer. Show your work. Possible work is shown.

- 1** Gabe sells 23 car wash tickets. Each ticket costs \$7. His dad gives him a \$15 donation. How much money does Gabe collect? Let c = amount collected. Let g = ticket sales.

Gabe collects \$176.

$$23 \times 7 = g$$

$$161 = g$$

$$161 + 15 = c$$

$$176 = c$$

- 2** Nicole has 48 stickers. She gives 5 stickers to each of her 6 friends. How many stickers does Nicole have left? Let s = stickers left. Let g = stickers given away.

Nicole has 18 stickers left.

$$5 \times 6 = g$$

$$30 = g$$

$$48 - 30 = s$$

$$18 = s$$

Multiply by Tens

One section of seating at an arena has 30 rows. Each row has 40 seats. How many seats in all are in that section?

Multiply. 30×40

Step 1 Think of each factor as a multiple of 10 and as a repeated addition.

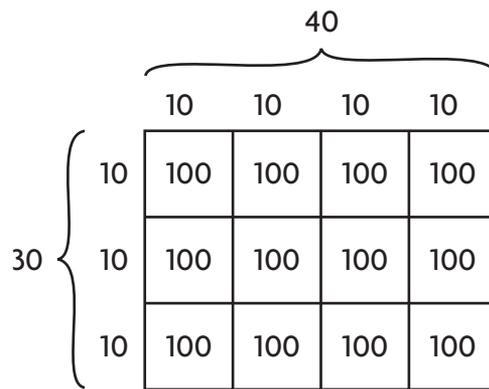
$$30 = \underline{3} \times \underline{10} \text{ or } \underline{10} + \underline{10} + \underline{10}$$

$$40 = \underline{4} \times \underline{10} \text{ or } \underline{10} + \underline{10} + \underline{10} + \underline{10}$$

Step 2 Draw a diagram to show the multiplication.

Step 3 Each small square in the diagram shows 10×10 , or 100. Count the squares.

There are 12 squares of 100.



Step 4 Use patterns and mental math to find 12×100 .

$$12 \times 1 = \underline{12}$$

$$12 \times 10 = \underline{120}$$

$$12 \times 100 = \underline{1,200}$$

There are 1,200 seats in that section.

Choose a method. Then find the product. *Methods will vary.*

1 $20 \times 90 = \underline{1,800}$ **2** $40 \times 40 = \underline{1,600}$ **3** $60 \times 70 = \underline{4,200}$

4 $50 \times 30 = \underline{1,500}$ **5** $80 \times 60 = \underline{4,800}$ **6** $90 \times 40 = \underline{3,600}$

Estimate Products by 2-Digit Numbers

You can use rounding and compatible numbers to estimate products.

Use mental math and rounding to estimate the product.

Estimate. $62 \times \$23$

Step 1 Round each factor to the nearest ten.

62 rounds to **60**.
\$23 rounds to **\$20**.

Step 2 Rewrite the problem using the rounded numbers. $60 \times \$20$

Step 3 Use mental math.

$6 \times \$2 = \12
 $6 \times \$20 = \120
 $60 \times \$20 = \$1,200$

So, $62 \times \$23$ is about \$1,200.

Use mental math and compatible numbers to estimate the product.

Estimate. 24×78

Step 1 Use compatible numbers. 25×80

Step 2 Use $25 \times 4 = 100$ to help find 25×8 .
 $25 \times 8 = 200$

Step 3 Since 80 has 1 zero, write 1 zero to the right of the product.

$$\begin{array}{r} 24 \times 78 \\ \downarrow \quad \downarrow \\ 25 \times 80 = 2,000 \end{array}$$

So, 24×78 is about 2,000.

Possible estimates are given.
Methods will vary.

Estimate the product. Choose a method.

1 78×21

1,600

2 $59 \times \$46$

\$3,000

3 81×33

2,400

4 67×21

1,400

5 $88 \times \$42$

\$3,600

6 51×36

2,000

7 73×73

4,900

8 $99 \times \$44$

\$4,000

9 92×19

1,800

10 26×37

1,000

11 89×18

1,800

12 58×59

3,600

Area Models and Partial Products

You can use area models to multiply 2-digit numbers by 2-digit numbers.

Use the model and partial products to solve.

Draw a rectangle to find 19×18 .

The rectangle is 19 units long and 18 units wide.

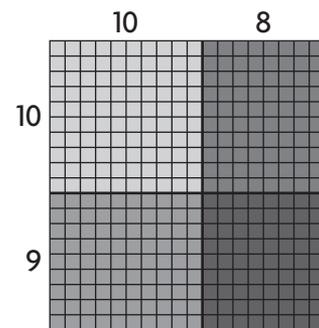
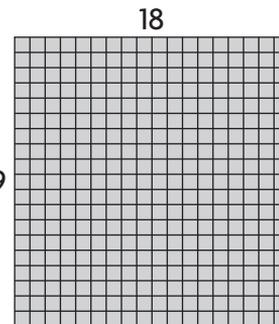
Step 1 Break apart the factors into tens and ones. Divide the area model into four smaller rectangles to show the factors.

Step 2 Find the products for each of the smaller rectangles.

$$10 \times 10 = 100 \quad 10 \times 8 = 80 \quad 9 \times 10 = 90 \quad 9 \times 8 = 72$$

Step 3 Find the sum of the products. $100 + 80 + 90 + 72 = 342$

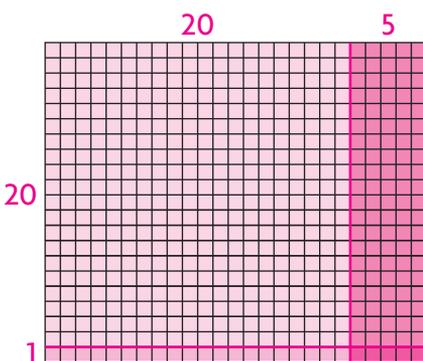
So, $19 \times 18 = 342$.



Models will vary. Possible models are given.

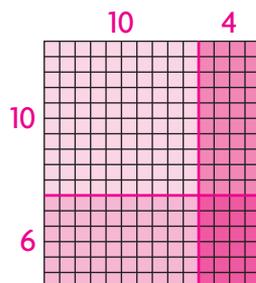
Draw a model to represent the product. Then record the product.

1 21×25



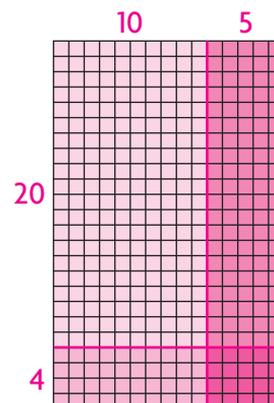
525

2 16×14



224

3 24×15



360

Multiply Using Partial Products

Multiply 25×43 . Record the product.

tens ones

Think: I can use partial products to find 25×43 .

Step 1 Multiply the tens by the tens.

$$20 \times 4 \text{ tens} = 80 \text{ tens, or } 800. \longrightarrow$$

$$\begin{array}{r} 43 \\ \times 25 \\ \hline 800 \end{array}$$

Step 2 Multiply the ones by the tens.

$$20 \times 3 \text{ ones} = 60 \text{ ones, or } 60. \longrightarrow$$

$$60$$

Step 3 Multiply the tens by the ones.

$$5 \times 4 \text{ tens} = 20 \text{ tens, or } 200. \longrightarrow$$

$$200$$

Step 4 Multiply the ones by the ones.

$$5 \times 3 \text{ ones} = 15 \text{ ones, or } 15. \longrightarrow$$

$$+ 15$$

Step 5 Add the partial products.

$$800 + 60 + 200 + 15 = 1,075. \longrightarrow$$

$$\begin{array}{r} 800 \\ 60 \\ 200 \\ + 15 \\ \hline 1,075 \end{array}$$

So, $25 \times 43 = \underline{1,075}$.

Record the product.

1

$$\begin{array}{r} 25 \\ \times 62 \\ \hline 1,200 \\ 300 \\ 40 \\ + 10 \\ \hline 1,550 \end{array}$$

2

$$\begin{array}{r} 59 \\ \times 38 \\ \hline 1,500 \\ 270 \\ 400 \\ + 72 \\ \hline 2,242 \end{array}$$

3

$$\begin{array}{r} 85 \\ \times 72 \\ \hline 5,600 \\ 350 \\ 160 \\ + 10 \\ \hline 6,120 \end{array}$$

4

$$\begin{array}{r} 46 \\ \times 52 \\ \hline 2,000 \\ 300 \\ 80 \\ + 12 \\ \hline 2,392 \end{array}$$

5

$$\begin{array}{r} 76 \\ \times 23 \\ \hline 1,400 \\ 120 \\ 210 \\ + 18 \\ \hline 1,748 \end{array}$$

6

$$\begin{array}{r} 38 \\ \times 95 \\ \hline 2,700 \\ 720 \\ 150 \\ + 40 \\ \hline 3,610 \end{array}$$

Multiply with Regrouping

Estimate. Then use regrouping to find 28×43 .

Step 1 Round to estimate the product. $30 \times 40 = 1,200$

Step 2 Think: $28 = 2$ tens 8 ones.
Multiply 43 by 8 ones.
 $8 \times 3 = 24$. Record the 4. Write
the regrouped 2 above the tens
place.

$$\begin{array}{r} \cancel{2} \\ 43 \\ \times 28 \\ \hline 344 \end{array} \longleftarrow 8 \times 43$$

Step 3 Multiply 43 by 2 tens.
 $20 \times 3 = 60$ and $20 \times 40 = 800$.
Record 860 below 344.

$$\begin{array}{r} \cancel{2} \\ 43 \\ \times 28 \\ \hline 344 \\ 860 \\ \hline 1,204 \end{array} \begin{array}{l} \longleftarrow 20 \times 43 \\ \longleftarrow 344 + 860 \end{array}$$

Step 4 Add the partial products.

So, $28 \times 43 = \underline{1,204}$. 1,204 is close to 1,200. The answer is **reasonable**.

Estimate. Then find the product.

Possible estimates are given.

1 Estimate: 400

2 Estimate: 1,200

3 Estimate: 2,500

$$\begin{array}{r} 36 \\ \times 12 \\ \hline 432 \end{array}$$

$$\begin{array}{r} 43 \\ \times 29 \\ \hline 1,247 \end{array}$$

$$\begin{array}{r} 51 \\ \times 47 \\ \hline 2,397 \end{array}$$

Choose a Multiplication Method

Estimate. Then use regrouping to find 47×89 .

$$\begin{array}{r} 89 \\ \times 47 \\ \hline \end{array}$$

Step 1 Estimate the product.

$$50 \times 90 = 4,500$$

Step 2 Multiply the 9 ones by the 7 ones. Regroup the 63 ones as 6 tens 3 ones.

$$\begin{array}{r} 6 \\ 89 \\ \times 47 \\ \hline 3 \end{array}$$

Step 3 Multiply the 8 tens, or 80, by the 7 ones, or 7. Add the regrouped tens. Regroup the 62 tens as 6 hundreds 2 tens.

$$\begin{array}{r} 6 \\ 89 \\ \times 47 \\ \hline 623 \end{array}$$

Step 4 Multiply the 9 ones by the 4 tens, or 40. Regroup the 36 tens as 3 hundreds 6 tens.

$$\begin{array}{r} 3 \\ \cancel{8}9 \\ \times 47 \\ \hline 623 \end{array}$$

Step 5 Multiply the 8 tens, or 80, by the 4 tens, or 40. Add the regrouped tens. Regroup the 35 hundreds as 3 thousands 5 hundreds.

$$\begin{array}{r} 60 \\ 3 \\ \cancel{8}9 \\ \times 47 \\ \hline 623 \\ 3,560 \end{array}$$

Step 6 Add the partial products.

$$\begin{array}{r} 3 \\ \cancel{8}9 \\ \times 47 \\ \hline 623 \\ + 3,560 \\ \hline 4,183 \end{array}$$

So, $47 \times 89 = 4,183$. Since 4,183 is close to the estimate of 4,500, it is reasonable.

Estimate. Then choose a method to find the product. Possible estimates are given.

1 Estimate: 2,400 **2** Estimate: 800 **3** Estimate: 300 **4** Estimate: 3,600

$$\begin{array}{r} 76 \\ \times 31 \\ \hline 2,356 \end{array}$$

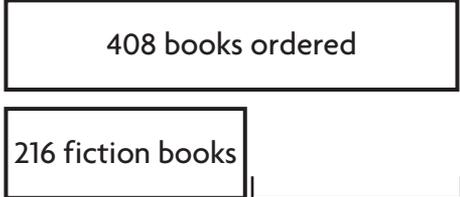
$$\begin{array}{r} 24 \\ \times 35 \\ \hline 840 \end{array}$$

$$\begin{array}{r} 14 \\ \times 28 \\ \hline 392 \end{array}$$

$$\begin{array}{r} 64 \\ \times 56 \\ \hline 3,584 \end{array}$$

Multiply by 2-Digit Numbers

A library ordered 17 cases with 24 books in each case. In 12 of the cases, 18 books were fiction books. The rest of the books were nonfiction. How many nonfiction books did the library order?

Read the Problem	Solve the Problem
<p>What do I need to find?</p> <p>I need to find <u>how many nonfiction books</u> were ordered.</p>	<ul style="list-style-type: none"> • First, find the total number of books ordered. $17 \times 24 = 408$ books ordered
<p>What information do I need to use?</p> <p><u>17</u> cases of <u>24</u> books each were ordered.</p> <p>In <u>12</u> cases, <u>18</u> books were fiction books.</p>	<ul style="list-style-type: none"> • Next, find the number of fiction books. $12 \times 18 = 216$ fiction books • Last, draw a bar model. I need to subtract. <div style="text-align: center;">  </div>
<p>How will I use the information?</p> <p>I can find the <u>total number of books ordered</u> and the <u>number of fiction books ordered</u>.</p> <p>Then I can draw a bar model to compare the <u>total number of books</u> to the <u>number of fiction books</u>.</p>	<p>$408 - 216 = 192$</p> <p>So, the library ordered <u>192</u> nonfiction books.</p>

1 A grocer ordered 32 cases with 28 small cans of fruit in each case. The grocer also ordered 24 cases with 18 large cans of fruit in each case. How many more small cans of fruit did the grocer order?

464 more small cans of fruit

2 Rebecca rode her bike 16 miles each day for 30 days. Michael rode his bike 25 miles for 28 days. Who rode farther? How much farther?

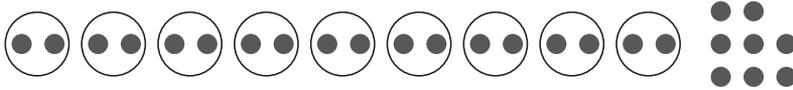
Michael rode 220 miles farther.

Investigate Remainders

Use counters to find the quotient and remainder.

$$9 \overline{)26}$$

- Use 26 counters to represent the dividend, 26.
- Since you are dividing 26 by 9, draw 9 circles. Divide the 26 counters into 9 equal-sized groups.



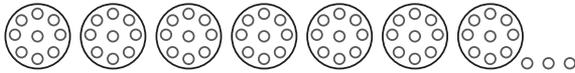
- There are 2 counters in each circle, so the quotient is 2. There are 8 counters left over, so the remainder is 8.

$$\begin{array}{r} 2 \text{ r}8 \\ 9 \overline{)26} \end{array}$$

Divide. Draw a quick picture to help.

$$7 \overline{)66}$$

- Use 66 counters to represent the dividend, 66.
- Since you are dividing 66 by 7, draw 7 circles. Divide 66 counters into 7 equal-sized groups.

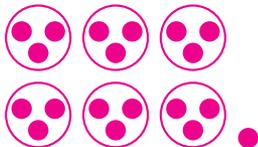


- There are 9 counters in each circle, so the quotient is 9. There are 3 counters left over, so the remainder is 3.

$$\begin{array}{r} 9 \text{ r}3 \\ 7 \overline{)66} \end{array}$$

Use counters to find the quotient and remainder.

$$1 \quad \begin{array}{r} 3 \text{ r}1 \\ 6 \overline{)19} \end{array}$$



$$2 \quad \begin{array}{r} 4 \text{ r}2 \\ 3 \overline{)14} \end{array}$$

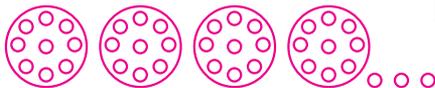


Check students' pictures.

Divide. Draw a quick picture to help.

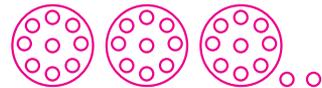
$$3 \quad 39 \div 4$$

$$\underline{\quad} \quad \begin{array}{r} 9 \text{ r}3 \end{array}$$



$$4 \quad 29 \div 3$$

$$\underline{\quad} \quad \begin{array}{r} 9 \text{ r}2 \end{array}$$



Interpret Remainders

When you solve a division problem with a remainder, the way you interpret the remainder depends on the situation and the question.

Way 1: Write the remainder as a fraction.

Callie has a board that is 60 inches long. She wants to cut 8 shelves of equal length from the board and use the entire board. How long will each shelf be?

Divide. $60 \div 8$ 7 r4

The remainder, 4 inches, can be divided into 8 equal parts.

$\frac{4}{8}$ ← remainder
 ← divisor

Write the remainder as a fraction.

Each shelf will be $7\frac{4}{8}$ inches long.

Way 2: Use only the quotient.

Callie has 60 beads. She wants to make 8 identical bracelets and use as many beads as possible on each bracelet. How many beads will be on each bracelet?

Divide. $60 \div 8$ 7 r4

The remainder is the number of beads left over. Those beads will not be used. Drop the remainder.

Callie will use 7 beads on each bracelet.

Way 3: Add 1 to the quotient.

Callie has 60 beads. She wants to put 8 beads in each container. How many containers will she need?

Divide. $60 \div 8$ 7 r4

The answer shows that Callie can fill 7 containers but will have 4 beads left over. She will need 1 more container for the 4 leftover beads. Add 1 to the quotient.

Callie will need 8 containers.

Way 4: Use only the remainder.

Callie has 60 stickers. She wants to give an equal number of stickers to 8 friends. She will give the leftover stickers to her sister. How many stickers will Callie give to her sister?

Divide. $60 \div 8$ 7 r4

The remainder is the number of stickers left over. Use the remainder as the answer.

Callie will give her sister 4 stickers.

- 1** There are 35 students going to the zoo. Each van can hold 6 students. How many vans are needed?

6 vans

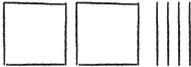
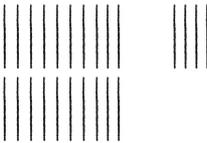
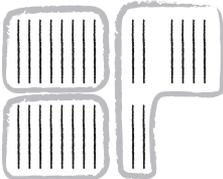
- 2** Sue has 55 inches of ribbon. She wants to cut the ribbon into 6 equal pieces. How long will each piece be?

$9\frac{1}{6}$ inches

Divide Tens, Hundreds, and Thousands

You can use base-ten blocks, place value, and basic facts to divide.

Divide. $240 \div 3$

Use base-ten blocks.	Use place value.
Step 1 Draw a quick picture to show 240. 	Step 1 Identify the basic fact to use. Use $24 \div 3$.
Step 2 You cannot divide 2 hundreds into 3 equal groups. Rename 2 hundreds as tens.  $240 = \underline{24}$ tens	Step 2 Use place value to rewrite 240 as tens. $240 = \underline{24}$ tens
Step 3 Separate the tens into 3 equal groups to divide.  There are 3 groups of $\underline{8}$ tens. Write the answer. $240 \div 3 = \underline{80}$	Step 3 Divide. $24 \text{ tens} \div 3 = \underline{8}$ tens $\quad \quad \quad = \underline{80}$ Write the answer. $240 \div 3 = \underline{80}$

Use basic facts and place value to find the quotient.

1 $280 \div 4$

What division fact can you use?

$\underline{28 \div 4 = 7}$

$280 = \underline{28}$ tens

$28 \text{ tens} \div 4 = \underline{7}$ tens

$280 \div 4 = \underline{70}$

2 $1,800 \div 9$

What division fact can you use?

$\underline{18 \div 9 = 2}$

$1,800 = 5 \underline{18}$ hundreds

$18 \text{ hundreds} \div 9 = \underline{2}$ hundreds

$1,800 \div 9 = \underline{200}$

3 $560 \div 7 = \underline{80}$

4 $180 \div 6 = \underline{30}$

5 $1,500 \div 5 = \underline{300}$

6 $3,200 \div 4 = \underline{800}$

Estimate Quotients Using Compatible Numbers

Compatible numbers are numbers that are easy to compute mentally. In division, one compatible number divides evenly into the other. Think of the multiples of a number to help you find compatible numbers.

Estimate. $6 \overline{)216}$

Step 1 Think of these multiples of 6:

6 12 18 24 30 36 42 48 54

Find multiples that are close to the first 2 digits of the dividend.

18 tens and 24 tens are both close to 21 tens. You can use either or both numbers to estimate the quotient.

Step 2 Estimate using compatible numbers.

$$216 \div 6$$



$$180 \div 6 = 30$$

$$216 \div 6$$



$$240 \div 6 = 40$$

So, $216 \div 6$ is between 30 and 40.

Step 3 Decide whether the estimate is closer to 30 or 40.

$$216 - 180 = 36 \qquad 240 - 216 = 24$$

216 is closer to 240, so use 40 as the estimate.

Use compatible numbers to estimate the quotient. Possible estimates are given.

1 $3 \overline{)252}$

80

2 $6 \overline{)546}$

90

3 $4 \overline{)2,545}$

600

4 $5 \overline{)314}$

60

5 $2 \overline{)1,578}$

800

6 $8 \overline{)289}$

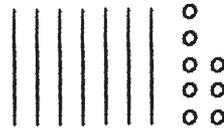
40

Division and the Distributive Property

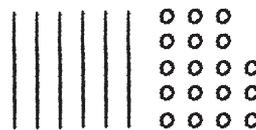
Divide. $78 \div 6$

Use the Distributive Property and quick pictures to break apart numbers to make them easier to divide.

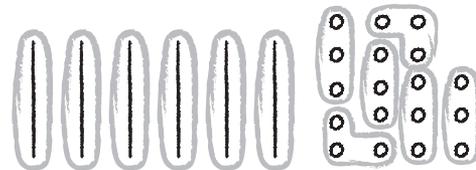
Step 1 Draw a quick picture to show 78.



Step 2 Think about how to break apart 78. You know $6 \text{ tens} \div 6 = 10$, so use $78 = 60 + 18$. Draw a quick picture to show 6 tens and 18 ones.



Step 3 Draw circles to show $6 \text{ tens} \div 6$ and $18 \text{ ones} \div 6$. Your drawing shows the use of the Distributive Property.
 $78 \div 6 = (60 \div 6) + (18 \div 6)$



Step 4 Add the quotients to find $78 \div 6$.

$$\begin{aligned} 78 \div 6 &= (60 \div 6) + (18 \div 6) \\ &= \underline{10} + \underline{3} \\ &= \underline{13} \end{aligned}$$

Use quick pictures to model the quotient.

Check students' pictures.

1 $84 \div 4 = \underline{21}$

2 $54 \div 3 = \underline{18}$

3 $68 \div 2 = \underline{34}$

4 $65 \div 5 = \underline{13}$

5 $96 \div 8 = \underline{12}$

6 $90 \div 6 = \underline{15}$

Divide Using Repeated Subtraction

You can use repeated subtraction to divide. Use repeated subtraction to solve the problem.

Nestor has 27 shells to make bracelets. He needs 4 shells for each bracelet. How many bracelets can he make?

Divide. $27 \div 4$

Write $4 \overline{)27}$.

Step 1

Subtract the divisor until the remainder is less than the divisor. Record a 1 each time you subtract.

$$\begin{array}{r}
 4 \overline{)27} \\
 \underline{-4} \quad 1 \\
 23 \\
 \underline{-4} \quad 1 \\
 19 \\
 \underline{-4} \quad 1 \\
 15 \\
 \underline{-4} \quad 1 \\
 11 \\
 \underline{-4} \quad 1 \\
 7 \\
 \underline{-4} \quad 1 \\
 3
 \end{array}$$

Step 2

Count the number of times you subtracted the divisor, 4.

4 is subtracted six times with 3 left.

$$\begin{array}{r}
 27 \div 4 \\
 \underline{6 \text{ r}3}
 \end{array}$$

So, Nestor can make 6 bracelets. He will have 3 shells left.

Use repeated subtraction to divide.

1 $30 \div 4$

7 r2

2 $24 \div 5$

4 r4

3 $47 \div 7$

6 r5

Divide Using Partial Quotients

You can use partial quotients to divide.

Divide. $492 \div 4$

Step 1 Subtract greater multiples of the divisor. Repeat if needed.

Step 2 Subtract lesser multiples of the divisor. Repeat until the remaining number is less than the divisor.

Step 3 Add the partial quotients.

Use rectangular models to record partial quotients.

$\overline{)492}$	Partial quotients	
$\underline{-400}$	↓	↓
92	100×4	100
$\underline{-80}$	20×4	20
12	3×4	$+ 3$
$\underline{-12}$		$\underline{+ 3}$
0		123

4	100	400	80	12	$\begin{array}{r} 492 \\ - 400 \\ \hline 92 \end{array}$
4	100	400	80	12	$\begin{array}{r} 92 \\ - 80 \\ \hline 12 \end{array}$
4	100	400	80	12	$\begin{array}{r} 12 \\ - 12 \\ \hline 0 \end{array}$

$$\underline{100} + \underline{20} + \underline{3} = \underline{123}$$

Divide. Use partial quotients.

$\begin{array}{r} 219 \\ 3 \overline{)657} \\ \underline{-300} \\ 357 \\ \underline{-300} \\ 57 \\ \underline{-30} \\ 27 \\ \underline{-27} \\ 0 \end{array}$	$100 \times \underline{3}$	100
$\underline{100}$	$100 \times \underline{3}$	$\underline{100}$
$\underline{10}$	$\underline{10} \times \underline{3}$	$\underline{10}$
$\underline{9}$	$\underline{9} \times \underline{3}$	$+ \underline{9}$
		219

Divide. Use rectangular models to record the partial quotients.

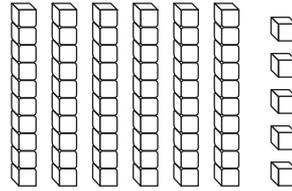
2	$852 \div 6 = \underline{142}$					
6	100	600	240	12	$\begin{array}{r} 852 \\ - 600 \\ \hline 252 \end{array}$	
6	100	600	40	240	12	$\begin{array}{r} 252 \\ - 240 \\ \hline 12 \end{array}$
6	100	600	40	2	12	$\begin{array}{r} 12 \\ - 12 \\ \hline 0 \end{array}$
		$100 + 40 + 2 = 142$				

Model Division with Regrouping

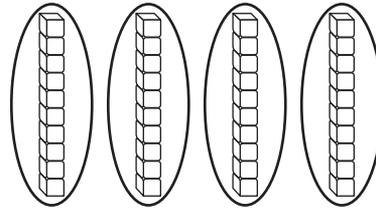
You can use base-ten blocks to model division with regrouping.

Use base-ten blocks to find the quotient $65 \div 4$.

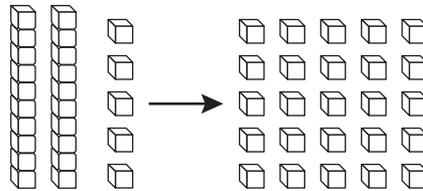
Step 1 Show 65 with base-ten blocks.



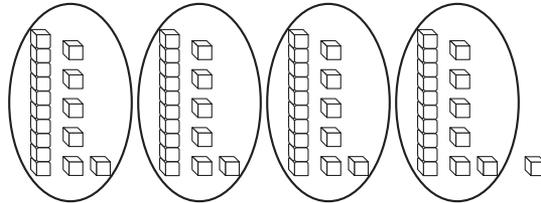
Step 2 Draw 4 circles to represent dividing 65 into 4 equal groups. Share the tens equally among the 4 groups.



Step 3 Regroup leftover tens as ones.



Step 4 Share the ones equally among the 4 groups.



There are 1 ten(s) and 6 one(s) in each group with 1 left over.

So, the quotient is 16 r1.

Divide. Use base-ten blocks.

1 $37 \div 2$

18 r1

2 $74 \div 3$

24 r2

3 $66 \div 5$

13 r1

Place the First Digit

Divide. $763 \div 3 = \blacksquare$

Step 1 Estimate. Then divide the hundreds.

Think: 3×1 hundred = 3 hundreds

$$3 \times 2 \text{ hundreds} = 6 \text{ hundreds}$$

$$3 \times 3 \text{ hundreds} = 9 \text{ hundreds}$$

3×3 hundreds is greater than 7 hundreds.

Use 2 hundreds as an estimate.

Step 2 There is 1 hundred left over. Regroup 1 hundred, now there are 16 tens. Divide the tens.

$$\begin{array}{r} 2 \\ 3 \overline{)763} \\ - 6 \downarrow \\ \hline 16 \end{array} \leftarrow 16 \text{ tens}$$

Step 3 There is 1 ten left over. Regroup 1 ten, now there are 13 ones. Divide the ones.

$$\begin{array}{r} 25 \\ 3 \overline{)763} \\ - 6 \downarrow \\ \hline 16 \\ - 15 \downarrow \\ \hline 13 \end{array} \leftarrow 13 \text{ ones}$$

Step 4 Check to make sure that the remainder is less than the divisor. Write the answer.

$$\begin{array}{r} 2 \\ 3 \overline{)763} \\ - 6 \\ \hline 1 \end{array} \begin{array}{l} \leftarrow \text{Divide 7 hundreds by 3.} \\ \leftarrow \text{Multiply. } 3 \times 2 \text{ hundreds} \\ \leftarrow \text{Subtract.} \end{array}$$

$$\begin{array}{r} 25 \\ 3 \overline{)763} \\ - 6 \\ \hline 16 \\ - 15 \\ \hline 1 \end{array} \begin{array}{l} \leftarrow \text{Divide 16 tens by 3.} \\ \leftarrow \text{Multiply. } 3 \times 5 \text{ tens} \\ \leftarrow \text{Subtract.} \end{array}$$

$$\begin{array}{r} 254 \\ 3 \overline{)763} \\ - 6 \\ \hline 16 \\ - 15 \\ \hline 13 \\ - 12 \\ \hline 1 \end{array} \begin{array}{l} \leftarrow \text{Divide 13 ones by 3.} \\ \leftarrow \text{Multiply. } 3 \times 4 \text{ ones} \\ \leftarrow \text{Subtract.} \end{array}$$

$$\begin{array}{r} 254 \text{ r}1 \\ 3 \overline{)763} \end{array} \quad 1 < 3$$

Divide.

1 $\begin{array}{r} 265 \text{ r}1 \\ 2 \overline{)531} \end{array}$

2 $\begin{array}{r} 157 \\ 4 \overline{)628} \end{array}$

3 $\begin{array}{r} 38 \text{ r}7 \\ 9 \overline{)349} \end{array}$

4 $\begin{array}{r} 113 \text{ r}3 \\ 7 \overline{)794} \end{array}$

Divide by 1-Digit Numbers

Divide. $766 \div 6 = \blacksquare$

Step 1 Use place value to place the first digit.
Think: 7 hundreds can be shared among 6 groups without regrouping.

$$\begin{array}{r} 1 \\ 6 \overline{)766} \end{array} \leftarrow \text{The first digit is in the hundreds place.}$$

Step 2 There is 1 hundred left over. Regroup 1 hundred, now there are 16 tens. Divide the tens.

$$\begin{array}{r} 1 \\ 6 \overline{)766} \\ - 6 \downarrow \\ \hline 16 \end{array} \leftarrow 16 \text{ tens}$$

$$\begin{array}{r} 12 \\ 6 \overline{)766} \\ - 6 \\ \hline 16 \\ - 12 \\ \hline 4 \end{array} \leftarrow \begin{array}{l} \text{Divide 16 tens by 6.} \\ \text{Multiply. } 6 \times 2 \text{ tens} \\ \text{Subtract.} \end{array}$$

Step 3 There are 4 tens left over. Regroup 4 tens, now there are 46 ones. Divide the ones.

$$\begin{array}{r} 12 \\ 6 \overline{)766} \\ - 6 \\ \hline 16 \\ - 12 \\ \hline 46 \end{array} \leftarrow 46 \text{ ones}$$

$$\begin{array}{r} 127 \\ 6 \overline{)766} \\ - 6 \\ \hline 16 \\ - 12 \\ \hline 46 \\ - 42 \\ \hline 4 \end{array} \leftarrow \begin{array}{l} \text{Divide 46 ones by 6.} \\ \text{Multiply. } 6 \times 7 \text{ ones} \\ \text{Subtract.} \end{array}$$

Step 4 Check to make sure that the remainder is less than the divisor. Write the answer.

$$\begin{array}{r} 127 \text{ r}4 \\ 6 \overline{)766} \end{array} \quad 4 < 6$$

Step 5 Use multiplication and addition to check your answer.

$$\begin{array}{r} 127 \leftarrow \text{quotient} \\ \times 6 \leftarrow \text{divisor} \\ \hline 762 \\ + 4 \leftarrow \text{remainder} \\ \hline 766 \leftarrow \text{dividend} \end{array}$$

Divide and check.

$$\begin{array}{r} 217 \\ 4 \overline{)868} \\ \times 4 \\ \hline 868 \end{array}$$

$$\begin{array}{r} 328 \text{ r}1 \\ 2 \overline{)657} \\ \times 2 \\ \hline 656 \\ + 1 \\ \hline 657 \end{array}$$

$$\begin{array}{r} 1,210 \text{ r}3 \\ 7 \overline{)8,743} \\ \times 7 \\ \hline 8,470 \\ + 3 \\ \hline 8,473 \end{array}$$

Multi-Step Division Problems

There are 72 third graders and 84 fourth graders going on a field trip. An equal number of students will ride on each of 4 buses. How many students will ride on each bus?

Read the Problem	Solve the Problem												
<p>What do I need to find?</p> <p>I need to find the number of <u>students</u> who will ride on each bus.</p>	<p>I can model the number of students in all using a bar model.</p> <table border="1" style="margin: 10px auto;"> <tr> <td style="text-align: center;">72</td> <td style="text-align: center;">84</td> </tr> <tr> <td colspan="2" style="text-align: center;"> $\underline{\hspace{2cm}}$ </td> </tr> <tr> <td colspan="2" style="text-align: center;">156</td> </tr> </table>	72	84	$\underline{\hspace{2cm}}$		156							
72	84												
$\underline{\hspace{2cm}}$													
156													
<p>What information do I need to use?</p> <p>There are <u>72</u> third graders and <u>84</u> fourth graders. There will be <u>4</u> buses.</p>	<p>I can model the number of buses and divide to find the number of students on each bus.</p> <table border="1" style="margin: 10px auto;"> <tr> <td style="text-align: center;"><u>39</u></td> <td style="text-align: center;"><u>39</u></td> <td style="text-align: center;"><u>39</u></td> <td style="text-align: center;"><u>39</u></td> </tr> <tr> <td colspan="4" style="text-align: center;"> $\underline{\hspace{4cm}}$ </td> </tr> <tr> <td colspan="4" style="text-align: center;">156</td> </tr> </table>	<u>39</u>	<u>39</u>	<u>39</u>	<u>39</u>	$\underline{\hspace{4cm}}$				156			
<u>39</u>	<u>39</u>	<u>39</u>	<u>39</u>										
$\underline{\hspace{4cm}}$													
156													
<p>How will I use the information?</p> <p>I will make a bar model for each step. I will add <u>72 and 84</u> to find the total number of students. I will divide by <u>4</u> to find how many students will ride on each bus.</p>	<p>So, <u>39</u> students will ride on each bus.</p>												

- 1** Miranda has 180 beads for making jewelry. She buys 240 more beads. She wants to store the beads in a case with 6 sections. She wants to put the same number of beads in each section. How many beads should Miranda put in each section?

70 beads

- 2** All 203 students at Polk School eat lunch at the same time. One day 19 students were absent. If 8 students sit at each table in the lunchroom, how many tables were used that day at lunch?

23 tables

Apply the Perimeter Formula

Perimeter is the distance around a shape. You can use grid paper to count the number of units around the outside of a rectangle to find its perimeter.

How many feet of ribbon are needed to go around the bulletin board?

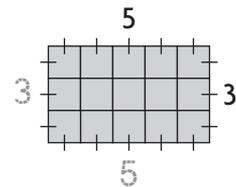
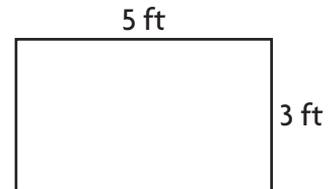
Step 1 On grid paper, draw a rectangle that has a length of 5 units and a width of 3 units.

Step 2 Find the length of each side of the rectangle. Mark each unit of length as you count.

Step 3 Add the side lengths. $5 + 3 + 5 + 3 = 16$

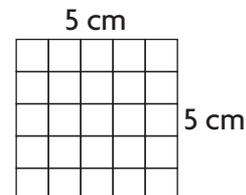
The perimeter is 16 feet.

So, 16 feet of ribbon are needed to go around the bulletin board.

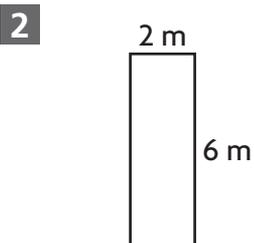


1 What is the perimeter of this square?

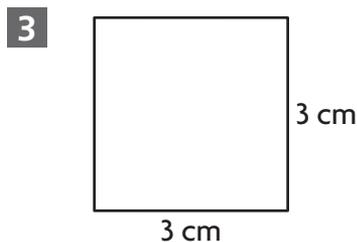
$$\underline{5} + \underline{5} + \underline{5} + \underline{5} = \underline{20} \text{ centimeters}$$



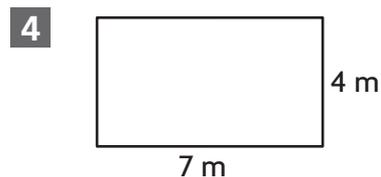
Find the perimeter of the rectangle or square.



$$\underline{16} \text{ meters}$$



$$\underline{12} \text{ centimeters}$$



$$\underline{22} \text{ meters}$$

Apply the Area Formula

Area is the measure of the number of **unit squares** needed to cover a surface. A unit square is a square with a side length of 1 unit. It has an area of 1 **square unit**.

Find the area of the rectangle at the right.

You can use the formula **Area = base \times height**.

Step 1 Identify one side as the base.

The base is 14 feet.

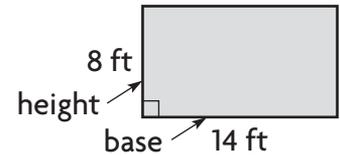
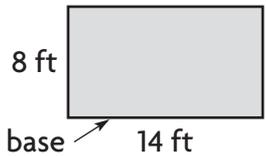
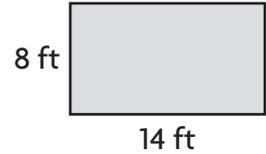
Step 2 Identify a perpendicular side as the height.

The height is 8 feet.

Step 3 Use the formula to find the area.

$$\begin{aligned} \text{Area} &= \text{base} \times \text{height} \\ &= 14 \times 8 \\ &= 112 \end{aligned}$$

So, the area of the rectangle is **112 square feet**.



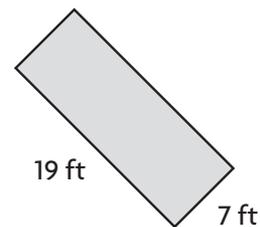
Find the area of the rectangle or square.

1



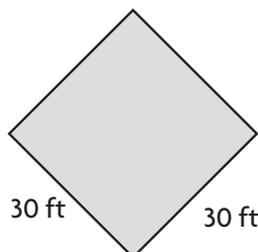
78 sq m

2



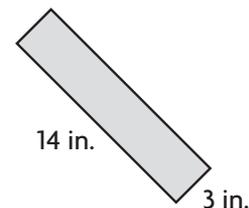
133 sq ft

3



900 sq ft

4

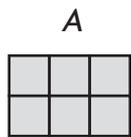


42 sq in.

Same Perimeter, Different Areas

You can use perimeter and area to compare rectangles.

Compare the perimeters of Rectangle *A* and Rectangle *B*.



Find the number of units around each rectangle.

Rectangle *A*: $3 + 2 + 3 + 2 = 10$ units

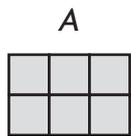
Rectangle *B*: $4 + 1 + 4 + 1 = 10$ units



Compare: 10 units = 10 units

So, Rectangle *A* has the same perimeter as Rectangle *B*.

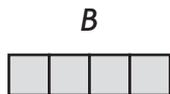
Compare the areas of Rectangle *A* and Rectangle *B*.



Find the number of unit squares needed to cover each rectangle.

Rectangle *A*: 2 rows of 3 = 2×3 , or 6 square units

Rectangle *B*: 1 row of 4 = 1×4 , or 4 square units

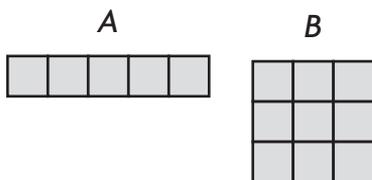


Compare: 6 square units $>$ 4 square units

So, Rectangle *A* has a greater area than Rectangle *B*.

Find the perimeter and the area. Tell which rectangle has a greater area.

1



A: Perimeter = 12 units;

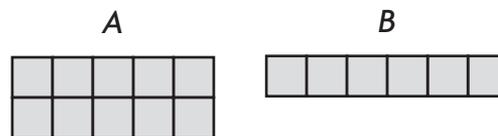
Area = 5 square units

B: Perimeter = 12 units;

Area = 9 square units

Rectangle B has a greater area.

2



A: Perimeter = 14 units;

Area = 10 square units

B: Perimeter = 14 units;

Area = 6 square units

Rectangle A has a greater area.

Same Area, Different Perimeters

Find the perimeter and area of Rectangles *A* and *B*.

Tell which rectangle has a greater perimeter.

Step 1 Find the area of each rectangle. You can multiply the number of unit squares in each row by the number of rows.

$$\text{Rectangle A: } 2 \times 6 = 12 \text{ square units}$$

$$\text{Rectangle B: } 3 \times 4 = 12 \text{ square units}$$

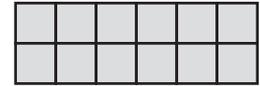
Step 2 Find the perimeter of each rectangle. You can add the sides.

$$\text{Rectangle A: } 6 + 2 + 6 + 2 = 16 \text{ units}$$

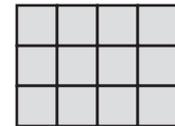
$$\text{Rectangle B: } 4 + 3 + 4 + 3 = 14 \text{ units}$$

Step 3 Compare the perimeters. $16 \text{ units} > 14 \text{ units}$.

So, Rectangle *A* has a greater perimeter.



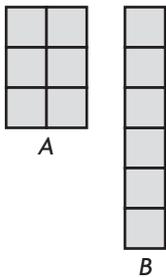
A



B

Find the perimeter and the area. Tell which rectangle has a greater perimeter.

1



$$\text{A: Area} = \underline{6 \text{ square units}};$$

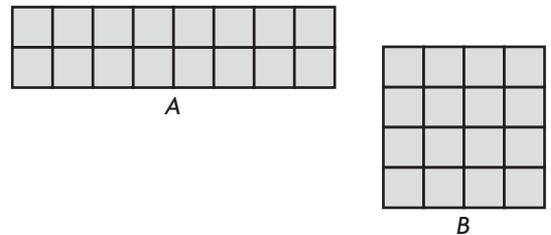
$$\text{Perimeter} = \underline{10 \text{ units}}$$

$$\text{B: Area} = \underline{6 \text{ square units}};$$

$$\text{Perimeter} = \underline{14 \text{ units}}$$

Rectangle B has a greater perimeter.

2



$$\text{A: Area} = \underline{16 \text{ square units}};$$

$$\text{Perimeter} = \underline{20 \text{ units}}$$

$$\text{B: Area} = \underline{16 \text{ square units}};$$

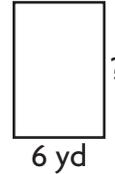
$$\text{Perimeter} = \underline{16 \text{ units}}$$

Rectangle A has a greater perimeter.

Find Unknown Measures

Fred has 30 yards of fencing to enclose a rectangular vegetable garden. He wants it to be 6 yards wide.

How long will his vegetable garden be?



Step 1 Decide whether this problem involves area or perimeter.

Think: The fencing goes *around the outside* of the garden. This is a measure of perimeter.

Step 2 Use a formula for perimeter. The width is 6 yards. The perimeter is 30 yards. The length is unknown.

$$P = (2 \times l) + (2 \times w)$$

$$30 = (2 \times l) + (2 \times 6)$$

$$30 = (2 \times l) + 12$$

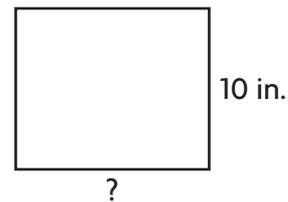
$$18 = 2 \times l, \text{ so the value of } l \text{ is } 9.$$

Step 3 Find the value of l .

The length of Fred's garden will be 9 yards.

Carol has 120 square inches of wood. The piece of wood is rectangular and has a height of 10 inches.

How long is the base?



Step 1 Decide whether this problem involves area or perimeter.

Think: *Square inches* is a measure of area.

Step 2 Use a formula for area. The height is 10 inches. The area is 120 square inches. The length is unknown.

$$A = b \times h$$

$$120 = b \times 10$$

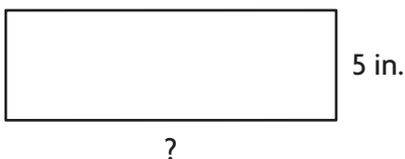
Step 3 Find the value of b .

Since $120 = 12 \times 10$, the value of b is 12.

The base of Carol's piece of wood is 12 inches.

Find the unknown measure.

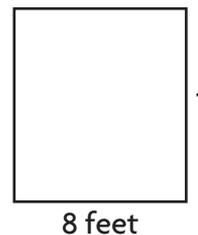
1



Perimeter = 40 inches

width = 15 inches

2



Area = 72 square feet

height = 9 feet

Find the Area

Use the strategy solve a simpler problem.

Marilyn is going to paint a wall in her bedroom. The wall is 15 feet long and 8 feet tall. The window takes up an area 6 feet long and 4 feet high. How many square feet of the wall will Marilyn have to paint?

Read the Problem	Solve the Problem
<p>What do I need to find?</p> <p>I need to find how many <u>square feet of the wall</u> Marilyn will paint.</p>	<p>First, find the area of the wall.</p> $A = b \times h$ $= 15 \times 8$ $= \underline{120} \text{ square feet}$
<p>What information do I need to use?</p> <p>The paint will cover the wall. The paint will not cover the <u>window</u>. The base of the wall is 15 feet and the height is <u>8 feet</u>. The base of the window is 6 feet and the height is <u>4 feet</u>.</p>	<p>Next, find the area of the window.</p> $A = b \times h$ $= 6 \times 4$ $= \underline{24} \text{ square feet}$ <p>Last, subtract the area of the window from the area of the wall.</p> $\begin{array}{r} 120 \\ - 24 \\ \hline \end{array}$
<p>How will I use the information?</p> <p>I can solve simpler problems. Find the area of the <u>wall</u>. Then, find the area of the window. Last, <u>subtract</u> the area of the <u>window</u> from the area of the wall.</p>	<p>So, Marilyn will paint <u>96 square feet</u> of her bedroom wall.</p>

- 1** Ned wants to wallpaper the wall of his bedroom that has the door. The wall is 14 feet wide and 9 feet high. The door is 3 feet wide and 7 feet high. How many square feet of wallpaper will Ned need for the wall?

105 square feet

- 2** Nicole has a rectangular canvas that is 12 inches long and 10 inches wide. She paints a blue square in the center of the canvas. The square is 3 inches on each side. How much of the canvas is NOT painted blue?

111 square inches

Factors and Divisibility

A number is divisible by another number if the quotient is a counting number and the remainder is 0.

You can decide if a number is divisible by 2, 3, 5, 6, or 9 by using divisibility rules instead of dividing. Divisibility rules help you decide if one number is a factor of another.

Is 39 divisible by 2, 3, 5, 6, or 9?

Divisibility Rules

$39 \div 2 = 19 \text{ r}1 \rightarrow$ 39 is not divisible by <u>2</u> .	The last digit, 9, is not even, so 39 is not divisible by 2.
$39 \div 3 = 13 \text{ r}0 \rightarrow$ 39 is divisible by <u>3</u> .	The sum of the digits, $3 + 9 = 12$, is divisible by 3, so 39 is divisible by 3.
$39 \div 5 = 7 \text{ r}4 \rightarrow$ 39 is not divisible by <u>5</u> .	The last digit, 9, is not a 0 or 5, so 39 is not divisible by 5.
$39 \div 6 = 6 \text{ r}3 \rightarrow$ 39 is not divisible by <u>6</u> .	39 is not divisible by both 2 and 3, so it is not divisible by 6.
$39 \div 9 = 4 \text{ r}3 \rightarrow$ 39 is not divisible by <u>9</u> .	The sum of the digits, $3 + 9 = 12$, is not divisible by 9, so 39 is not divisible by 9.

39 is divisible by 3.
3 is a factor of 39.

Tell whether 30 is divisible by 2, 3, 5, 6, or 9. Show your work.

- $30 \div 2$ _____ yes; 30 is even
- $30 \div 3$ _____ yes; $30 \div 3 = 10$
- $30 \div 5$ _____ yes; $30 \div 5 = 6$
- $30 \div 6$ _____ yes; $30 \div 6 = 5$
- $30 \div 9$ _____ no; $30 \div 9 = 3 \text{ r}3$

Is 4 a factor of the number? Write yes or no.

- | | | |
|-------------------------|--------------------------|--------------------------|
| 6 81
_____ no | 7 24
_____ yes | 8 56
_____ yes |
|-------------------------|--------------------------|--------------------------|

Factors and Multiples

You know that $1 \times 10 = \underline{10}$ and $2 \times 5 = \underline{10}$.

So, 1, 2, 5, and 10 are all **factors** of 10.

You can skip count to find **multiples** of a number:

Count by 1s: 1, 2, 3, 4, 5, 6, 7, 8, 9, **10**, . . .

Count by 2s: 2, 4, 6, 8, **10**, 12, . . .

Count by 5s: 5, **10**, 15, 20, 25, . . .

Count by 10s: **10**, 20, 30, 40, . . .

Note that **10** is a multiple of 1, 2, 5, and 10. A number is a multiple of all of its factors.

A **common multiple** is a multiple of two or more numbers. So, 10 is a common multiple of 1, 2, 5, and 10.

1 Multiply to list the next five multiples of 3.

3, 6, 9, 12, 15, 18

2 Multiply to list the next five multiples of 7.

7, 14, 21, 28, 35, 42

Is the number a factor of 8? Write yes or no.

3 2

yes

4 8

yes

5 15

no

6 20

no

Is the number a multiple of 4? Write yes or no.

7 2

no

8 12

yes

9 16

yes

10 18

no

Prime and Composite Numbers

A **prime number** is a whole number greater than 1 that has exactly two factors, 1 and the number itself.

A **composite number** is a whole number greater than 1 that has more than two factors.

You can use division to find the factors of a number and tell whether the number is prime or composite.

Tell whether 55 is *prime* or *composite*.

Use division to find all the numbers that divide into 55 without a remainder. Those numbers are the factors of 55.

$55 \div 1 = 55$, so 1 and 55 are factors.

$55 \div 5 = 11$, so 5 and 11 are factors.

The factors of 55 are 1, 5, 11, and 55.

Because 55 has more than two factors, 55 is a composite number.

Tell whether 61 is *prime* or *composite*.

Use division to find all the numbers that divide into 61 without a remainder. Those numbers are the factors of 61.

$61 \div 1 = 61$, so 1 and 61 are factors.

There are no other numbers that divide into 61 evenly without a remainder.

The factors of 61 are 1 and 61.

Because 61 has exactly two factors, 61 is a prime number.

Tell whether the number is *prime* or *composite*.

1 44

Think: Is 44 divisible by any number other than 1 and 44?

composite

2 53

Think: Does 53 have other factors besides 1 and itself?

prime

3 12

4 50

5 24

6 67

composite

composite

composite

prime

7 83

8 27

9 34

10 78

prime

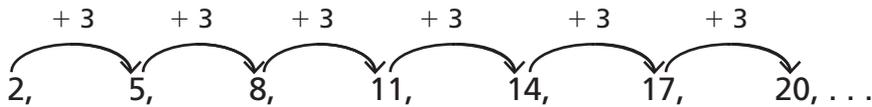
composite

composite

composite

Number Patterns

A pattern is an ordered set of numbers or objects, called terms. The numbers below form a pattern. The first term in the pattern is 2.

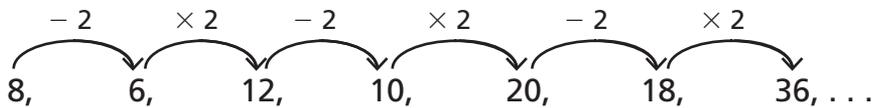


First term

A rule is used to describe a pattern. The rule for this pattern is add 3.

You can describe other patterns in the numbers. Notice that the terms in the pattern shown alternate between even and odd numbers.

For some patterns, the rule may have two operations.



The rule for this pattern is subtract 2, multiply by 2. The first term is 8. Notice that all of the terms in this pattern are even numbers.

Use the rule to write the numbers in the pattern.

1 Rule: Add 7. First term: 12

12, 19, 26, 33, 40, ...

2 Rule: Multiply by 3, subtract 1. First term: 2

2, 6, 5, 15, 14, ...

Use the rule to write the numbers in the pattern. Describe another pattern in the numbers.

3 Rule: Subtract 5. First term: 50

50, 45, 40, 35, 30, ...

Possible answer: The ones digit alternates between 0 and 5.

4 Rule: Multiply by 2, add 1. First term: 4

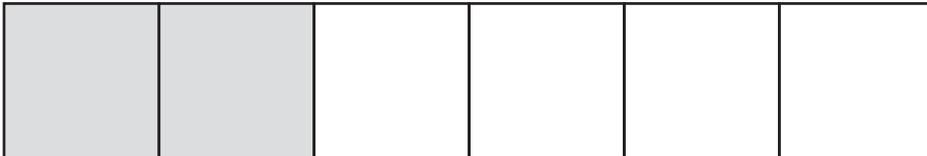
4, 8, 9, 18, 19, ...

Possible answer: After the first term, the ones digit alternates between 8 and 9.

Equivalent Fractions

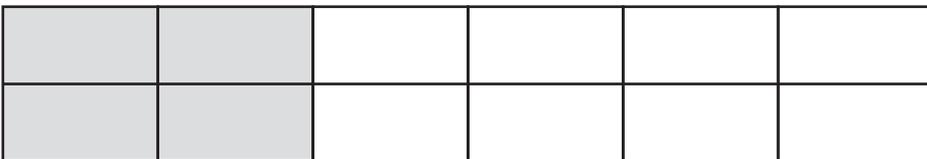
Write two fractions that are equivalent to $\frac{2}{6}$.

Step 1 Make a model to represent $\frac{2}{6}$.



The rectangle is divided into 6 equal parts, with 2 parts shaded.

Step 2 Divide the rectangle from Step 1 in half.



The rectangle is now divided into 12 equal parts, with 4 parts shaded.

The model shows the fraction $\frac{4}{12}$. So, $\frac{2}{6}$ and $\frac{4}{12}$ are equivalent.

Step 3 Draw the same rectangle as in Step 1, but with only 3 equal parts. Keep the same amount of the rectangle shaded.



The rectangle is now divided into 3 equal parts, with 1 part shaded.

The model shows the fraction $\frac{1}{3}$. So, $\frac{2}{6}$ and $\frac{1}{3}$ are equivalent.

Use models to write two equivalent fractions.

1 $\frac{2}{4}$

$\frac{1}{2}, \frac{4}{8}$

2 $\frac{4}{6}$

$\frac{2}{3}, \frac{8}{12}$

Check students' models.
Possible answers are given.

Generate Equivalent Fractions

Write an equivalent fraction for $\frac{4}{5}$.

Step 1 Choose a whole number, like 2.

Step 2 Create a fraction using 2 as the numerator and denominator: $\frac{2}{2}$.
This fraction is equal to 1. You can multiply a number by 1 without changing the value of the number.

Step 3 Multiply $\frac{4}{5}$ by $\frac{2}{2}$: $\frac{4 \times 2}{5 \times 2} = \frac{8}{10}$.

So, $\frac{4}{5}$ and $\frac{8}{10}$ are equivalent.

Write another equivalent fraction for $\frac{4}{5}$.

Step 1 Choose a different whole number, like 20.

Step 2 Create a fraction using 20 as the numerator and denominator: $\frac{20}{20}$.

Step 3 Multiply $\frac{4}{5}$ by $\frac{20}{20}$: $\frac{4 \times 20}{5 \times 20} = \frac{80}{100}$.

So, $\frac{4}{5}$ and $\frac{80}{100}$ are equivalent.

Write two equivalent fractions.

1 $\frac{2}{6}$

$\frac{6}{18}$ $\frac{8}{24}$

Possible answers are given.

2 $\frac{4}{10}$

$\frac{8}{20}$ $\frac{12}{30}$

3 $\frac{3}{8}$

$\frac{9}{24}$ $\frac{18}{48}$

4 $\frac{3}{5}$

$\frac{12}{20}$ $\frac{21}{35}$

Use Division to Generate Equivalent Fractions

A fraction is in **simplest form** when 1 is the only factor that the numerator and denominator have in common.

Tell whether the fraction $\frac{7}{8}$ is in simplest form.

Look for common factors in the numerator and the denominator.

Step 1 The numerator of $\frac{7}{8}$ is 7. List all the factors of 7.	$1 \times 7 = 7$ The factors of 7 are 1 and 7.
Step 2 The denominator of $\frac{7}{8}$ is 8. List all the factors of 8.	$1 \times 8 = 8$ $2 \times 4 = 8$ The factors of 8 are 1, 2, 4, and 8.
Step 3 Check if the numerator and denominator of $\frac{7}{8}$ have any common factors greater than 1.	The only common factor of 7 and 8 is 1.
So, $\frac{7}{8}$ is in simplest form.	

Tell whether the fraction is in simplest form. Write yes or no.

1 $\frac{4}{10}$

no

2 $\frac{2}{8}$

no

3 $\frac{3}{5}$

yes

Write the fraction in simplest form.

4 $\frac{4}{12}$

$\frac{1}{3}$

5 $\frac{6}{10}$

$\frac{3}{5}$

6 $\frac{3}{6}$

$\frac{1}{2}$

Find Equivalent Fractions

Kyle's mom bought bunches of balloons for a family party.

Each bunch has 4 balloons, and $\frac{1}{4}$ of the balloons are blue.

If Kyle's mom bought 5 bunches of balloons, how many balloons did she buy? How many of the balloons are blue?

Read the Problem

What do I need to find?

I need to find how many balloons Kyle's mom bought and how many of the balloons are blue.

What information do I need to use?

Each bunch has 1 out of 4 balloons that are blue, and there are 5 bunches.

How will I use the information?

I will make a table to find the total number of balloons Kyle's mom bought and the fraction of balloons that are blue.

Solve the Problem

I can make a table.

Number of Bunches	1	2	3	4	5
<u>Total Number of Blue Balloons</u>	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{4}{16}$	$\frac{5}{20}$
<u>Total Number of Balloons</u>	4	8	12	16	20

Kyle's mom bought 20 balloons. 5 of the balloons are blue.

Make a table to solve.

Check students' tables.

- 1** Jackie is making a beaded bracelet. The bracelet will have no more than 12 beads. $\frac{1}{3}$ of the beads on the bracelet will be green. What other fractions could represent the part of the beads on the bracelet that will be green?
- 2** Ben works in his dad's bakery packing bagels. Each package can have no more than 16 bagels. $\frac{3}{4}$ of the bagels in each package are plain. What other fractions could represent the part of the bagels in each package that will be plain?

$$\frac{2}{6}, \frac{3}{9}, \frac{4}{12}$$

$$\frac{6}{8}, \frac{9}{12}, \frac{12}{16}$$

Write Fractions as Sums

A **unit fraction** tells the part of the whole that 1 piece represents.

A unit fraction always has a numerator of 1.

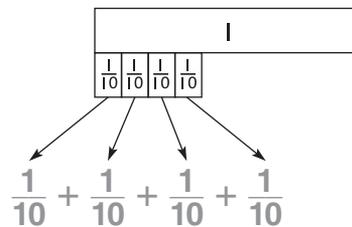
Bryan has $\frac{4}{10}$ pound of clay for making clay figures. He wants to use $\frac{1}{10}$ pound of clay for each figure. How many clay figures can he make?

Use fraction strips to write $\frac{4}{10}$ as a sum of unit fractions.

Step 1 Represent $\frac{4}{10}$ with fraction strips.

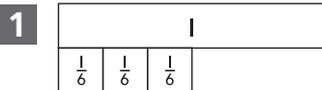
Step 2 Each $\frac{1}{10}$ is a unit fraction. Write a $\frac{1}{10}$ addend for each $\frac{1}{10}$ -strip you used to show $\frac{4}{10}$.

Step 3 Count the number of addends. The number of addends represents the number of clay figures Bryan can make.

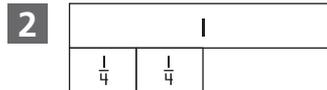


So, Bryan can make 4 clay figures.

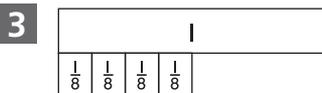
Write the fraction as the sum of unit fractions.



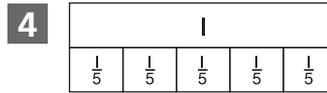
$$\frac{3}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$$



$$\frac{2}{4} = \frac{1}{4} + \frac{1}{4}$$



$$\frac{4}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$



$$\frac{5}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$$

Rename Fractions and Mixed Numbers

A **mixed number** is made up of a whole number and a fraction. You can use multiplication and addition to rename a mixed number as a fraction greater than 1.

Rename $2\frac{5}{6}$ as a fraction.

First, multiply the denominator, or the number of parts in the whole, by the whole number.

$$6 \times 2 = 12$$

Then, add the numerator to your product.

$$12 + 5 = 17$$

$$\text{So, } 2\frac{5}{6} = \frac{17}{6}.$$

$$2 \frac{5}{6} = \frac{\boxed{17}}{6}$$

total number
of parts
number of
parts in the whole

You can use division to write a fraction greater than 1 as a mixed number.

Rename $\frac{16}{3}$ as a mixed number.

To rename $\frac{16}{3}$ as a mixed number, divide the numerator by the denominator.

Use the quotient and remainder to write a mixed number.

$$\text{So, } \frac{16}{3} = 5\frac{1}{3}.$$

$$\begin{array}{r} 5 \\ 3 \overline{)16} \\ - 15 \\ \hline 1 \end{array}$$

Write the mixed number as a fraction.

$$1 \quad 3\frac{2}{3} = \underline{\frac{11}{3}}$$

$$2 \quad 4\frac{3}{5} = \underline{\frac{23}{5}}$$

$$3 \quad 4\frac{3}{8} = \underline{\frac{35}{8}}$$

$$4 \quad 2\frac{1}{6} = \underline{\frac{13}{6}}$$

Write the fraction as a mixed number.

$$5 \quad \frac{32}{5} = \underline{6\frac{2}{5}}$$

$$6 \quad \frac{19}{3} = \underline{6\frac{1}{3}}$$

$$7 \quad \frac{15}{4} = \underline{3\frac{3}{4}}$$

$$8 \quad \frac{51}{10} = \underline{5\frac{1}{10}}$$

Compare Fractions Using Benchmarks

A **benchmark** is a known size or amount that helps you understand a different size or amount. You can use $\frac{1}{2}$ as a benchmark.

Sara reads for $\frac{3}{6}$ hour every day after school. Connor reads for $\frac{2}{3}$ hour. Who reads for a longer amount of time?

Compare the fractions. $\frac{3}{6}$ ● $\frac{2}{3}$

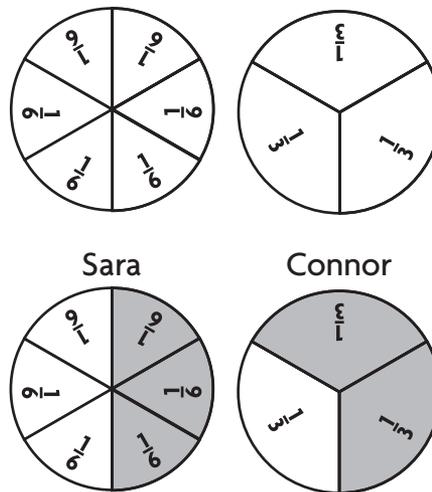
Step 1 Divide one circle into 6 equal parts. Divide another circle into 3 equal parts.

Step 2 Shade $\frac{3}{6}$ of the first circle. How many parts will you shade? **3 parts**

Step 3 Shade $\frac{2}{3}$ of the second circle.

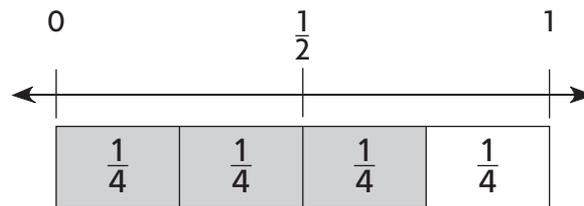
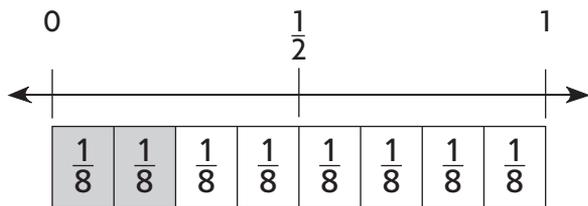
How many parts will you shade? **2 parts**

Step 4 Compare the shaded parts of each circle. Half of Sara's circle is shaded. More than half of Connor's circle is shaded.



$\frac{3}{6}$ is less than $\frac{2}{3}$. Since $\frac{3}{6} < \frac{2}{3}$, Connor reads for a longer amount of time.

1 Compare $\frac{2}{8}$ and $\frac{3}{4}$. Write $<$ or $>$.



Compare. Write $<$ or $>$.

$$\frac{2}{8} < \frac{3}{4}$$

2 $\frac{1}{4} < \frac{8}{10}$

3 $\frac{7}{8} > \frac{1}{3}$

4 $\frac{5}{12} < \frac{1}{2}$

5 $\frac{2}{8} < \frac{8}{12}$

6 $\frac{4}{6} > \frac{4}{8}$

7 $\frac{7}{12} > \frac{2}{4}$

Compare Fractions

Theo filled a beaker $\frac{2}{4}$ full with water. Angelica filled a beaker $\frac{3}{8}$ full with water. Whose beaker has more water?

Compare $\frac{2}{4}$ and $\frac{3}{8}$.

Step 1 Divide one beaker into 4 equal parts.
Divide another beaker into 8 equal parts.

Step 2 Shade $\frac{2}{4}$ of the first beaker.

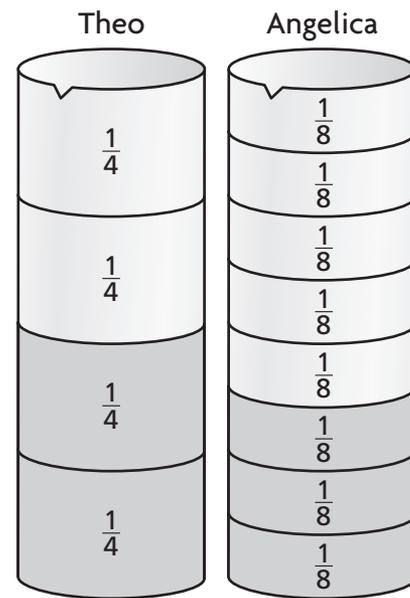
Step 3 Shade $\frac{3}{8}$ of the second beaker.

Step 4 Compare the shaded parts of each beaker.
Half of Theo's beaker is shaded. Less than half of Angelica's beaker is shaded.

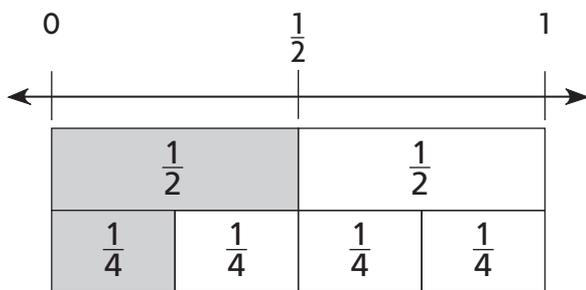
$\frac{2}{4}$ is greater than $\frac{3}{8}$.

$$\frac{2}{4} > \frac{3}{8}$$

So, Theo's beaker has more water.

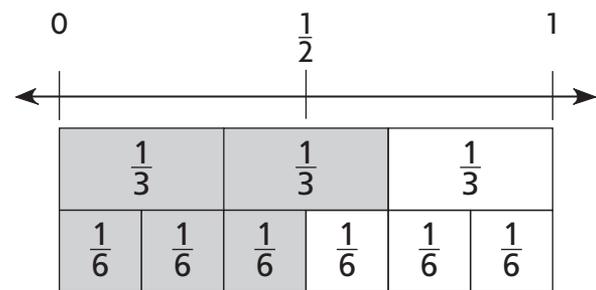


1 Compare $\frac{1}{2}$ and $\frac{1}{4}$.



Which is greater? $\frac{1}{2}$

2 Compare $\frac{2}{3}$ and $\frac{3}{6}$.



Which is less? $\frac{3}{6}$

Compare. Write $<$, $>$, or $=$.

3 $\frac{1}{2} < \frac{3}{4}$

4 $\frac{6}{12} < \frac{5}{8}$

5 $\frac{2}{3} = \frac{4}{6}$

6 $\frac{3}{8} > \frac{1}{4}$

Compare and Order Fractions

Write $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ in order from least to greatest.

Step 1 Identify a common denominator. Multiples of 8: $\textcircled{8}$, 16, 24

Multiples of 4: 4, $\textcircled{8}$, 16

Multiples of 2: 2, 4, 6, $\textcircled{8}$

Use 8 as a common denominator.

Step 2 Use the common denominator to write equivalent fractions.

$$\frac{3}{8}$$

$$\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$$

$$\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$

Step 3 Compare the numerators.

$$2 < 3 < 4$$

Step 4 Order the fractions from least to greatest, using $<$ or $>$ symbols.

$$\frac{2}{8} < \frac{3}{8} < \frac{4}{8}$$

So,

$$\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$$

Write the fraction with the greatest value.

1 $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{6}$

$$\frac{2}{3}$$

2 $\frac{3}{10}$, $\frac{1}{2}$, $\frac{2}{5}$

$$\frac{1}{2}$$

3 $\frac{1}{8}$, $\frac{5}{12}$, $\frac{9}{10}$

$$\frac{9}{10}$$

Write the fractions in order from least to greatest.

4 $\frac{9}{10}$, $\frac{1}{2}$, $\frac{4}{5}$

$$\frac{1}{2} < \frac{4}{5} < \frac{9}{10}$$

5 $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{2}$

$$\frac{1}{2} < \frac{3}{4} < \frac{7}{8}$$

6 $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$

$$\frac{2}{3} < \frac{3}{4} < \frac{5}{6}$$

Add and Subtract Parts of a Whole

Justin has $\frac{3}{8}$ pound of cheddar cheese and $\frac{2}{8}$ pound of brick cheese.
How much cheese does he have in all?

Step 1 Use fraction strips to model the problem. Use three $\frac{1}{8}$ -strips to represent $\frac{3}{8}$ pound of cheddar cheese.

Step 2 Join two more $\frac{1}{8}$ -strips to represent the amount of brick cheese.

Step 3 Count the number of $\frac{1}{8}$ -strips. There are five $\frac{1}{8}$ -strips. Write the amount as a fraction. Justin has $\frac{5}{8}$ pound of cheese.

$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

Step 4 Use the model to write an equation.

Suppose Justin eats $\frac{1}{8}$ pound of cheese. How much cheese is left?

Step 1 Use five $\frac{1}{8}$ -strips to represent the $\frac{5}{8}$ pound of cheese.

Step 2 Remove one $\frac{1}{8}$ -strip to show the amount eaten.

Step 3 Count the number of $\frac{1}{8}$ -strips left. There are four $\frac{1}{8}$ fraction strips. There is $\frac{4}{8}$ pound left.

$$\frac{5}{8} - \frac{1}{8} = \frac{4}{8}$$

Step 4 Write an equation for the model.

Use the model to write an equation.

1

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$$

2

$$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

3

$$\frac{3}{4} + \frac{1}{4} = \frac{4}{4}$$

4

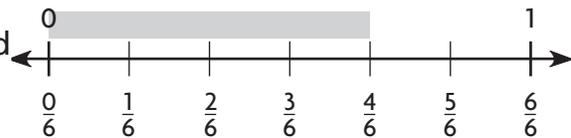
$$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$$

Add Fractions Using Models

Fractions with like denominators have the same denominator. You can add fractions with like denominators using a number line.

Model $\frac{4}{6} + \frac{1}{6}$.

Step 1 Draw a number line labeled with sixths. Model the fraction $\frac{4}{6}$ by starting at 0 and shading 4 sixths.



Step 2 Add the fraction $\frac{1}{6}$ by shading 1 more sixth.

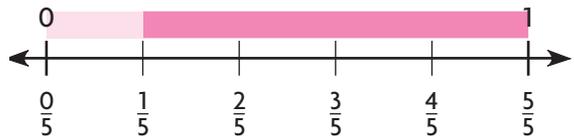


Step 3 How many sixths are there in all? 5 sixths
Write the number of sixths as a fraction.

$$5 \text{ sixths} = \frac{5}{6} \quad \frac{4}{6} + \frac{1}{6} = \frac{5}{6}$$

1 Model $\frac{1}{5} + \frac{4}{5}$.

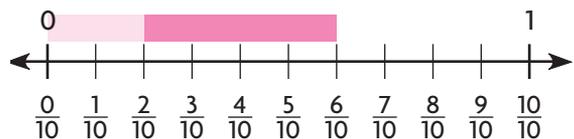
$$\frac{1}{5} + \frac{4}{5} = \underline{\hspace{2cm}} \quad \frac{5}{5}$$



Find the sum. Use a model to help.

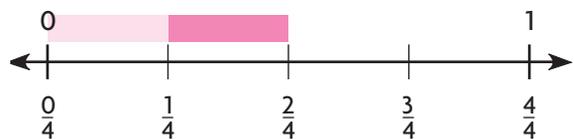
2 $\frac{2}{10} + \frac{4}{10}$

$$\underline{\hspace{2cm}} \quad \frac{6}{10}$$



3 $\frac{1}{4} + \frac{1}{4}$

$$\underline{\hspace{2cm}} \quad \frac{2}{4}$$

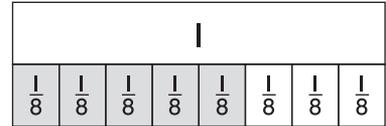


Subtract Fractions Using Models

You can subtract fractions with like denominators using fraction strips.

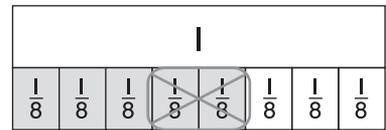
Model $\frac{5}{8} - \frac{2}{8}$.

Step 1 Shade the eighths you start with.
Shade 5 eighths.



Step 2 Subtract $\frac{2}{8}$.

Think: How many eighths are taken away?
Cross out 2 of the shaded eighths.



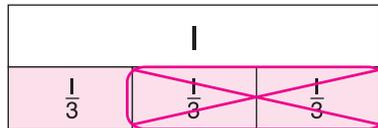
Step 3 Count the shaded eighths that remain.
There are 3 eighths remaining.

Step 4 Write the number of eighths that remain as a fraction.

$$3 \text{ eighths} = \frac{3}{8} \qquad \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$

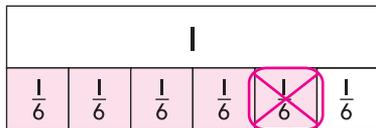
1 Model $\frac{3}{3} - \frac{2}{3}$.

$$\frac{3}{3} - \frac{2}{3} = \underline{\hspace{2cm} \frac{1}{3} \hspace{2cm}}$$



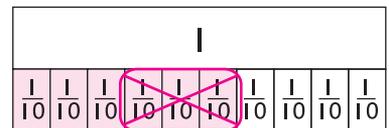
Subtract. Use fraction strips to help.

2 $\frac{5}{6} - \frac{1}{6}$



$$\frac{5}{6} - \frac{1}{6} = \underline{\hspace{2cm} \frac{4}{6} \hspace{2cm}}$$

2 $\frac{6}{10} - \frac{3}{10}$



$$\frac{6}{10} - \frac{3}{10} = \underline{\hspace{2cm} \frac{3}{10} \hspace{2cm}}$$

Use Benchmarks to Determine Reasonableness

Mr. Rodriguez mixes red, blue, and white paint for a project in art class. He uses $\frac{5}{6}$ of a jar of red paint, $\frac{2}{6}$ of a jar of blue paint, and $\frac{1}{6}$ of a jar of white paint. About how many jars of paint does he use?

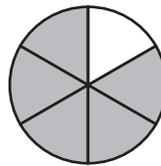
A. Write an expression to represent the situation. Possible answer: $\frac{5}{6} + \frac{2}{6} + \frac{1}{6}$

B. Is the problem asking for an exact answer?

No, it says "about how many."

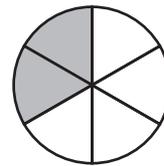
C. Use benchmarks to estimate each fraction.

- $\frac{5}{6}$ is close to 1 whole because almost the whole circle is shaded.



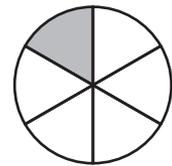
$\frac{5}{6}$

- $\frac{2}{6}$ is close to $\frac{1}{2}$ because about $\frac{1}{2}$ of the circle is shaded.



$\frac{2}{6}$

- $\frac{1}{6}$ is close to 0 because very little of the circle is shaded.



$\frac{1}{6}$

D. About how much of each color paint does Mr. Rodriguez use?

about 1 jar of red, about $\frac{1}{2}$ jar of blue, and almost no white

E. Write an equation to estimate the total jars of paint

Mr. Rodriguez uses. $1 + \frac{1}{2} + 0 = 1\frac{1}{2}$

Mr. Rodriguez uses about $1\frac{1}{2}$ jars of paint.

Use benchmarks to estimate the sum or difference.

- 1** Adam buys a bag of dog food with a weight of $\frac{7}{8}$ pound and a bag of cat food with a weight of $1\frac{1}{8}$ pounds. Estimate the total weight of the pet food he buys.

$1 + 1 = 2$; about 2 pounds

2 $\frac{5}{8} + \frac{7}{8}$

$1\frac{4}{8}$

3 $\frac{4}{5} - \frac{3}{5}$

$\frac{1}{5}$

4 $\frac{4}{10} + \frac{3}{10}$

1

Add and Subtract Fractions

You can find and record the sums and the differences of fractions.

Add. $\frac{2}{6} + \frac{4}{6}$

Step 1 Model it.



Step 2 Think: How many sixths are there in all?

There are 6 sixths.

$$6 \text{ sixths} = \frac{6}{6}$$

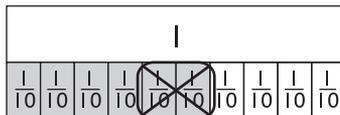
Step 3 Record it.

Write the sum as an addition equation.

$$\frac{2}{6} + \frac{4}{6} = \frac{6}{6}$$

Subtract. $\frac{6}{10} - \frac{2}{10}$

Step 1 Model it.



Step 2 Think: There are 6 tenths. I take away 2 tenths. How many tenths are left?

There are 4 tenths left.

$$4 \text{ tenths} = \frac{4}{10}$$

Step 3 Record it.

Write the difference as a subtraction equation.

$$\frac{6}{10} - \frac{2}{10} = \frac{4}{10}$$

Find the sum or difference.

1 7 eighth-size parts – 4 eighth-size parts = 3 eighth-size parts

$$\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$$

2 $\frac{11}{12} - \frac{4}{12} = \frac{7}{12}$ **3** $\frac{2}{10} + \frac{2}{10} = \frac{4}{10}$ **4** $\frac{6}{8} - \frac{4}{8} = \frac{2}{8}$

5 $\frac{2}{4} + \frac{2}{4} = \frac{4}{4}$ **6** $\frac{4}{5} - \frac{3}{5} = \frac{1}{5}$ **7** $\frac{1}{3} + \frac{2}{3} = \frac{3}{3}$

Add and Subtract Mixed Numbers

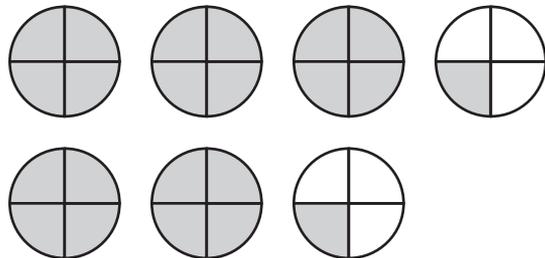
Find the sum. $3\frac{1}{4} + 2\frac{1}{4}$

Add the whole number and fraction parts.

- Add the whole numbers: $3 + 2 = 5$
- Add the fractions: $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

Write the sum as a mixed number, so the

fractional part is less than 1. $3\frac{1}{4} + 2\frac{1}{4} = 5\frac{2}{4}$

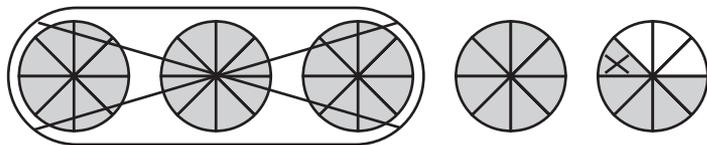


Find the difference. $4\frac{5}{8} - 3\frac{1}{8}$

Subtract the fraction and the whole number parts.

- Subtract the fractions: $\frac{5}{8} - \frac{1}{8} = \frac{4}{8}$
- Subtract the whole numbers: $4 - 3 = 1$

$$4\frac{5}{8} - 3\frac{1}{8} = 1\frac{4}{8}$$



Find the sum or difference.

$$\begin{array}{r} \mathbf{1} \quad 3\frac{4}{5} \\ + 4\frac{3}{5} \\ \hline 8\frac{2}{5} \end{array}$$

$$\begin{array}{r} \mathbf{2} \quad 7\frac{2}{3} \\ - 3\frac{1}{3} \\ \hline 4\frac{1}{3} \end{array}$$

$$\begin{array}{r} \mathbf{3} \quad 4\frac{7}{12} \\ + 6\frac{5}{12} \\ \hline 11 \end{array}$$

$$\begin{array}{r} \mathbf{4} \quad 12\frac{3}{4} \\ - 6\frac{1}{4} \\ \hline 6\frac{2}{4} \end{array}$$

$$\begin{array}{r} \mathbf{5} \quad 2\frac{3}{8} \\ + 8\frac{1}{8} \\ \hline 10\frac{4}{8} \end{array}$$

$$\begin{array}{r} \mathbf{6} \quad 11\frac{9}{10} \\ - 3\frac{7}{10} \\ \hline 8\frac{2}{10} \end{array}$$

$$\begin{array}{r} \mathbf{7} \quad 7\frac{3}{5} \\ + 4\frac{3}{5} \\ \hline 12\frac{1}{5} \end{array}$$

$$\begin{array}{r} \mathbf{8} \quad 8\frac{3}{6} \\ - 3\frac{1}{6} \\ \hline 5\frac{2}{6} \end{array}$$

Use Properties of Addition

Properties of addition can help you group and order addends so you can use mental math to find sums.

The **Commutative Property of Addition** states that when the order of two addends is changed, the sum is the same. $6 + 3 = 3 + 6$

The **Associative Property of Addition** states that when the grouping of addends is changed, the sum is the same. $(3 + 6) + 4 = 3 + (6 + 4)$

Use the properties and mental math to add $10\frac{3}{8} + 4\frac{7}{8} + 6\frac{5}{8}$.

Step 1 Look for fractions that combine to make 1. $10\frac{3}{8} + 4\frac{7}{8} + 6\frac{5}{8}$

Step 2 Use the Commutative Property to order the addends so that the fractions with a sum of 1 are together. $10\frac{3}{8} + 4\frac{7}{8} + 6\frac{5}{8} = 10\frac{3}{8} + 6\frac{5}{8} + 4\frac{7}{8}$

Step 3 Use the Associative Property to group the addends that you can add mentally. $= (10\frac{3}{8} + 6\frac{5}{8}) + 4\frac{7}{8}$

Step 4 Add the grouped numbers and then add the other mixed number. $= (17) + 4\frac{7}{8}$

Step 5 Write the sum. $= 21\frac{7}{8}$

Use the properties and mental math to find the sum.

1 $(3\frac{1}{5} + 1\frac{2}{5}) + 4\frac{4}{5}$

$9\frac{4}{5}$

2 $(5\frac{7}{10} + 1\frac{4}{10}) + 6\frac{3}{10}$

$13\frac{4}{10}$

3 $7\frac{3}{4} + (5 + 3\frac{1}{4})$

16

4 $(2\frac{5}{12} + 3\frac{11}{12}) + 1\frac{7}{12}$

$7\frac{11}{12}$

5 $4\frac{7}{8} + (6\frac{3}{8} + \frac{1}{8})$

$11\frac{3}{8}$

6 $9\frac{2}{6} + (4\frac{1}{6} + 7\frac{4}{6})$

$21\frac{1}{6}$

Multiples of Unit Fractions

A unit fraction is a fraction with a numerator of 1. You can write a fraction as the product of a whole number and a unit fraction.

Write $\frac{7}{10}$ as the product of a whole number and a unit fraction.

Write $\frac{7}{10}$ as the sum of unit fractions.

$$\frac{7}{10} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10}$$

Use multiplication to show repeated addition.

$$\frac{7}{10} = \underline{7} \times \frac{1}{10}$$

So, $\frac{7}{10} = \underline{7} \times \underline{\frac{1}{10}}$.

The product of a number and a counting number is a multiple of the number. You can find multiples of unit fractions.

List the next 4 multiples of $\frac{1}{8}$.

Make a table and use repeated addition.

$1 \times \frac{1}{8}$	$2 \times \frac{1}{8}$	$3 \times \frac{1}{8}$	$4 \times \frac{1}{8}$	$5 \times \frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8} + \frac{1}{8}$	$\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$	$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$	$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
$\frac{1}{8}$	$\underline{\frac{2}{8}}$	$\underline{\frac{3}{8}}$	$\underline{\frac{4}{8}}$	$\underline{\frac{5}{8}}$

The next 4 multiples of $\frac{1}{8}$ are $\underline{\frac{2}{8}}$, $\underline{\frac{3}{8}}$, $\underline{\frac{4}{8}}$, and $\underline{\frac{5}{8}}$.

Write the fraction as the product of a whole number and a unit fraction.

1 $\frac{2}{5} = \underline{2} \times \underline{\frac{1}{5}}$

2 $\frac{5}{12} = \underline{5} \times \underline{\frac{1}{12}}$

3 $\frac{7}{2} = \underline{7} \times \underline{\frac{1}{2}}$

List the next four multiples of the unit fraction.

4 $\frac{1}{4}$, $\underline{\frac{2}{4}}$, $\underline{\frac{3}{4}}$, $\underline{\frac{4}{4}}$, $\underline{\frac{5}{4}}$

5 $\frac{1}{6}$, $\underline{\frac{2}{6}}$, $\underline{\frac{3}{6}}$, $\underline{\frac{4}{6}}$, $\underline{\frac{5}{6}}$

Multiples of Fractions

You have learned to write multiples of unit fractions. You can also write multiples of other fractions.

Write the next 4 multiples of $\frac{2}{5}$.

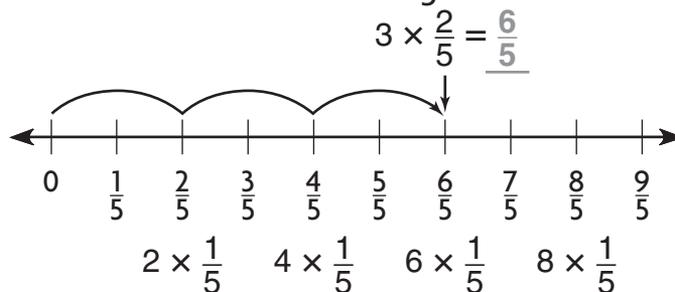
Make a table.

$1 \times \frac{2}{5}$	$2 \times \frac{2}{5}$	$3 \times \frac{2}{5}$	$4 \times \frac{2}{5}$	$5 \times \frac{2}{5}$
$\frac{2}{5}$	$\frac{2}{5} + \frac{2}{5}$	$\frac{2}{5} + \frac{2}{5} + \frac{2}{5}$	$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$	$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$
$\frac{2}{5}$	$\frac{4}{5}$	$\frac{6}{5}$	$\frac{8}{5}$	$\frac{10}{5}$

So, the next 4 multiples of $\frac{2}{5}$ are $\frac{4}{5}$, $\frac{6}{5}$, $\frac{8}{5}$, and $\frac{10}{5}$.

Write $3 \times \frac{2}{5}$ as the product of a whole number and a unit fraction.

Use a number line. Make three jumps of $\frac{2}{5}$.



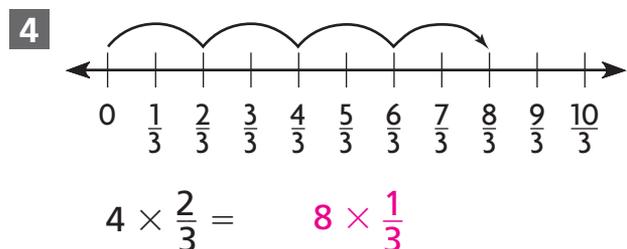
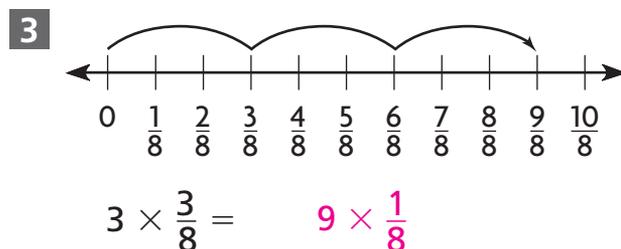
So, $3 \times \frac{2}{5} = \frac{6}{5}$, or $6 \times \frac{1}{5}$.

List the next four multiples of the fraction.

1 $\frac{3}{4}$, $\frac{6}{4}$, $\frac{9}{4}$, $\frac{12}{4}$, $\frac{15}{4}$

2 $\frac{5}{6}$, $\frac{10}{6}$, $\frac{15}{6}$, $\frac{20}{6}$, $\frac{25}{6}$

Write as the product of a whole number and a unit fraction.



Multiply a Fraction by a Whole Number Using Models

You can use a model to multiply a fraction by a whole number.

Find the product of $4 \times \frac{3}{5}$.

Use fraction strips. Show 4 groups of $\frac{3}{5}$ each.



1 group of $\frac{3}{5} = \frac{3}{5}$



2 groups of $\frac{3}{5} = \frac{6}{5}$



3 groups of $\frac{3}{5} = \frac{9}{5}$

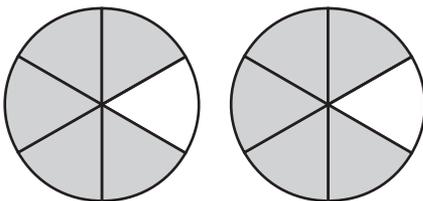


4 groups of $\frac{3}{5} = \frac{12}{5}$

So, $4 \times \frac{3}{5} = \frac{12}{5}$.

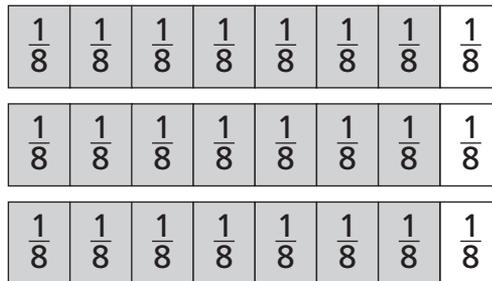
Multiply.

1



$$2 \times \frac{5}{6} = \underline{\frac{10}{6}}$$

2



$$3 \times \frac{7}{8} = \underline{\frac{21}{8}}$$

3 $6 \times \frac{2}{3} = \underline{\frac{12}{3}}$

4 $2 \times \frac{9}{10} = \underline{\frac{18}{10}}$

5 $5 \times \frac{3}{4} = \underline{\frac{15}{4}}$

6 $4 \times \frac{5}{8} = \underline{\frac{20}{8}}$

7 $7 \times \frac{2}{5} = \underline{\frac{14}{5}}$

8 $8 \times \frac{4}{6} = \underline{\frac{32}{6}}$

Find Part of a Group

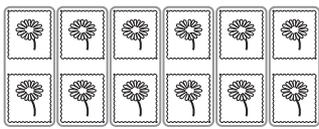
Lauren bought 12 stamps for postcards. She gave Brianna $\frac{1}{6}$ of them. How many stamps did Lauren give to Brianna?



Find $\frac{1}{6}$ of 12.

Step 1 What is the denominator in the fraction of the stamps Lauren gave to Brianna? 6

So, divide the 12 stamps into 6 equal groups. Circle the groups.



Step 2 Each group represents $\frac{1}{6}$ of the stamps.

How many stamps are in 1 group? 2

So, $\frac{1}{6}$ of 12 is 2, or $\frac{1}{6} \times 12$ is 2.

So, Lauren gave Brianna 2 stamps.

Use a model to solve.

Check students' models.

1 $\frac{3}{4} \times 12 = \underline{9}$

2 $\frac{1}{3} \times 9 = \underline{3}$

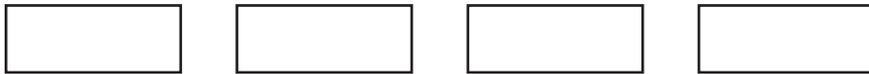
3 $\frac{3}{5} \times 20 = \underline{12}$

4 $\frac{4}{6} \times 18 = \underline{12}$

Multiply Fractions and Whole Numbers

Find the product. $\frac{3}{8} \times 4$

Step 1 Draw 4 rectangles to represent the factor 4.



Step 2 The denominator of the factor $\frac{3}{8}$ is 8. So, divide the 4 rectangles into 8 equal parts.



Step 3 The numerator of the factor $\frac{3}{8}$ is 3. So, shade 3 of the parts.



Step 4 The 4 rectangles have 3 shaded parts. Each rectangle is divided into 2 equal parts. So, $\frac{3}{2}$ of the rectangles are shaded.

So, $\frac{3}{8} \times 4$ is $\frac{3}{2}$, or $1\frac{1}{2}$.

Find the product.

1 $\frac{5}{12} \times 4 = \underline{\frac{5}{3}, \text{ or } 1\frac{2}{3}}$ 2 $8 \times \frac{3}{4} = \underline{6}$ 3 $\frac{7}{9} \times 3 = \underline{\frac{7}{3}, \text{ or } 2\frac{1}{3}}$

4 $5 \times \frac{4}{7} = \underline{\frac{20}{7}, \text{ or } 2\frac{6}{7}}$ 5 $\frac{9}{10} \times 5 = \underline{\frac{9}{2}, \text{ or } 4\frac{1}{2}}$ 6 $3 \times \frac{3}{4} = \underline{\frac{9}{4}, \text{ or } 2\frac{1}{4}}$

7 $\frac{7}{12} \times 6 = \underline{\frac{7}{2}, \text{ or } 3\frac{1}{2}}$ 8 $12 \times \frac{2}{9} = \underline{\frac{24}{9}, \text{ or } 2\frac{2}{3}}$ 9 $\frac{2}{9} \times 3 = \underline{\frac{2}{3}}$

Fraction and Whole Number Multiplication

Find the product. $3 \times \frac{5}{6}$

$$3 \times \frac{5}{6} = \frac{3}{\boxed{1}} \times \frac{5}{6}$$

$$= \frac{3 \times \boxed{5}}{1 \times 6}$$

$$= \frac{\boxed{15}}{6}$$

$$= \boxed{2} \frac{3}{6}, \text{ or } 2 \frac{\boxed{1}}{\boxed{2}}$$

So, $3 \times \frac{5}{6}$ is $2\frac{1}{2}$.

Write the whole-number factor, 3, as $\frac{3}{1}$.

Multiply the numerators. Then multiply the denominators.

Write the product as a mixed number in simplest form.

Find the product. Write the product in simplest form.

$$\mathbf{1} \quad \frac{2}{3} \times 8 = \frac{2}{3} \times \frac{8}{\boxed{1}}$$

$$= \frac{\boxed{2} \times \boxed{8}}{\boxed{3} \times \boxed{1}}$$

$$= \frac{\boxed{16}}{\boxed{3}}, \text{ or } \underline{5\frac{1}{3}}$$

$$\mathbf{2} \quad 4 \times \frac{2}{9} = \underline{\frac{8}{9}}$$

$$\mathbf{3} \quad 6 \times \frac{3}{4} = \underline{4\frac{1}{2} \text{ or } \frac{9}{2}}$$

$$\mathbf{4} \quad \frac{4}{9} \times 3 = \underline{1\frac{1}{3} \text{ or } \frac{4}{3}}$$

$$\mathbf{5} \quad 5 \times \frac{3}{8} = \underline{1\frac{7}{8} \text{ or } \frac{15}{8}}$$

$$\mathbf{6} \quad 9 \times \frac{2}{3} = \underline{6}$$

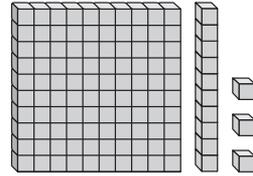
$$\mathbf{7} \quad 2 \times \frac{5}{6} = \underline{1\frac{2}{3} \text{ or } \frac{5}{3}}$$

$$\mathbf{8} \quad 7 \times \frac{4}{10} = \underline{2\frac{4}{5} \text{ or } \frac{14}{5}}$$

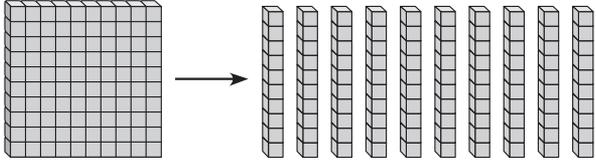
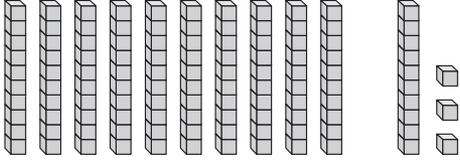
Model Tenths and Hundredths

The flat represents one unit, the long represents $\frac{1}{10}$ of one unit, and the small cube represents $\frac{1}{100}$ of one unit.

Model the decimal in another way. Then tell which blocks you used.



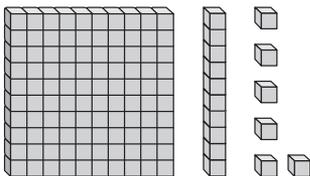
1.13

Step 1 Describe the model.	1 flat 1 long 3 small cubes
Step 2 Describe the value of one of the larger units using smaller units.	Think: I can replace 1 flat with 10 longs. 
Step 3 Replace the larger unit with the equivalent number of smaller units. Describe the new model.	 There are now 11 longs in the model. The model has 11 longs and 3 small cubes.

Check students' models. Possible answers are given.

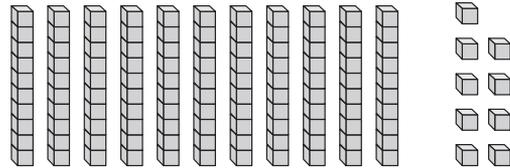
Write the decimal shown by the model. The flat represents 1 unit. Then model the decimal in another way. Tell which blocks you used.

1



1.16; 11 longs 6
small cubes

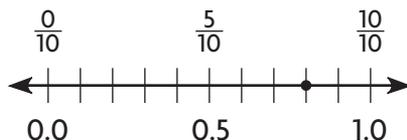
2



1.19; 1 flat 1 long
9 small cubes

Relate Tenths and Decimals

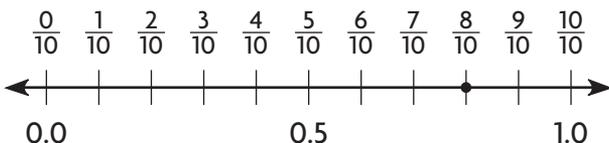
Write the fraction and the decimal that are shown by the point on the number line.



Step 1 Count the number of equal parts of the whole shown on the number line. There are ten equal parts.

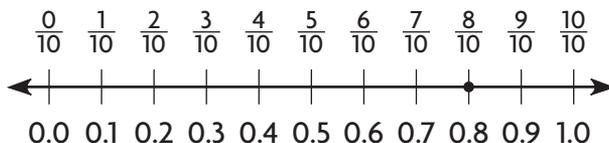
This tells you that the number line shows tenths.

Step 2 Label the number line with the missing fractions. What fraction is shown by the point on the number line?



The fraction shown by the point on the number line is $\frac{8}{10}$.

Step 3 Label the number line with the missing decimals. What decimal is shown by the point on the number line?

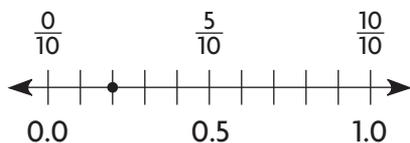


The decimal shown by the point on the number line is **0.8**.

So, the fraction and decimal shown by the point on the number line are $\frac{8}{10}$ and **0.8**.

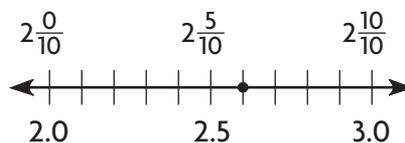
Write the fraction or mixed number and the decimal shown by the model.

1



$\frac{2}{10}$; 0.2

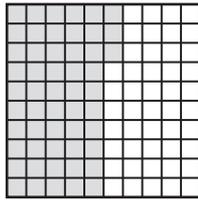
2



$2\frac{6}{10}$; 2.6

Relate Hundredths and Decimals

Write the fraction or mixed number and the decimal shown by the model.



Step 1 Count the number of shaded squares in the model and the total number of squares in the whole model.

Number of shaded squares: 53

Total number of squares: 100

Step 2 Write a fraction to represent the part of the model that is shaded.

$\frac{\text{Number of Shaded Squares}}{\text{Total Number of Squares}} = \frac{53}{100}$

The fraction shown by the model is $\frac{53}{100}$.

Step 3 Write the fraction in decimal form.

Think: The fraction shown by the model is _____.

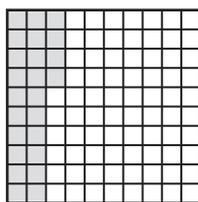
0.53 names the same amount as $\frac{53}{100}$. $\frac{53}{100}$

The decimal shown by the model is 0.53.

The fraction and decimal shown by the model are $\frac{53}{100}$ and 0.53.

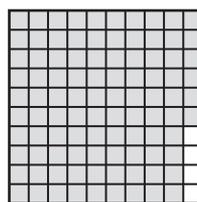
Write the fraction or mixed number and the decimal shown by the model.

1



$\frac{24}{100}$, 0.24

2



$\frac{96}{100}$, 0.96

Equivalent Fractions and Decimals

Lori ran $\frac{20}{100}$ mile. How many tenths of a mile did she run?

Write $\frac{20}{100}$ as an equivalent fraction with a denominator of 10.

Step 1 Think: 10 is a common factor of the numerator and the denominator.

Step 2 Divide the numerator and denominator by 10.

$$\frac{20}{100} = \frac{20 \div 10}{100 \div 10} = \frac{2}{10}$$

So, Lori ran $\frac{2}{10}$ mile.

Use a place-value chart.

Step 1 Write $\frac{20}{100}$ as an equivalent decimal.

Ones	.	Tenths	Hundredths
0	.	2	0

Step 2 Think: 20 hundredths is 2 tenths 0 hundredths

Ones	.	Tenths
0	.	2

So, Lori ran 0.2 mile.

Write the number as hundredths in fraction form and decimal form.

1 $\frac{9}{10}$

2 0.6

3 4 $\frac{\quad}{10}$

$\frac{90}{100}$; 0.90

$\frac{60}{100}$; 0.60

$\frac{40}{100}$; 0.40

Write the number as tenths in fraction form and decimal form.

4 70 $\frac{\quad}{100}$

5 80 $\frac{\quad}{100}$

6 0.50

$\frac{7}{10}$; 0.7

$\frac{8}{10}$; 0.8

$\frac{5}{10}$; 0.5

Relate Fractions, Decimals, and Money

Write the total money amount. Then write the amount as a fraction and as a decimal in terms of a dollar.



Step 1 Count the value of coins from greatest to least. Write the total money amount.



\$0.25 → \$0.35 → \$0.40 → \$0.45 → \$0.50

Step 2 Write the total money amount as a fraction of a dollar.

The total money amount is \$0.50, which is the same as 50 cents.

Think: There are 100 cents in a dollar.

So, the total amount written as a fraction of a dollar is:

$$\frac{50 \text{ cents}}{100 \text{ cents}} = \frac{50}{100}$$

Step 3 Write the total money amount as a decimal.

Think: I can write \$0.50 as 0.50.

The total money amount is $\frac{50}{100}$ written as a fraction of a dollar, and 0.50 written as a decimal.

Write the total money amount. Then write the amount as a fraction or a mixed number and as a decimal in terms of a dollar.

1



\$0.80; $\frac{80}{100}$; 0.80

2



\$1.45; $1\frac{45}{100}$; 1.45

Add Fractional Parts of 10 and 100

Sam uses 100 glass beads for a project. Of the beads, $\frac{35}{100}$ are gold and $\frac{4}{10}$ are silver. What fraction of the glass beads are gold or silver?

Add $\frac{35}{100}$ and $\frac{4}{10}$.

Step 1 Decide on a common denominator. Use 100.

Step 2 Write $\frac{4}{10}$ as an equivalent fraction with a denominator of 100.

$$\frac{4}{10} = \frac{4 \times 10}{10 \times 10} = \frac{40}{100}$$

Step 3 Add $\frac{35}{100}$ and $\frac{40}{100}$.

$$\frac{35}{100} + \frac{40}{100} = \frac{75}{100}$$

← Add the numerators.

← Use 100 as the denominator.

So, $\frac{75}{100}$ of the glass beads are gold or silver.

Add \$0.26 and \$0.59.

Step 1 Write each amount as a fraction of a dollar.

$$\$0.26 = \frac{26}{100} \text{ of a dollar} \qquad \$0.59 = \frac{59}{100} \text{ of a dollar}$$

Step 2 Add $\frac{26}{100}$ and $\frac{59}{100}$.

$$\frac{26}{100} + \frac{59}{100} = \frac{85}{100}$$

← Add the numerators.

← 100 is the common denominator.

Step 3 Write the sum as a decimal.

$$\frac{85}{100} = 0.85$$

So, $\$0.26 + \$0.59 = \underline{\$0.85}$.

Find the sum.

$$1 \quad \frac{75}{100} + \frac{2}{10} = \frac{77}{100}$$

$$2 \quad \$0.73 + \$0.25 = \$ \underline{0.98}$$

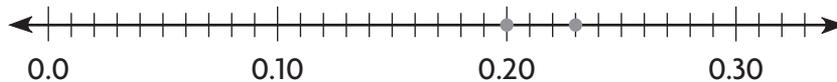
$$\frac{73}{100} + \frac{25}{100} = \frac{98}{100}$$

Compare Decimals

Alfie found 0.2 of a dollar and Gemma found 0.23 of a dollar.
Which friend found more money?

To compare decimals, you can use a number line.

Step 1 Locate each decimal on a number line.



Step 2 The number farther to the right is greater.

$0.23 > 0.2$, so Gemma found more money.

To compare decimals, you can compare equal-size parts.

Step 1 Write 0.2 as a decimal in hundredths.

0.2 is 2 tenths, which is equivalent to 20 hundredths.

$$0.2 = \underline{0.20}$$

Step 2 Compare.

23 hundredths is greater than 20 hundredths,
so $0.23 > 0.2$.

So, Gemma found more money.

Compare. Write $<$, $>$, or $=$.

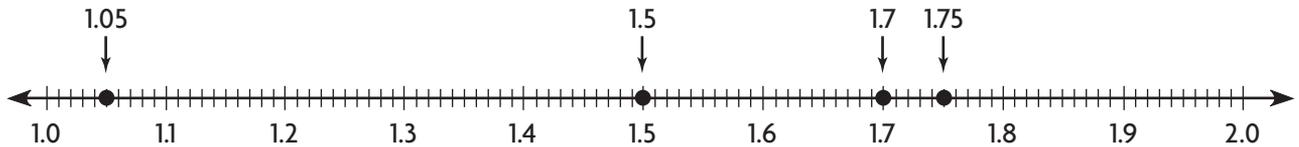
1 $0.17 > 0.13$ **2** $0.8 > 0.08$ **3** $0.36 < 0.63$ **4** $0.4 = 0.40$

5 $0.75 > 0.69$ **6** $0.3 < 0.7$ **7** $0.45 > 0.37$ **8** $0.96 > 0.78$

Order Decimals

Use the number line to order the decimals from least to greatest.

1.7, 1.75, 1.5, 1.05



Locate and label 1.7, 1.75, 1.5, and 1.05 on the number line.

1.05 is the farthest to the left on the number line, so it is the **least**.

1.75 is the farthest to the right on the number line, so it is the **greatest**.

So, the decimals in order from least to greatest are **1.05, 1.5, 1.7, and 1.75**.

Order the decimals from greatest to least.

\$1.89, \$2.15, \$1.09

Step 1 Line up the decimal places.

\$1.89

\$2.15

\$1.09

Step 2 Compare the digits beginning with the greatest place, the ones digit.

Because the 2 in \$2.15 has the greatest value, \$2.15 is the **greatest number**.

Step 3 Compare the other two numbers: \$1.89 and \$1.09.

They have the same digit in the ones place, so compare the tenths place.

\$1.89 is greater than \$1.09 because it has a greater digit in the tenths place.

So, the decimals in order from greatest to least are **\$2.15, \$1.89, and \$1.09**.

Use the number line above to order the decimals from least to greatest.

1 1.6, 1.06, 1.61, 1.66

1.06, 1.6, 1.61, 1.66

2 1.2, 1.23, 1.12, 1.21

1.12, 1.2, 1.21, 1.23

3 1.7, 1.77, 1.07, 1.01

1.01, 1.07, 1.7, 1.77

Use the number line above to order the decimals from greatest to least.

4 1.2, 1.02, 1.32, 1.23

1.32, 1.23, 1.2, 1.02

5 1.18, 1.38, 1.08, 1.88

1.88, 1.38, 1.18, 1.08

6 1.5, 1.75, 1.05, 1.65

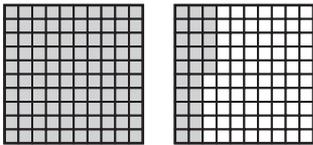
1.75, 1.65, 1.5, 1.05

Decimal Addition

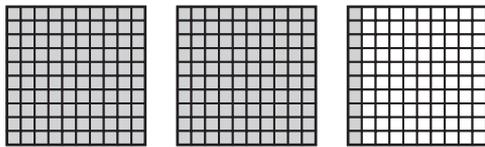
You can use decimal models to help you add decimals.

Add. $1.25 + 0.85$

Step 1 Shade squares to represent 1.25.



Step 2 Shade additional squares to represent adding 0.85.



Remember:

Since there are only 75 squares left in the second model, you need to add another whole model for the remaining 10 squares.

Step 3 Count the total number of shaded squares.

There are 2 whole squares and 10 one-hundredths squares shaded. So, 2.10 wholes in all are shaded.

So, $1.25 + 0.85 = \underline{2.10}$.

Add. Use decimal models. Draw a picture to show your work. *Check students' drawings.*

1 $2.1 + 0.59$

2.69

2 $1.4 + 0.22$

1.62

3 $1.27 + 1.15$

2.42

4 $0.81 + 0.43$

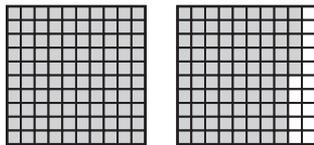
1.24

Decimal Subtraction

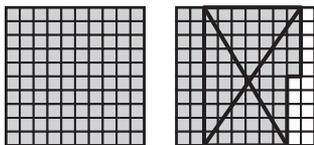
You can use decimal models to help you subtract decimals.

Subtract. $1.85 - 0.65$

Step 1 Shade squares to represent 1.85.



Step 2 Circle and cross out 65 of the shaded squares to represent subtracting 0.65.



Step 3 Count the shaded squares that are not crossed out. Altogether, 1 whole square and 20 one-hundredths squares, or 1.20 wholes, are NOT crossed out.

So, $1.85 - 0.65 = \underline{1.20}$.

Remember:

By circling and crossing out shaded squares, you can see how many squares are taken away, or subtracted.

Subtract. Use decimal models. Draw a picture to show your work.

1 $1.4 - 0.61$

0.79

3 $0.84 - 0.17$

0.67

2 $1.6 - 1.08$

0.52

4 $1.39 - 1.14$

0.25

Check students' drawings.

Add Decimals

Add. $4.17 + 9.8$

Step 1 Estimate the sum.

$$\begin{array}{r} 4.17 + 9.8 \\ \downarrow \quad \downarrow \\ \text{Estimate: } 4 + 10 = 14 \end{array}$$

Step 2 Line up the place values for each number in a place-value chart. Then add.

	Ones	Tenths	Hundredths	
	4	• 1	7	
+	9	• 8		
	13	• 9	7	← sum

Step 3 Use your estimate to determine if your answer is reasonable.

Think: 13.97 is close to the estimate, 14. The answer is reasonable.

So, $4.17 + 9.8 = \underline{13.97}$.

Estimate. Then find the sum.

Possible estimates are given.

1 Estimate: 1

$$\begin{array}{r} 1.20 \\ + 0.34 \\ \hline 1.54 \end{array}$$

2 Estimate: 3

$$\begin{array}{r} 1.52 \\ + 1.21 \\ \hline 2.73 \end{array}$$

3 Estimate: 23

$$\begin{array}{r} 12.25 \\ + 11.25 \\ \hline 23.50 \text{ or } 23.5 \end{array}$$

4 Estimate: 12

$$\begin{array}{r} 10.75 \\ + 1.11 \\ \hline 11.86 \end{array}$$

5 Estimate: 41

$$\begin{array}{r} 22.65 \\ + 18.01 \\ \hline 40.66 \end{array}$$

6 Estimate: 49

$$\begin{array}{r} 34.41 \\ + 15.37 \\ \hline 49.78 \end{array}$$

Subtract Decimals

Subtract. $6.56 - 4.33$

Step 1 Estimate the difference.

$$\begin{array}{r} 6.56 - 4.33 \\ \downarrow \quad \downarrow \\ \text{Estimate: } 7 - 4 = 3 \end{array}$$

Step 2 Line up the place values for each number in a place-value chart. Then subtract.

	Ones	Tenths	Hundredths	
	6	• 5	6	
—	4	• 3	3	
	2	• 2	3	← difference

Step 3 Use your estimate to determine if your answer is reasonable.

Think: 2.23 is close to the estimate, 3. The answer is reasonable.

So, $6.56 - 4.33 = \underline{2.23}$.

Estimate. Then find the difference. Possible estimates are given.

1 Estimate: 1

$$\begin{array}{r} 1.97 \\ - 0.79 \\ \hline 1.18 \end{array}$$

2 Estimate: 3

$$\begin{array}{r} 4.42 \\ - 1.26 \\ \hline 3.16 \end{array}$$

3 Estimate: 2

$$\begin{array}{r} 10.25 \\ - 8.25 \\ \hline 2.00 \text{ or } 2 \end{array}$$

Find the difference. Check your answer.

4

$$\begin{array}{r} 5.75 \\ - 1.11 \\ \hline 4.64 \end{array}$$

5

$$\begin{array}{r} 25.21 \\ - 19.05 \\ \hline 6.16 \end{array}$$

6

$$\begin{array}{r} 42.14 \\ - 25.07 \\ \hline 17.07 \end{array}$$

Solve Multi-Step Money Problems

Solve. Use the table to solve 1–3.

- 1** Dorian and Jack decided to go bowling. They each need to rent shoes. Jack pays to rent 1 lane and their shoes at member's cost with \$20. What change should he receive?

Bowl-a-Rama		
	Regular Cost	Member's Cost
Lane Rental (up to 4 people)	\$9.75	\$7.50
Shoe Rental	\$3.95	\$2.95

Calculate the cost:

$$\$2.95 + \$2.95 + \$7.50 = \$13.40$$

Calculate the change: $\$20 - \$13.40 = \$6.60$

\$6.60

- 2** Natalie and her friends decided to rent 4 lanes at regular cost for a party. Ten people need to rent shoes, and 4 people are members. What is the total cost for the party?

\$74.50

- 3** Warren is a member and paid \$22.30 to bowl with some non-member friends. Warren rented 1 lane and shoes for himself at member's cost and shoes for each of his friends at regular cost. How many pairs of shoes did he rent?

4 pairs of shoes

Use the following information to solve 4–5.

Check students' tables.

At the soccer game concession stand, bottles of water cost \$1.25 and large muffins cost \$2.50.

- 4** How many bottles of water can be purchased with \$20? Make a table to show your answer.

as many as 16 bottles

- 5** Quon bought 2 bottles of water and 2 large muffins. He paid with \$20. What amount of change should he have received?

\$12.50

Explore Angles

Find how many $\frac{1}{6}$ turns make a complete circle.

Materials: fraction circles

Step 1 Place a $\frac{1}{6}$ piece so the tip of the fraction piece is on the center of the circle. Trace the fraction piece by drawing along the dashed lines in the circle.

Step 2 Shade and label the angle formed by the $\frac{1}{6}$ piece.

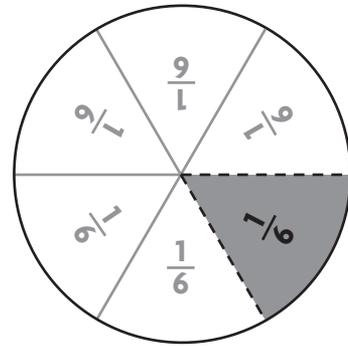
Step 3 Place the $\frac{1}{6}$ piece on the shaded angle. Turn it clockwise (in the direction that the hands on a clock move). Turn the fraction piece to line up directly beside the shaded section.

Step 4 Trace the fraction piece. Shade and label it. You have traced 2 sixths in all.

Step 5 Repeat until you have shaded the entire circle.

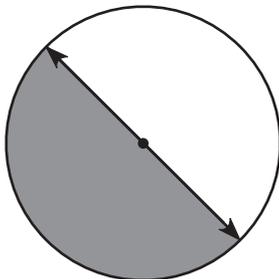
There are six angles that come together in the center of the circle.

So, you need six $\frac{1}{6}$ turns to make a circle.



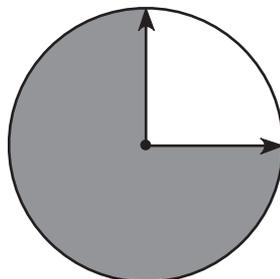
Tell what fraction of the circle the shaded angle represents.

1



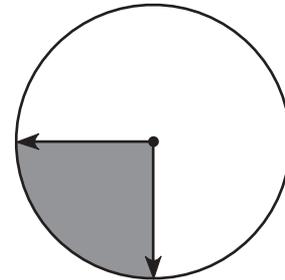
$\frac{1}{2}$

2



$\frac{3}{4}$

3

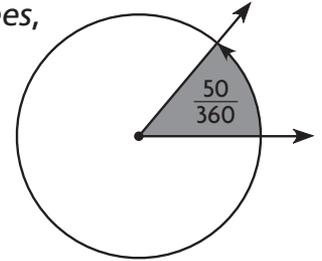


$\frac{1}{4}$

Degrees

Angles are measured in units called **degrees**. The symbol for degrees is $^\circ$. If a circle is divided into 360 equal parts, called *degrees*, then an angle that turns through 1 part of the 360 measures 1° .

An angle that turns through $\frac{50}{360}$ of a circle measures 50° .



Find the measure of an angle that turns through $\frac{1}{6}$ of a circle.

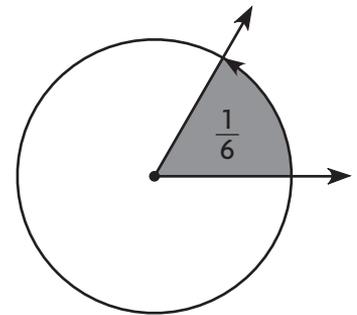
Step 1 Find a fraction that is equivalent to $\frac{1}{6}$ with 360 in the denominator. **Think:** $6 \times 60 = 360$.

$$\frac{1}{6} = \frac{1 \times 60}{6 \times 60} = \frac{60}{360}$$

Step 2 Look at the numerator of $\frac{60}{360}$.

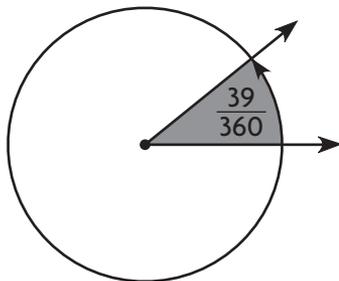
The numerator tells how many degrees are in $\frac{1}{6}$ of a circle.

So, an angle that turns through $\frac{1}{6}$ of a circle measures 60° .



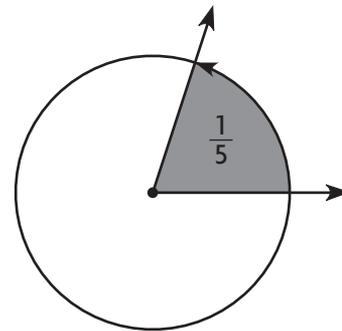
Tell the measure of the angle in degrees.

1



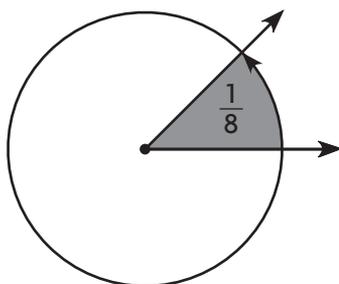
39°

2



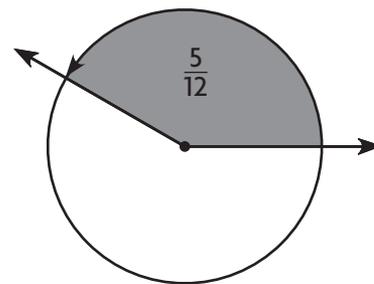
72°

3



45°

4



150°

Measure and Draw Angles

A **protractor** is a tool for measuring the size of an angle.

Follow the steps below to measure $\angle ABC$.

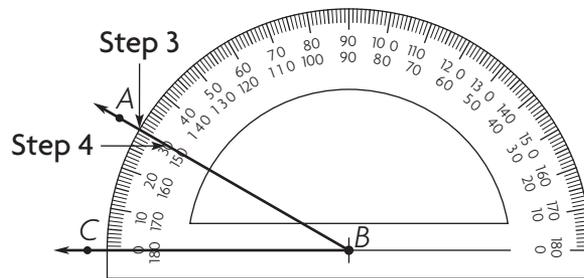
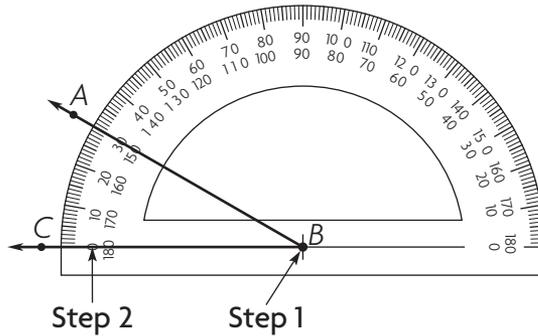
Step 1 Place the center point of the protractor on vertex B of the angle.

Step 2 Align the 0° mark on the protractor with ray BC . Note: For this angle, use the outer scale.

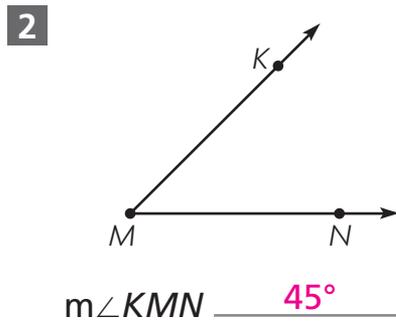
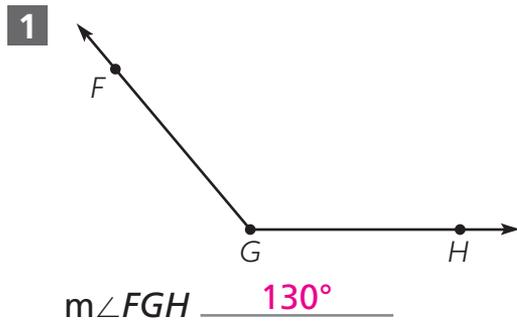
Step 3 Find where ray BA intersects the same scale.

Step 4 Read the angle measure on the scale.

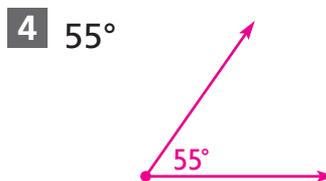
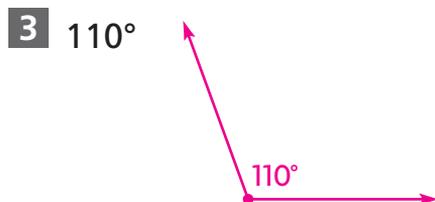
The $m\angle ABC = \underline{30^\circ}$.



Use a protractor to find the angle measure.



Use a protractor to draw the angle.



Check students' drawings.

Join and Separate Angles

The measure of an angle equals the sum of the measures of its parts.

Use your protractor and the angles at the right.

Step 1 Measure $\angle ABC$ and $\angle CBD$. Record the measures.

$$m\angle ABC = \underline{35^\circ}; m\angle CBD = \underline{40^\circ}$$

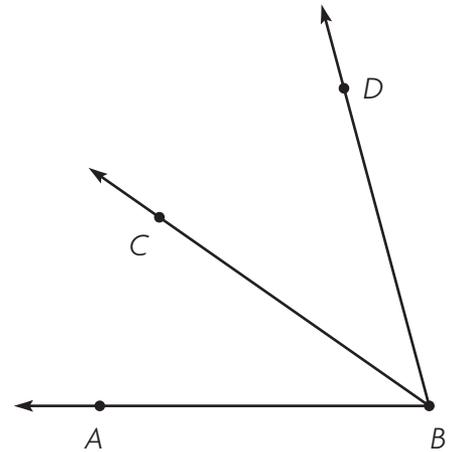
Step 2 Find the sum of the measures.

$$\underline{35^\circ} + \underline{40^\circ} = \underline{75^\circ}$$

Step 3 Measure $\angle ABD$. Record the measure.

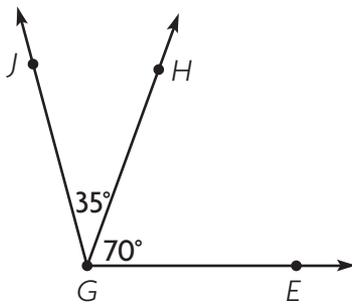
$$m\angle ABD = \underline{75^\circ}$$

So, $m\angle ABC + m\angle CBD = m\angle ABD$.



Add to find the measure of the angle. Write an equation to record your work.

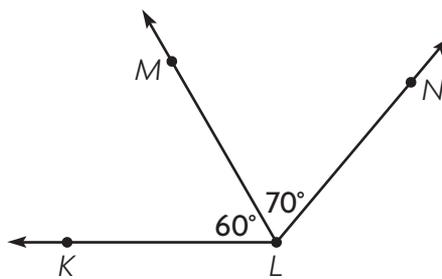
1



$$\underline{35^\circ + 70^\circ = 105^\circ}$$

$$m\angle EGJ = \underline{105^\circ}$$

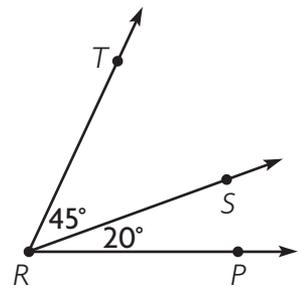
2



$$\underline{60^\circ + 70^\circ = 130^\circ}$$

$$m\angle KLN = \underline{130^\circ}$$

3



$$\underline{45^\circ + 20^\circ = 65^\circ}$$

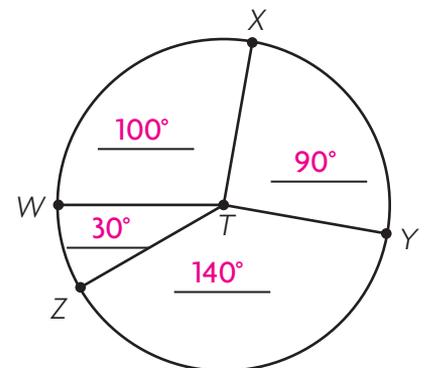
$$m\angle PRT = \underline{65^\circ}$$

Use a protractor and the art at the right.

4 Find the measure of each angle. Label each angle with its measure.

5 Write the sum of the angle measures as an equation.

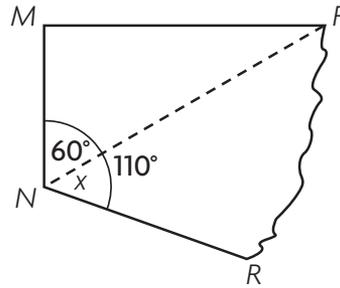
$$\underline{100^\circ + 90^\circ + 140^\circ + 30^\circ = 360^\circ}$$



Unknown Angle Measures

Use the strategy *draw a diagram*.

Mrs. Allen is cutting a piece of wood for a set for the school play. She needs a piece of wood with a 60° angle. After the cut, what is the angle measure of the part left over?



Read the Problem

What do I need to find?

I need to find the angle measure of the part left over, or $m\angle PNR$.

What information do I need to use?

I can use the angle measures I know:
 $m\angle MNP = 60^\circ$ and
 $m\angle MNR = 110^\circ$

How will I use the information?

I can draw a bar model to find the unknown angle measure, or $m\angle PNR$.

Solve the Problem

I can draw a bar model to represent the problem.

Then I can write an equation to solve the problem.

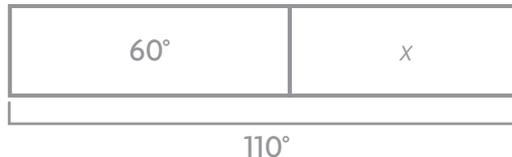
$$m\angle MNP + m\angle PNR = m\angle MNR$$

$$\underline{60^\circ} + x = \underline{110^\circ}$$

$$x = \underline{110^\circ} - \underline{60^\circ}, \text{ or } \underline{50^\circ}$$

$$\text{So, } m\angle PNR = \underline{50^\circ}$$

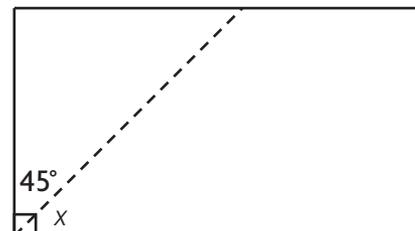
The angle measure of the part left over is 50° .



- 1** Cal is cutting a rectangular board as shown. What is the angle measure of the part left over? 45°

- 2** What equation did you use to solve?

Possible answer: $x + 45^\circ = 90^\circ$



Name _____

Measurement Benchmarks

You can use benchmarks to estimate measurements.

The chart shows benchmarks for customary units of measurement.

Benchmarks for Some Customary Units					
 1 ft about 1 foot	 1 yd about 1 yard	 about 1 cup	 about 1 gallon	 about 1 ounce	 about 1 pound

Here are some more examples of estimating with customary units.

- The width of a professional football is about 1 foot.
- A large fish bowl holds about 1 gallon of water.
- A box of cereal weighs about 1 pound.

The chart shows benchmarks for metric units of measurement.

Benchmarks for Some Metric Units					
 about 1 centimeter	 about 1 meter	 about 1 milliliter	 about 1 liter	 about 1 gram	 about 1 kilogram

Here are some more examples of estimating with metric units.

- The width of a large paper clip is about 1 centimeter.
- A pitcher holds about 1 liter of juice.
- Three laps around a track is about 1 kilometer.

Use benchmarks to choose the customary unit you would use to measure each.

1 length of a school bus

yard

2 weight of a computer

pound

Use benchmarks to choose the metric unit you would use to measure each.

3 the amount of liquid a bottle of detergent holds

liter

4 distance between two cities

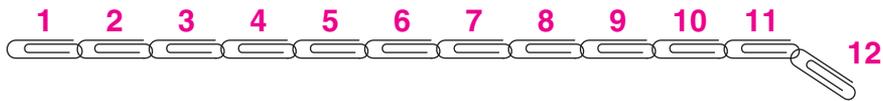
kilometer

Customary Units of Length

A ruler is used to measure length. A ruler that is 1 foot long shows 12 inches in 1 foot. A ruler that is 3 feet long is called a yardstick. There are 3 feet in 1 yard.

How does the size of a foot compare to the size of an inch?

Step 1 A small paper clip is about 1 inch long. Below is a drawing of a chain of paper clips that is about 1 foot long. Number each paper clip, starting with 1.



Step 2 Complete this sentence.

In the chain of paper clips shown, there are 12 paper clips.

Step 3 Compare the size of 1 inch to the size of 1 foot.

There are 12 inches in 1 foot.

So, 1 foot is 12 times as long as 1 inch.

Complete.

1 5 feet = 60 inches

2 3 yards = 9 feet

3 5 yards = 15 feet

4 4 feet = 48 inches

5 6 feet = 72 inches

6 8 yards = 24 feet

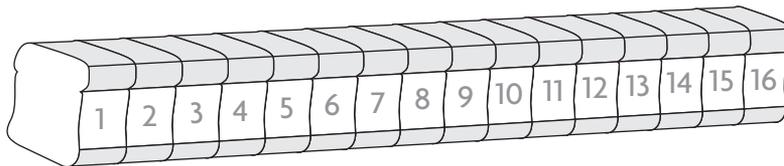
Customary Units of Weight

Ounces and **pounds** are customary units of weight. A **ton** is a unit of weight that is equal to 2,000 pounds.

A slice of bread weighs about 1 ounce. Some loaves of bread weigh about 1 pound.

How does the size of 1 ounce compare to the size of 1 pound?

Step 1 You know a slice of bread weighs about 1 ounce. Below is a drawing of a loaf of bread that weighs about 1 pound. Number each slice of bread, starting with 1.



Step 2 Complete this sentence.

In the loaf of bread shown above, there are 16 slices of bread.

Step 3 Compare the size of 1 ounce to the size of 1 pound.

There are 16 ounces in 1 pound.

So, 1 pound is 16 times as heavy as 1 ounce.

Complete.

1 2 pounds = 32 ounces

2 2 tons = 4,000 pounds

Think: $2 \times 16 = 32$

3 7 pounds = 112 ounces

4 4 pounds = 64 ounces

5 3 tons = 6,000 pounds

6 10 pounds = 160 ounces

Customary Units of Liquid Volume

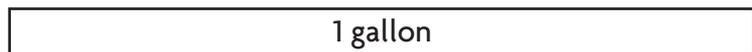
Liquid volume is the measure of the space a liquid occupies. Some basic units for measuring liquid volume are **gallons, half gallons, quarts, pints, cups,** and **fluid ounces**. The table at the right shows the relationships among some units of liquid volume.

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 half gallon = 2 quarts
1 gallon = 4 quarts

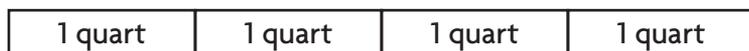
How does the size of a gallon compare to the size of a pint?

Step 1 Use the information in the table.

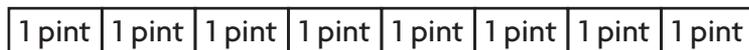
Draw a bar to represent
1 gallon.



Step 2 The table shows that 1 gallon is equal to 4 quarts. Draw a bar to show 4 quarts.



Step 3 The table shows that 1 quart is equal to 2 pints. Draw a bar to show 2 pints for each of the 4 quarts.



Step 4 Compare the size of 1 gallon to the size of 1 pint.

There are 8 pints in 1 gallon.

So, 1 gallon is 8 times as much as 1 pint.

Complete. Draw a model to help.

1 2 quarts = 4 pints

2 1 gallon = 16 cups

3 1 pint = 16 fluid ounces

4 3 pints = 6 cups

5 3 quarts = 12 cups

6 1 half gallon = 4 pints

Mixed Measures

Gabrielle's puppy weighs 2 pounds 7 ounces. What is the weight of the puppy in ounces?

Step 1 Think of 2 pounds 7 ounces as 2 pounds + 7 ounces.

Step 2 Change the pounds to ounces.

Think: 1 pound = 16 ounces

So, 2 pounds = 2×16 ounces, or 32 ounces.

Step 3 Add like units to find the answer.

$$\begin{array}{r} 32 \text{ ounces} \\ + 7 \text{ ounces} \\ \hline 39 \text{ ounces} \end{array}$$

So, Gabrielle's puppy weighs 39 ounces.

Gabrielle played with her puppy for 2 hours 10 minutes yesterday and 1 hour 25 minutes today. How much longer did she play with the puppy yesterday than today?

Step 1 Subtract the mixed measures. Write the subtraction with like units lined up.

Think: 25 minutes is greater than 10 minutes.

$$\begin{array}{r} 2 \text{ hr } 10 \text{ min} \\ - 1 \text{ hr } 25 \text{ min} \\ \hline \end{array}$$

Step 2 Rename 2 hours 10 minutes to subtract.

1 hour = 60 minutes

So, 2 hr 10 min = 1 hr + 60 min + 10 min, or 1 hr 70 min.

$$\begin{array}{r} 1 \quad 70 \\ \cancel{2} \text{ hr } \cancel{10} \text{ min} \\ - 1 \text{ hr } 25 \text{ min} \\ \hline 0 \text{ hr } 45 \text{ min} \end{array}$$

Step 3 Subtract like units.

1 hr - 1 hr = 0 hr; 70 min - 25 min = 45 min

So, she played with the puppy 45 minutes longer yesterday than today.

Complete.

1 4 yd 2 ft = 14 ft **2** 1 hr 20 min = 80 min **3** 4 qt 1 pt = 9 pt

Add or subtract.

4 2 gal 1 qt
+ 3 gal 2 qt

5 gal 3 qt

5 3 lb 12 oz
- 1 lb 8 oz

2 lb 4 oz

6 4 yr 9 mo
- 1 yr 10 mo

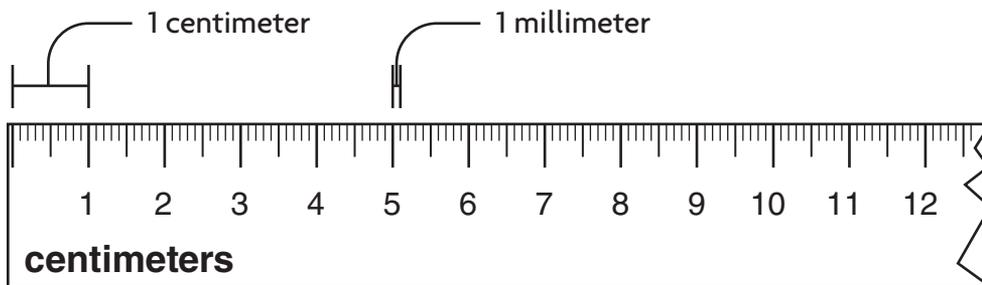
2 yr 11 mo

Metric Units of Length

Meters (m), **decimeters** (dm), centimeters (cm), and **millimeters** (mm) are all metric units of length. You can use a ruler and a meterstick to find out how these units are related.

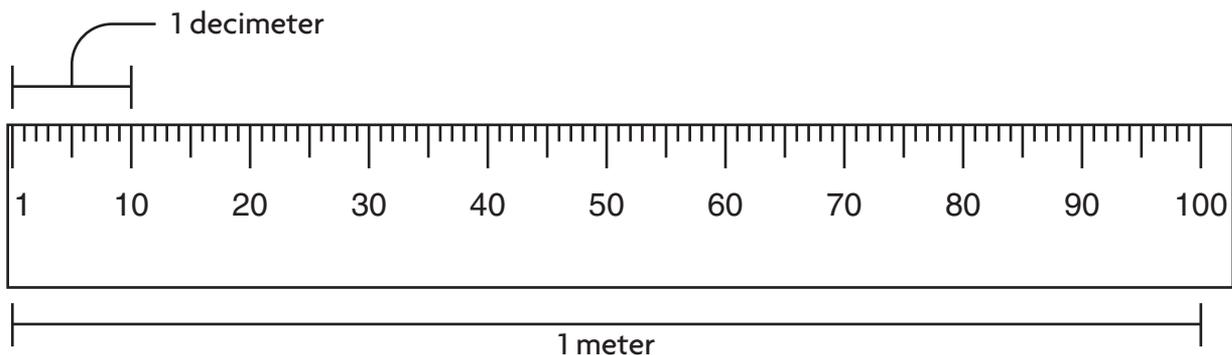
Materials: ruler, meterstick

Step 1 Look at a metric ruler. Most look like the one below.



The short marks between each centimeter mark show millimeters. 1 centimeter has the same length as a group of 10 millimeters.

Step 2 Look at a meterstick. Most look like the one below.



1 decimeter has the same length as a group of 10 centimeters.

Step 3 Use the ruler and the meterstick to compare metric units of length.

1 centimeter = 10 millimeters

1 decimeter = 10 centimeters

1 meter = 10 decimeters

1 meter = 100 centimeters

Complete.

1 3 meters = 30 decimeters

2 5 meters = 500 centimeters

3 4 centimeters = 40 millimeters

4 9 decimeters = 90 centimeters

Metric Units of Mass and Liquid Volume

Mass is the amount of matter in an object. Metric units of mass include grams (g) and kilograms (kg). 1 kilogram represents the same mass as 1,000 grams.

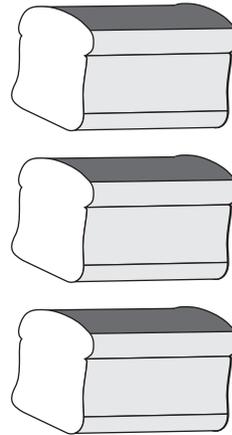
One large loaf of bread has a mass of about 1 kilogram. Jacob has 3 large loaves of bread. About how many grams is the mass of the loaves?

$$\begin{aligned} 3 \text{ kilograms} &= 3 \times \underline{1,000} \text{ grams} \\ &= \underline{3,000} \text{ grams} \end{aligned}$$

Liters (L) and milliliters (mL) are metric units of liquid volume. 1 liter represents the same liquid volume as 1,000 milliliters.

A large bowl holds about 2 liters of juice. Carmen needs to know the liquid volume in milliliters.

$$\begin{aligned} 2 \text{ liters} &= 2 \times \underline{1,000} \text{ milliliters} \\ &= \underline{2,000} \text{ milliliters} \end{aligned}$$



Complete.

1 4 kilograms = 4,000 grams

2 9 liters = 9,000 milliliters

3 3 liters = 3,000 milliliters

4 7 kilograms = 7,000 grams

5 5 kilograms = 5,000 grams

6 8 liters = 8,000 milliliters

Temperature

Temperature is measured in degrees with a thermometer. Fahrenheit ($^{\circ}\text{F}$) is the customary temperature scale. Celsius ($^{\circ}\text{C}$) is the metric temperature scale.

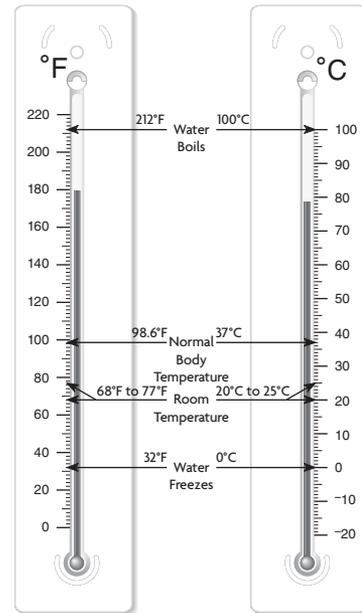
Use the thermometers at the right. Knowing some benchmark temperatures will help you estimate.

Which is a better estimate for a hot day, 95°C or 40°C ?

Step 1 Look at the benchmarks on the thermometer. Water boils at 100°C . Normal body temperature is 37°C .

Step 2 Which is more reasonable? Is the temperature of a hot day closer to the temperature when water boils or closer to normal body temperature? **closer to normal body temperature**

So, the better estimate for a hot day is 40°C .



Circle the best estimate of the temperature.



10°C **30°C** 85°C



15°F 55°F 80°F



0°C **15°C** 65°C

Units of Time

Some analog clocks have an hour hand, a minute hand, and a **second** hand.

There are 60 seconds in a minute. The second hand makes 1 full turn every minute. There are 60 minutes in an hour. The minute hand makes 1 full turn every hour. The hour hand makes 1 full turn every 12 hours.



You can think of the clock as unrolling to become a number line.



The hour hand moves from one number to the next in 1 hour.



The minute hand moves from one number to the next in 5 minutes.

Use the table at the right to change between units of time.

1 hour = 60 minutes, or 60×60 seconds, or 3,600 seconds.

So, 1 hour is 3,600 times as long as 1 second.

1 day = 24 hours, so 3 days = 3×24 hours, or 72 hours.

1 year = 12 months, so 5 years = 5×12 months, or 60 months.

Units of Time

1 minute = 60 seconds
 1 hour = 60 minutes
 1 day = 24 hours
 1 week = 7 days
 1 year = 12 months
 1 year = 52 weeks

Complete.

1 3 hours = 180 minutes

2 2 years = 104 weeks

3 6 days = 144 hours

4 5 weeks = 35 days

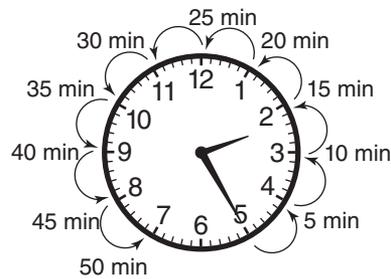
5 8 minutes = 480 seconds

6 7 years = 84 months

Elapsed Time

Opal finished her art project at 2:25 p.m. She spent 50 minutes working on her project. What time did she start working on her project?

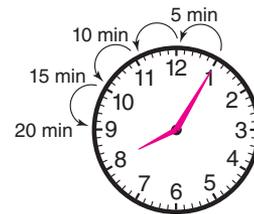
Read the Problem		
What do I need to find?	What information do I need to use?	How will I use the information?
I need to find Opal's start time.	End time: <u>2:25 p.m.</u> Elapsed time: <u>50</u> minutes	I can draw a diagram of a clock. I can then count back 5 minutes at a time until I reach 50 minutes.
Solve the Problem		
<p>I start by showing 2:25 p.m. on the clock. Then I count back 50 minutes by 5s.</p> <p>Think: As I count back, I go past the 12. The hour must be 1 hour less than the ending time. The hour will be <u>1 o'clock</u>.</p> <p>So, Opal started on her project at <u>1:35 p.m.</u></p>		



Draw hands on the clock to help you solve the problem.

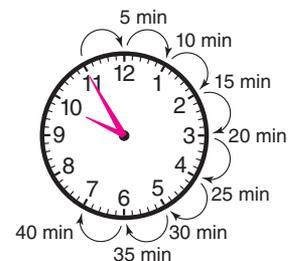
- 1** Bill wants to be at school at 8:05 a.m. It takes him 20 minutes to walk to school. At what time should Bill leave his house?

Bill should leave his house at 7:45 a.m.



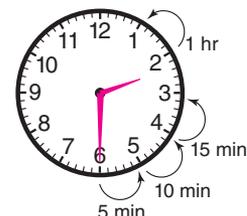
- 2** Mr. Gleason's math class lasts 40 minutes. Math class starts at 9:55 a.m. At what time does math class end?

Math class ends at 10:35 a.m.



- 3** Hannah rode her bike for 1 hour and 15 minutes until she got a flat tire at 2:30 p.m. What time did Hannah start riding her bike?

Hannah started riding her bike at 1:15 p.m.



Frequency Tables

A **frequency table** is a way to show how often each number in a set of numbers occurs. The first column of the table shows all of the different numbers in the set. The second column shows how many times each number occurs.

An example of a set of numbers is shown below. The numbers are quiz scores earned by the students in Ms. Coe's math class.

80 100 90 90 100 80
100 80 100 100 80 90

Math Quiz	
Score	Frequency
80	4
90	3
100	5

A frequency table of the quiz scores is shown at the right. Complete the first column of the table to show that the scores were 80, 90, and 100. Then complete the second column to show that there were 4 scores of 80, 3 scores of 90, and 5 scores of 100.

The quiz scores for ten students in Mr. Domingo's social studies class are shown below. Use the scores to complete the frequency table at the right.

90 90 95 95 95 95 95 100 100 100

- How many times is a score of 90 present?
- What score is present 5 times?
- How many times is a score of 100 present?

Social Studies Quiz	
Score	Frequency
90	2
95	5
100	3

Use Frequency Tables

A **frequency table** is a way to show how often each number in a set of numbers occurs. The first column of a frequency table shows all of the different numbers in the set. The second column shows how many times each number occurs.

The table at the right shows the length of time it usually takes each student in a class to travel to school in the morning. How many more students take $\frac{1}{6}$ hour to travel than take $\frac{1}{12}$ hour?

Morning Travel Time	
Time (in hours)	Frequency
$\frac{1}{12}$	5
$\frac{1}{6}$	7
$\frac{1}{4}$	4

Read	Solve
<p>What do I need to find?</p> <p>How many more students take $\frac{1}{6}$ hour to travel to school than take $\frac{1}{12}$ hour?</p>	<p>In the table, find the number of students who take $\frac{1}{6}$ hour and $\frac{1}{12}$ hour.</p> <p>Think: 7 students take $\frac{1}{6}$ hour. 5 students take $\frac{1}{12}$ hour.</p> <p>Subtract the number of students who take $\frac{1}{12}$ hour from the number who take $\frac{1}{6}$ hour.</p> $7 - 5 = 2$ <p>So, 2 more students take $\frac{1}{6}$ hour to travel than take $\frac{1}{12}$ hour.</p>
<p>What information am I given?</p> <p>data about the lengths of time it takes all of the students to travel</p>	
<p>Plan</p>	
<p>What is my plan or strategy?</p> <p>Subtraction is used to compare. I will subtract the number of students who take $\frac{1}{12}$ hour from the number of students who take $\frac{1}{6}$ hour.</p>	

- 1** How many more students take $\frac{1}{12}$ hour to travel than take $\frac{1}{4}$ hour?

1 more student

- 2** How many fewer students take $\frac{1}{4}$ hour to travel than take $\frac{1}{6}$ hour?

3 fewer students

Determine Mode, Median, and Range

Median	Mode	Range
The median is the middle value when the data are arranged in order. If there are two middle values, add them and then divide by 2.	The mode is the value or values that occur most often. A set of data can have more than one mode or no mode.	The range is the difference between the greatest value and least value in a set of data.

The median, mode, and range can be used to describe a set of data.

Step 1 Order the data values from least to greatest.

16, 16, 18, 18, 20, 22, 23

Step 2 Find the median. 16, 16, 18, **18**, 20, 22, 23

- Find the **middle** value. The median is **18**.

About half of Jim's friends did more than **18** push-ups; about half of his friends did fewer than **18** push-ups.

Step 3 Find the mode. 16, 16, **18, 18**, 20, 22, 23

- Find the value(s) that **occur most often**. The modes are **16** and **18**.

More friends did **16** or **18** push-ups.

Step 4 Find the range. 16, 16, 18, 18, 20, 22, 23

- Subtract the **least** value from the **greatest** value. $23 - 16 = 7$

The range is **7**. The spread of the data is **7** push-ups.

Jim's Friends	Push-Ups
Ernie	18
Max	22
Ben	23
Luz	16
Jess	18
Sara	16
Vika	20

Find the median, mode, and range.

1 Bowling scores: 92, 56, 80, 62, 66, 59, 100, 58

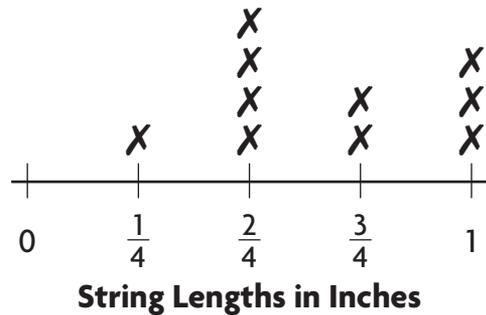
median: 64 mode: no mode range: 44

Line Plots

A **line plot** is a graph that shows data above a number line.

Xenia measures the length of some strings. She records the data in a line plot. How many strings have a length of $\frac{2}{4}$ inch?

- Locate $\frac{2}{4}$ on the number line.
- Count the number of Xs above $\frac{2}{4}$.



Four strings have a length of $\frac{2}{4}$ inch.

Use the line plot above.

- | | |
|--|---|
| <p>1 How many strings does Xenia measure?
<u>10 strings</u></p> | <p>2 What is the length of the shortest string?
<u>$\frac{1}{4}$ inch</u></p> |
| <p>3 How many more strings have a length of 1 inch than a length of $\frac{3}{4}$ inch?
<u>1 more string</u></p> | <p>4 How many strings have a length of less than 1 inch?
<u>7 strings</u></p> |
| <p>5 How many fewer strings have a length of $\frac{1}{4}$ inch than a length of $\frac{2}{4}$ inch?
<u>3 fewer strings</u></p> | <p>6 Which length of string appears most often?
<u>$\frac{2}{4}$ inch</u></p> |
| <p>7 Explain why there are no Xs above the 0 on the line plot.
<u>Possible explanation: The 0 on the line plot represents a string 0 inches long. A string cannot be 0 inches long.</u></p> | |

Use Line Plots

A **line plot** shows the frequency of data along a number line. The table shows the heights of the plants in Taylor's garden.

Heights (in yards)
$\frac{3}{4}$, $1\frac{1}{8}$, $\frac{1}{2}$, $1\frac{1}{8}$, $\frac{3}{4}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{1}{4}$

Make a line plot to display the data.

- Make a tally table. Order the different heights from least to greatest.
- Make a tally mark for each plant of each height.
- Draw a number line and label the fraction lengths. Write a title and label the units.
- Plot an X above the number line for each data point. For example, one plant is $\frac{1}{4}$ yard tall. So, draw one X above $\frac{1}{4}$.

Plant Heights	
Heights (in yards)	Tally
$\frac{1}{4}$	
$\frac{1}{2}$	
$\frac{3}{4}$	
$\frac{7}{8}$	
$1\frac{1}{8}$	

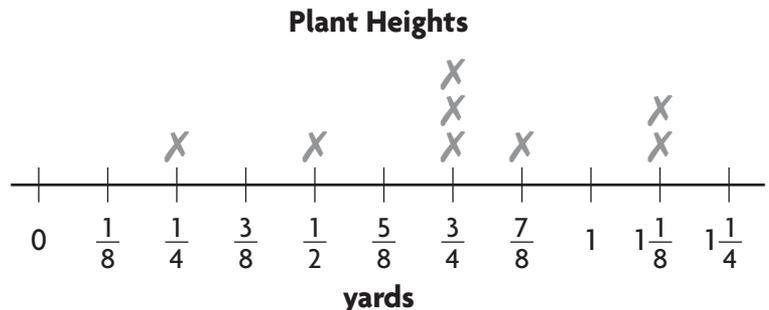
1 What is the most common height? $\frac{3}{4}$ yard

2 What is the height of the shortest plant? $\frac{1}{4}$ yard

3 What is the height of the tallest plant? $1\frac{1}{8}$ yards

4 How much taller is the most common height plant than the shortest plant? Write an equation and solve.

Possible equation: $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$; $\frac{2}{4}$ yard or $\frac{1}{2}$ yard



Stem-and-Leaf Plots

A **stem-and-leaf** plot is a frequency display that shows groups of whole numbers arranged by place value. Look at the stem-and-leaf plot at the right. The first row has a stem of 4 and a leaf of 8, and represents the number 48.

Stem	Leaves
4	8
5	1 1 7

Think: 4 | 8 represents 48.

The numbers displayed by a stem-and-leaf plot are always arranged from least to greatest. The leaves column displays the ones digits of the numbers. The stem column displays all of the other place values.

The second row of the stem-and-leaf plot above represents three numbers because it has three leaves. What three numbers does the second row represent? (Hint: Two numbers are the same.)

51, 51, and 57

The list at the right shows the number of students in each of ten classes at Carter Elementary School. Use the data for 1–4 and complete the stem-and-leaf plot.

Number of Students				
18	20	21	19	19
19	17	22	16	21

- Write the numbers in order from least to greatest.
16, 17, 18, 19, 19, 19, 20, 21, 21, 22
- Write the tens digit of the numbers 16, 17, 18, 19, 19, and 19 in the Stem column of the first row.
- Write the tens digit of the numbers 20, 21, 21, and 22 in the Stem column of the second row.
- Write the ones digits in the Leaves column of each row.

Number of Students	
Stem	Leaves
1	6 7 8 9 9 9
2	0 1 1 2

Key: 1 | 6 represents 16 students.

Use Stem-and-Leaf Plots

A **stem-and-leaf** plot is a frequency display that shows groups of whole numbers arranged by place value, and ordered from least to greatest. The leaves column displays the ones digits of the numbers. The stem column displays all of the other digits.

A girls fifth- and sixth-grade basketball team plays games every season. The number of points the team scored in each game last season is shown in the stem-and-leaf plot at the right.

How many times did the team score more than 20 points?

Think: 20 is represented by 2 | 0.

The points scored that are more than 20 are

21, 23, 26, 27, 30.

So, the team scored more than 20 points 5 times.

Points Scored Last Season

Stem	Leaves
0	9
1	2 4 5 8 8 9
2	1 3 6 7
3	0

Key: 2 | 1 represents 21 points.

Use the stem-and-leaf plot for 1–4.

1 How many times did the team score fewer than 15 points?

3 times

2 How many times was the number of points scored a 2-digit number?

11 times

3 What number of points was scored most often by the team?

18 points

4 How many games did the team play last season? **Explain.**

12 games; Possible explanation: each leaf represents 1 game. There are 12 leaves.