Alg 2 Honors

Garcia

Week 3 & 4

2

9.1 Inverse Variation & 2.3 Direct Variation

Inverse Variation – as one quantity increases, the other decreases

$$y = \frac{k}{x}$$

$$k \neq 0$$

So k = xy

'k' is the constant of variation

Direct Variation – when one quantity increases, the other increases

a linear equation in the form y = kx where k cannot = 0

$K = \frac{y}{x}$

Ex Direct, inverse, or neither?

x	y
3	0.7
6	0.35
21	0.1

x	y
-2	6
-1.3	5
7	-4

x	y
-2	5
4	-10
6	-15

- Your math class has decided to pick up litter each weekend in a local park. Each week there is approximately the same amount of litter. The table shows the number of students who worked each of the first four weeks of the project and the time needed for the pickup. Write a function to model this data.

# of students (n)	3	5	17
time in minutes (t)	85	51	15

- How many students should there be to complete the project in at most 30 minutes?

Ex

Suppose that x and y vary inversely. If $x = 7$ and $y = 4$, write a function.

Are these direct variations?

- $3y = 7x + 7$
- $5x = 2y$

Ex

A dripping faucet wastes a cup of water if it drips for three minutes. The amount of water wasted varies directly with the amount of time the faucet drips. Write an equation.

How long must it drip to waste 4.5 cups?

Combined variation – when one quantity varies with respect to two or more quantities.

- y varies directly with the square of x: $y = kx^2$
- y varies inversely with the cube of x: $y = \frac{k}{x^3}$
- z varies jointly with x and y and inversely with w: $z = \frac{kxy}{w}$
- z varies directly with x and inversely with the product of w and y: $z = \frac{kx}{wy}$

Ex

Mass m of a moving object is related to its kinetic energy k and its velocity v by $m = 2k/v^2$. Describe the relationship using combined variation.

Ex

Describe using combined variation: $A = \frac{1}{2}h(b_1 + b_2)$

EX

- Z varies directly as x and inversely as the square of y. When x = 35, y = 7, and z = 50, write a function and find z when x = 5 and y = 10.

- The number of bags of grass seed n needed to reseed a yard varies directly with the area a to be seeded and inversely with the weight w of a bag of seed. If it takes two 3-lb bags to seed an area of 3600 ft^2, how many 3-lb bags will seed 9000 ft^2?

9.1 & 2.3 VARIATION

$y = Kx$

Inverse Variation – as one quantity increases, the other decreases

$$y = \frac{k}{x}$$

$$k \neq 0$$

So k = xy

'k' is the constant of variation

as you get taller, your weight increases. Both go ↑

Direct Variation – when one quantity increases, the other increases

a linear equation in the form y = kx where k cannot = 0

$K = \frac{y}{x}$ $K \neq 0$

as your speed increases the time it takes you to get somewhere decreases. One goes ↑ one goes ↓

Find K' for each point - must be the same

Ex Direct, inverse, or neither?

$K = \frac{y}{x}$ $K = xy$

x	y
3	0.7
6	0.35
21	0.1

dir: .23, .05, stop; inv: 2.1, 2.1, 2.1 — inv $y = \frac{2.1}{x}$

x	y
-2	6
-1.3	5
7	-4

Neither

x	y
-2	5
4	-10
6	-15

$K = \frac{y}{x}$: -2.5, -2.5, -2.5 — Direct $y = -\frac{5}{2}x$

Inverse

- Your math class has decided to pick up litter each weekend in a local park. Each week there is approximately the same amount of litter. The table shows the number of students who worked each of the first four weeks of the project and the time needed for the pickup. Write a function to model this data.

# of students (n) x	3	5	17
time in minutes (t) y	85	51	15

$y = \frac{255}{x}$

- How many students should there be to complete the project in at most 30 minutes?

$y = \frac{255}{30} = 8.5$ so 9 students

Ex

Suppose that x and y vary inversely. If x = 7 and y = 4, write a function.

$y = \frac{K}{x}$ $4 = \frac{K}{7}$

$K = 28$

$y = \frac{28}{x}$

Are these direct variations?

- 3y = 7x + 7 NO
- 5x = 2y yes $y = \frac{5}{2}x$

- The equation is y = Kx + 0 - there cannot be a constant if it is a direct variation

9.1 & 2.3 VARIATION (time, cups) (3, 1) $K = \frac{1}{3}$

You must Know these words

Ex $y = Kx$

A dripping faucet wastes a cup of water if it drips for three minutes. The amount of water wasted varies directly with the amount of time the faucet drips. Write an equation

$y = \frac{1}{3}x$

How long must it drip to waste 4.5 cups?

$\frac{9}{2} = \frac{1}{3}x$

$x = \frac{27}{2}$ min

We will never talk @ the #

Combined variation – when one quantity varies with respect to two or more quantities.

- y varies directly with the square of x: $y = kx^2$
- y varies inversely with the cube of x: $y = \frac{k}{x^3}$
- z varies jointly with x and y and inversely with w. $z = \frac{kxy}{w}$
- z varies directly with x and inversely with the product of w and y. $z = \frac{kx}{wy}$

jointly is when there is a product in the numerator

Ex

Mass m of a moving object is related to its kinetic energy k and its velocity v by $m = 2k/v^2$. Describe the relationship using combined variation.

m varies directly with K & inversely with the square of v (or v^2)

Ex

Describe using combined variation: $A = \frac{1}{2}h(b_1 + b_2)$

A varies jointly with h & the sum of b_1 & b_2

EX

- Z varies directly as x and inversely as the square of y. When x = 35, y = 7, and z = 50, write a function and find z when x = 5 and y = 10

① $Z = \frac{Kx}{y^2}$

② find K $50 = \frac{35K}{49}$ $K = 70$

③ $Z = \frac{70x}{y^2}$

④ use red equation to answer question

$Z = \frac{70(5)}{100}$ = 3.5

- The number of bags of grass seed n needed to reseed a yard varies directly with the area a to be seeded and inversely with the weight w of a bag of seed. If it takes two 3-lb bags to seed an area of 3600 ft², how many 3-lb bags will seed 9000 ft²?

① $n = \frac{Ka}{w}$

② $2 = \frac{3600K}{3}$ $K = \frac{1}{600}$

③ $n = \frac{\frac{1}{600}a}{w}$ ← no ok so simplify

$\frac{a}{600} \div w$ $\frac{a}{600} \cdot \frac{1}{w}$

$n = \frac{a}{600w}$

③ $n = \frac{9000}{600(3)}$ = 5 bags

9.4 Rational Expressions

The quotient of two polynomials.

Simplest form:

- When the numerator and denominator are polynomials with NO common divisors.

Ex 1 Simplify:

$$\frac{36}{16} \bullet \frac{10}{4}$$

$$\frac{2xy}{5x^2z^3} \bullet \frac{10x^5}{14y^9}$$

Ex 2 Simplify.

$$\frac{x^2-6x-16}{x^2+5x+6}$$

Ex 3

Multiply, state any restrictions on the variables.

$$\frac{3x^2+5x-2}{x-5} \bullet \frac{x^2-25}{3x^2-7x+2}$$

******Simplify******

$$\frac{r-s}{r-s} \qquad \frac{r-s}{s-r}$$

$$\frac{r+s}{s+r} \qquad \frac{r-s}{r+s}$$

Ex 4

Divide, state any restrictions on the variables.

$$\frac{3-y}{2x^2+9x-5} \div \frac{6y-18}{2x^2-15x+7}$$

The width of a rectangle is $\frac{a+10}{3a+24}$. The area of the rectangle is $\frac{2a+20}{3a+15}$. Find the length in simplest form.

Simplify: $\dfrac{\dfrac{x^2-1}{x^2-9}}{\dfrac{x^2+3x-4}{x^2+8x+15}}$

9.5 Adding & Subtracting Rational Expressions

$$\frac{2}{x}+\frac{3}{x}=$$

Ex 1

$$\frac{4}{3x}+\frac{2}{3x}=$$

Ex 2

$$\frac{3c}{2c-1}-\frac{5c+1}{2c-1}=$$

Ex 3

- Find the LCM of $2x^2 - 8x + 8$ and $15x^2 - 60$
- Find prime factors of each expression.
- Write each prime factor the greatest number of times it appears in either expression.

Ex 4

- Find the LCM of:

$5x^2 + 15x + 10$ and $2x^2 - 8$

Ex 5

$$\frac{1}{3x^2+21x+30}+\frac{4x}{3x+15}$$

Ex 6
Simplify

$$\frac{2x}{x^2-2x-3}-\frac{3}{4x+4}$$

Ex 7
simplify

Complex fractions

- Has a fraction in the numerator, denominator, or both.

Ex 8

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{2}{y}-\frac{1}{x}}$$

Ex 9

$$\frac{\frac{x-2}{x}-\frac{2}{x+1}}{\frac{3}{x-1}-\frac{1}{x+1}}$$

Write two rational expressions that simplify to $\frac{x+1}{x-5}$

Need common denominators

9.5 Adding & Subtracting Rational Expressions

Ex 1

$\frac{2}{x}+\frac{3}{x}=$ $\frac{5}{x}$

Ex 2

$\frac{4}{3x}+\frac{2}{3x}=$ $\frac{6}{3x}$

$\frac{2}{x}$

Ex 3

$\frac{3c}{2c-1}-\frac{5c+1}{2c-1}=$

$\frac{3c-(5c+1)}{2c-1}=\frac{3c-5c-1}{2c-1}$

$\frac{-2c-1}{2c-1}$ not opposites

Ex 4

- Find the LCM of $2x^2 - 8x + 8$ and $15x^2 - 60$
- Find prime factors of each expression.
- Write each prime factor the greatest number of times it appears in either expression.

$2(x-2)(x-2)$

$15(x-2)(x+2)$

common denom =

$30(x-2)^2(x+2)$

b/c there's two in blue poly

Ex 5

- Find the LCM of:

$5x^2 + 15x + 10$ and $2x^2 - 8$

$5(x+2)(x+1)$

$2(x-2)(x+2)$

$10(x+2)(x-2)(x+1)$

$$\frac{1}{3x^2+21x+30}+\frac{4x}{3x+15}$$

$3(x+5)(x+2) \quad 3(x+5)(x+2)$

$$\frac{1+4x^2+8x}{3(x+5)(x+2)}$$

Ex 6 Simplify

$$\frac{4x^2+8x+1}{3(x+5)(x+2)}$$

$$\frac{2x}{x^2-2x-3}-\frac{3}{4x+4}$$

$4 \cdot (x-3)(x+1) \quad 4(x+1)(x-3)$

$$\frac{8x-3x+9}{4(x+1)(x-3)}$$

Ex 7 simplify

$$\frac{5x+9}{4(x+1)(x-3)}$$

- Has a fraction in the numerator, denominator, or both.

Complex fractions

$$\frac{\frac{1}{x}+\frac{1}{y}}{\frac{2}{y}-\frac{1}{x}}$$

$$\frac{\frac{y+x}{xy}}{\frac{2x-y}{xy}}$$

Ex 8

$$\frac{y+x}{2x-y}$$

$$\frac{\frac{x-2}{x}-\frac{2}{x+1}}{\frac{3}{x-1}-\frac{1}{x+1}}$$

$$\frac{\frac{x^2-x-2-2x}{x(x+1)}}{\frac{3x+3-x+1}{(x-1)(x+1)}}$$

Ex 9

$$\frac{x^2-3x-2}{x(x+1)} \div \frac{2x+4}{(x-1)(x+1)}$$

$$\frac{x^2-3x-2}{x(x+1)} \cdot \frac{(x-1)(x+1)}{2x+4}$$

$$\frac{(x^2-3x-2)(x-1)}{x(2x+4)}$$

Write two rational expressions that simplify to $\frac{x+1}{x-5}$

9.6 SOLVING RATIONAL EQUATIONS & INEQUALITIES

9.6 Solving Rational Equations

AN EQUATION THAT CONTAINS AT LEAST ONE RATIONAL EXPRESSION.

- An extraneous solution can be introduced when multiplying both sides of an equation by the same algebraic expression.
- An extraneous solution is a solution of the derived equation but not the original.
- CHECK for extraneous solutions.

Ex 1 $\frac{1}{x-3}=\frac{6x}{x^2-9}$

Ex 2 $\frac{3}{5x}-\frac{4}{3x}=\frac{1}{3}$

Ex 3

$$\frac{4}{x} - \frac{3}{x+1} = 1$$

Ex 4

$$r + \frac{r^2 - 5}{r^2 - 1} = \frac{r^2 + r + 2}{r + 1}$$

Flight A flight across the U.S. takes longer east to west than it does west to east. Assume that winds are constant in the eastward direction. When flying westward, the headwind decreases the airplane's speed. When flying eastward, the tailwind increases its speed. The time for a round trip shown at the right is $7\frac{3}{4}$ h. If the airplane cruises at 480 mi/h, what is the speed of the wind?

Ex 5

- Rose can jog 5 mi downhill in the same time it takes her to jog 3 mi uphill. She jogs downhill 4mi/h faster than she jogs uphill. Find her jogging rate each way.

Ex 6

- Jim and Al have to paint 6000 square feet of hallway in an office building. Al works twice as fast as Jim. Working together, they can complete the job in 15 hours. How long would it take each of them working alone?

Ex 7

- George can mow his lawn in 4 hours. Kate can do it in 3 hours. How long will it take them to do the job together?

Rational Inequalities

1. State the excluded values.
2. Solve the related equation.
3. Use the values from steps 1 & 2 to divide a number line into regions. Test the values in each region to see which satisfy the original inequality.

Ex 8

$$8+\frac{3}{y}>\frac{19}{y}$$

Ex 9

$$\frac{1}{4a}+\frac{5}{8a}>\frac{1}{2}$$

ALGEBRA 2 HONORS WS 9.1, 2.3, 9.4 **Name:**________________________

Is the relationship a direct variation, inverse variation or neither? Write equations to model the direct and inverse variations.

1. (1, 1) (2, 4) (3, 9)
2. (-1, -3) (1, 3) (3, 9)
3. (-2, -3) (6, 1) (4,1.5)

Describe the combined variation:

4. $I = \frac{2xw}{yz}$
5. $h = \frac{3x}{y^2}$
6. $V = \frac{1}{3}\pi r^2 h$

Write the function that models the relationship.

7. z varies jointly with x an y. When x = 7 and y = 3, z = 28. Find z when x = 6 and y = 4.

8. Suppose z varies directly as the 4th power of x and inversely as the cube of y when x = 1, y = 1 and z = 3. Find z when x = 3 and y = 3.

9. A 15-minute long distance phone call costs \$0.90. The cost varies directly as the length of the call. Write an equation that relates the cost to the length of the call and find out how long is a call that costs \$1.32?

Write in simplest form and state any restrictions.

10. $\frac{x^2-5x+4}{x^2-1} \bullet \frac{x^2+5x+4}{x^2-9}$

11. $\frac{x^2-4}{x^2+6x+9} \bullet \frac{x^2-9}{x^2+4x+4}$

12. $\frac{x^2+10x+16}{x^2-6x-16} \div \frac{x+8}{x^2-64}$

13. $\frac{6x^2-32x+10}{3x^2-15x} \div \frac{3x^2+11x-4}{2x^2-32}$

14. $\frac{x^3+8}{x-2} \bullet \frac{x^2-4x+4}{x^2-2x+4}$

15. $\frac{3x^2-2xy+6x-4y}{3x^2+xy-2y^2} \div \frac{x^2-4}{x-2}$

16. $\frac{x^2+3x}{x^2+6x+8} \bullet \frac{-(x^2+x-2)}{4x^3+12x^2}$

17. $\frac{3x-12}{2x^2-8x} \div \frac{x^2+x-6}{x^3-4x}$

18. $\frac{9-a^2}{a^2+5a+6} \div \frac{2a-6}{5a+10}$

19. $\frac{c^2-3c}{c^2-25} \bullet \frac{c^2+4c-5}{c^2-4c+3}$

Name ______________________ Class ______________ Date ______________

Practice 9-5

Adding and Subtracting Rational Expressions

Find the least common multiple of each pair of polynomials.

1. $3x(x + 2)$ and $6x(2x - 3)$
2. $2x^2 - 8x + 8$ and $3x^2 + 27x - 30$
3. $4x^2 + 12x + 9$ and $4x^2 - 9$
4. $2x^2 - 18$ and $5x^3 + 30x^2 + 45x$

Simplify.

5. $\frac{x^2}{5} + \frac{x^2}{5}$
6. $\frac{x^2 - 2}{12} + \frac{x}{6}$
7. $\frac{12}{xy^3} - \frac{9}{xy^3}$
8. $-\frac{2}{n + 4} - \frac{n^2}{n^2 - 16}$
9. $\frac{x}{9} - \frac{2x}{9}$
10. $\frac{2y + 1}{3y} + \frac{5y + 4}{3y}$
11. $\frac{6y - 4}{y^2 - 5} + \frac{3y + 1}{y^2 - 5}$
12. $\frac{6}{5x^2y} + \frac{5}{10xy^2}$
13. $\frac{3}{8x^3y^3} - \frac{1}{4xy}$
14. $\frac{4}{x^2 - 25} + \frac{6}{x^2 + 6x + 5}$
15. $\frac{3}{7x^2y} + \frac{4}{21xy^2}$
16. $\frac{xy - y}{x - 2} - \frac{y}{x + 2}$
17. $\frac{x + 2}{x^2 + 4x + 4} + \frac{2}{x + 2}$
18. $\frac{3}{x^2 - x - 6} + \frac{2}{x^2 + 6x + 5}$
19. $\frac{1}{6x^2 - 11x + 3} + \frac{1}{8x^2 - 18}$
20. $\frac{4}{x^2 - 3x} + \frac{6}{3x - 9}$
21. $\frac{3}{x^2 + 3x - 10} + \frac{1}{x^2 + 6x + 5}$
22. $\frac{3}{x - 9} + 4x$
23. $3 - \frac{1}{x^2 + 5}$
24. $5 + \frac{1}{x^2 - 5x + 6}$
25. $1 + \frac{2x + 7}{3x - 1}$
26. $\frac{2a}{a + 2} + \frac{3a}{a - 2}$
27. $\frac{4c}{c - 3} + \frac{4c}{c + 3}$
28. $\frac{f + 1}{fgh} + \frac{f - 1}{fgh}$
29. $\frac{2 - t}{t - 5} + \frac{2 + t}{t + 5}$
30. $\frac{4r}{r - 2} + \frac{4r}{r + 2}$
31. $\frac{x - y}{x + y} + \frac{y}{x}$
32. $\frac{\frac{2}{x}}{\frac{3}{y}}$
33. $\frac{1 + \frac{2}{x}}{4 - \frac{6}{x}}$
34. $\frac{\frac{1}{x - 2}}{2 + \frac{1}{x}}$
35. $\frac{y}{4y + 8} - \frac{1}{y^2 + 2y}$
36. $\frac{1 + \frac{2}{3}}{\frac{4}{9}}$
37. $\frac{6x^2}{3x - 2} + \frac{5x - 6}{3x - 2}$
38. $\frac{\frac{3}{x + 1}}{\frac{5}{x - 1}}$
39. $\frac{\frac{2}{x} + 6}{\frac{1}{y}}$
40. $\frac{2y}{y^2 - 4y - 12} + \frac{y}{y^2 - 10y + 24}$

41. The total resistance for a parallel circuit is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$.
   a. If $R = 1$ ohm, $R_2 = 6$ ohms, and $R_3 = 8$ ohms, find R_1.
   b. If $R_1 = 3$ ohms, $R_2 = 4$ ohms, and $R_3 = 6$ ohms, find R.

NAME ______________________ DATE __________ PERIOD _____

9-6 Practice

Solving Rational Equations and Inequalities

Solve each equation or inequality. Check your solutions.

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$

2. $\frac{x}{x-1} - 1 = \frac{x}{2}$

3. $\frac{p+10}{p^2-2} = \frac{4}{p}$

4. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$

5. $\frac{5}{y-5} = \frac{y}{y-5} - 1$

6. $\frac{1}{3x-2} + \frac{5}{x} = 0$

7. $\frac{5}{t} < \frac{9}{2t+1}$

8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$

9. $\frac{4}{w-2} = \frac{-1}{w+3}$

10. $5 - \frac{3}{a} < \frac{7}{a}$

11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$

12. $8 + \frac{3}{y} > \frac{19}{y}$

13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$

14. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$

15. $g + \frac{g}{g-2} = \frac{2}{g-2}$

16. $b + \frac{2b}{b-1} = 1 - \frac{b-3}{b-1}$

17. $2 = \frac{x+2}{x-3} + \frac{x-2}{x-6}$

18. $5 - \frac{3d+2}{d-1} = \frac{2d-4}{d+2}$

19. $\frac{1}{n+2} + \frac{1}{n-2} = \frac{3}{n^2-4}$

20. $\frac{c+1}{c-3} = 4 - \frac{12}{c^2-2c-3}$

21. $\frac{3}{k-3} + \frac{4}{k-4} = \frac{25}{k^2-7k+12}$

22. $\frac{4v}{v-1} - \frac{5v}{v-2} = \frac{2}{v^2-3v+2}$

23. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$

24. $\frac{x^2+4}{x^2-4} + \frac{x}{2-x} = \frac{2}{x+2}$

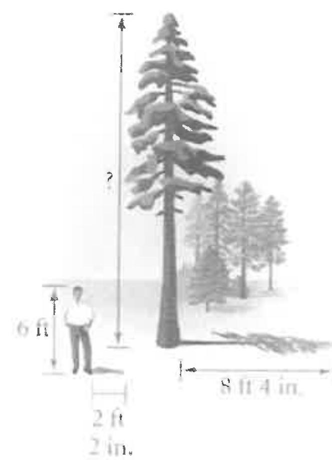
25. $\frac{r}{r+4} + \frac{4}{r-4} = \frac{r^2+16}{r^2-16}$

26. $3 = \frac{6a-1}{2a+7} + \frac{22}{a+5}$

27. BASKETBALL Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next x free throws in a row, the function $f(x) = \frac{9+x}{19+x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%?

28. OPTICS The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance p of an object from a lens, the distance q of the image of the object from the lens, and the focal length f of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters?

23. **Environment** Suppose you work on a tree farm and you need to find the height of each tree. You know that the length of an object's shadow varies directly with its height. Refer to the diagram.
 a. Find the constant of variation.
 b. Write an equation to calculate the height of the tree.
 c. Find the height of a tree with a shadow 8 ft 4 in. long.



Example 4 (page 74)

For Exercises 24–27, y varies directly with x.

24. If $y = 4$ when $x = -2$, find x when $y = 6$.

25. If $y = 6$ when $x = 2$, find x when $y = 12$.

26. If $y = 7$ when $x = 2$, find y when $x = 3$.

27. If $y = 5$ when $x = -3$, find y when $x = -1$.

28. **Aviation** A speed of 60 mi/h is equal to a speed of 88 ft/s. Find the speed in miles per hour of an aircraft travelling 1000 ft/s.

B Apply Your Skills

For each function, determine whether y varies directly with x. If so, find the constant of variation and write the equation.

29.

x	y
9	6
12	8
15	10

30.

x	y
4	1
6	2
8	3

31.

x	y
23	24
55	56
66	67

32.

x	y
2	2.6
3	3.9
4	5.2

Write an equation for a direct variation with a graph that passes through each point.

33. $(1, 2)$ 34. $(-3, -7)$ 35. $(2, -9)$ 36. $(-0.1, 50)$

37. $(-5, -3)$ 38. $(9, -1)$ 39. $(7, 2)$ 40. $(-3, 14)$

In Exercises 41–45, y varies directly with x.

41. If $y = 7$ when $x = 3$, find x when $y = 21$.

42. If $y = 25$ when $x = 15$, find x when $y = 10$.

43. If $y = 30$ when $x = -3$, find y when $x = -9$.

44. If $y = -20$ when $x = 2$, find y when $x = 14$.

45. If $y = 0.9$ when $x = 4.8$, find y when $x = 6.4$.

Determine whether a line with the given slope through the given point represents a direct variation. Explain.

46. $m = -1.7, (9, -9)$ 47. $m = -\frac{5}{6}, \left(15, -12\frac{1}{2}\right)$ 48. $m = \frac{7}{2}, \left(6\frac{1}{2}, 22\frac{3}{4}\right)$

Open-Ended In Exercises 49–51, choose a value of k within the given range. Then write and graph a direct variation using your value for k.

49. $0 < k < 1$ 50. $3 < k < 4.5$ 51. $-1 < k < -\frac{1}{2}$

It takes more effort for an engine to propel a car with underinflated tires. Cars with properly inflated tires get better gas mileage.

52. **Gas Mileage** Suppose you drive a car 392 mi on one tank of gas. The tank holds 14 gallons. The number of miles traveled varies directly with the number of gallons of gas you use.
   a. Write an equation that relates miles traveled to gallons of gas used.
   b. You only have enough money to buy 3.7 gallons of gas. How far can you drive before refueling?
   c. Last year you drove 11,700 mi. About how many gallons of gas did you use?
   d. Suppose the price of gas averaged $1.57 per gallon last year. Find the cost per mile.

53. **Writing** Suppose you use the origin to test whether a linear equation is a direct variation. Does this method work? Support your answer with an example.

54. **Error Analysis** Find the error in the following computation: If y varies directly with x^2, and $y = 2$ when $x = 4$, then $y = 3$ when $x = 9$.

Challenge

In Exercises 55–58, y varies directly with x.

55. If x is doubled, what happens to y?

56. If x is halved, what happens to y?

57. If x is divided by 7, what happens to y?

58. If x is multiplied by 10, what happens to y?

59. If z varies directly with the product of x and y ($z = kxy$), then z is said to vary jointly with x and y.
   a. **Geometry** The area of a triangle varies jointly with its base and height. What is the constant of variation?
   b. Suppose q varies jointly with v and s, and $q = 24$ when $v = 2$ and $s = 3$. Find q when $v = 4$ and $s = 2$.
   c. **Critical Thinking** Suppose z varies jointly with x and y, and x varies directly with w. Show that z varies jointly with w and y.

FCAT Practice

Multiple Choice

60. Which equation does NOT represent a direct variation?
   A. $y - 3x = 0$ B. $y + 2 = \frac{1}{2}x$ C. $\frac{y}{x} = \frac{2}{3}$ D. $y = \frac{x}{17}$

61. Suppose y varies directly with x. If x is 30 when y is 10, what is x when y is 9?
   F. 3 G. 27 H. 29 I. $\frac{300}{9}$

62. Suppose y varies directly with x. If x is −7 when y is 3, what is x when y is −5?
   A. $-11\frac{2}{3}$ B. $-4\frac{1}{5}$ C. $4\frac{1}{5}$ D. $11\frac{2}{3}$

FCAT Online
FCAT Format quiz at
www.PHSchool.com
Web Code: aga-0203

63. Which equation represents the direct variation in the table at the right?

x	3	4	9
y	8.1	10.8	24.3

   F. $4y - 10x = 0$ G. $8x = 3y$
   H. $y + 8.1x = 0$ I. $10y = 27x$

Short Response

64. Do the values in the table below represent a direct variation? Explain.

x	4	5	7
y	13.1	16.3	22.6

475-481

EXERCISES

[illegible] ee *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 478)

Suppose that x [illegible] each inverse variati[illegible]

1. $x = 1$ whe[illegible] 1 when $y = 1$

4. $x = 28$ wh[illegible] 2.5 when $y = 100$

Example 2 (page 479)

Is the relation[illegible] tion, an inverse variation, or neither? Write equations to model the direct and inverse variations.

7.

x	3	8	10	22
y	15	40	50	110

8.

x	3	5	7	10.5
y	14	8.4	6	4

9.

x	0.5	2.1	3.5	11
y	1	4.2	7	22

10.

x	0.1	3	6	24
y	3	0.1	0.05	0.0125

11.

[illegible]	7	3	1	$\frac{1}{5}$
[illegible]	$\frac{1}{7}$	$\frac{1}{3}$	1	5

12.

[illegible]	10	12	20	23
[illegible]	2	$2\frac{2}{5}$	4	$5\frac{3}{5}$

Example 3 (page 479)

Suppose that x and y vary inversely. Write a function that models each inverse variation and find y when $x = 10$.

13. $x = 20$ when $y = 5$ 14. $x = 20$ when $y = -4$ 15. $x = 5$ when $y = -\frac{1}{3}$

Example 4 (page 480)

Describe the combined variation that is modeled by each formula.

16. $A = \pi r^2$ 17. $A = 0.5bh$ 18. $h = \frac{2A}{b}$ 19. $V = \frac{Bh}{3}$

20. $V = \pi r^2 h$ 21. $h = \frac{V}{\pi r^2}$ 22. $V = \ell wh$ 23. $\ell = \frac{V}{wh}$

Example 5 (page 480)

Write the function that models each relationship. Find z when $x = 4$ and $y = 9$.

24. z varies directly with x and inversely with y. When $x = 6$ and $y = 2$, $z = 15$.

25. z varies jointly with x and y. When $x = 2$ and $y = 3$, $z = 60$.

26. z varies directly with the square of x and inversely with y. When $x = 2$ and $y = 4$, $z = 3$.

27. z varies inversely with the product of x and y. When $x = 2$ and $y = 4$, $z = 0.5$.

B Apply Your Skills

28. a. The spreadsheet shows data that could be modeled by an equation of the form $PV = k$. Estimate the value of k.
b. Estimate P when $V = 62$.

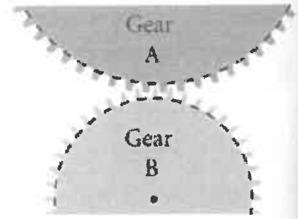
	A	B
1	P	V
2	140.00	100
3	147.30	95
4	155.60	90
5	164.70	85
6	175.00	80
7	186.70	75

Each ordered pair is from an inverse variation. Find the constant of variation.

29. $(6, 3)$ 30. $(0.9, 4)$ 31. $\left(\frac{3}{8}, \frac{2}{3}\right)$

32. $(\sqrt{2}, \sqrt{18})$ 33. $(\sqrt{3}, \sqrt{27})$ 34. $(\sqrt{8}, \sqrt{32})$

35. **Mechanics** Gear A drives Gear B. Gear A has a teeth and speed r_A in revolutions per minute (rpm). Gear B has b teeth and speed r_B. The quantities are related by the formula $ar_A = br_B$. Gear A has 60 teeth and speed 5400 rpm. Gear B has 45 teeth. Find the speed of Gear B.



36. **Physics** The force F of gravity on a rocket varies directly with its mass m and inversely with the square of its distance d from Earth. Write a model for this combined variation.

Each pair of values is from a direct variation. Find the missing value.

37. (3, 7), (8, y)
38. (2, 5), (4, y)
39. (4, 6), (x, 3)
40. (9, 5), (x, 3)
41. (8.3, 7.1), (5, y)
42. (2.6, 4.5), (x, 6.3)

Each pair of values is from an inverse variation. Find the missing value.

43. (3, 7), (8, y)
44. (2, 5), (4, y)
45. (4, 6), (x, 3)
46. (9, 5), (x, 3)
47. (8.3, 7.1), (5, y)
48. (2.6, 4.5), (x, 6.3)

49. Suppose that y varies inversely with the square of x, and $y = 50$ when $x = 4$. Find y when $x = 5$.

50. Suppose that c varies jointly with d and the square of g, and $c = 30$ when $d = 15$ and $g = 2$. Find d when $c = 6$ and $g = 8$.

51. Suppose that d varies jointly with r and t, and $d = 110$ when $r = 55$ and $t = 2$. Find r when $d = 40$ and $t = 3$.

Exercise 52

52. **Construction** A concrete supplier sells premixed concrete in 300-ft^3 truckloads. The area A that the concrete will cover is inversely proportional to the depth d of the concrete.
   a. Write a model for the relationship between the area and the depth of a truckload of poured concrete.
   b. What area will the concrete cover if it is poured to a depth of 0.5 ft? A depth of 1 ft? A depth of 1.5 ft?
   c. When the concrete is poured into a circular area, the depth of the concrete is inversely proportional to the square of the radius r. Write a model for this relationship.

53. Suppose that y varies directly with x and inversely with z^2, and $x = 48$ when $y = 8$ and $z = 3$. Find x when $y = 12$ and $z = 2$.

54. Suppose that t varies directly with s and inversely with the square of r. How is the value of t changed when the value of s is doubled? Is tripled?

55. Suppose that x varies directly with the square of y and inversely with z. How is the value of x changed if the value of y is halved? Is quartered?

C Challenge

56. **Writing** Explain why 0 cannot be in the domain of an inverse variation.

57. **Critical Thinking** Suppose that (x_1, y_1) and (x_2, y_2) are values from an inverse variation. Show that $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

58. **Open-Ended** The height h of a cylinder varies directly with its volume V and inversely with the square of its radius r. Find at least four ways to change the volume and radius of a cylinder so that its height is quadrupled.

Honors Algebra 2
9.1, 2.3, 9.4 – 9.6 Review WS

Write the function that models the relationship.

1. p varies jointly with the square of x and y and inversely with the product of w and z. When x = 4 and y = 2, w = 3, z = 8 and p = 64. Find p when x = 1, y = 8, w = 2, z = 6.

Write in simplest form and state any restrictions.

2. $\frac{5x-9}{42x^3-48x^2} \div \frac{45x^2-81x}{7x^2+27x-40}$

3. $\frac{7r^2+51r-40}{r^2-r-72} \div \frac{49r^2-21r-10}{7r+2}$

Simplify.

4. $\frac{4}{x-1}-\frac{3}{1-x}+\frac{2}{x+1}$

5. $\frac{x-3+\frac{12}{x+2}}{x-8+\frac{42}{x+5}}$

6. $\frac{r^2}{r-t}+\frac{t^2}{t-r}\,\frac{r^2}{r-t}+\frac{t^2}{t-r}$

Solve

7. $\frac{4}{x-1}+\frac{5}{x}<2$

8. $1+\frac{2}{x+1}\le\frac{2}{x}$

9. $\frac{2}{x+3}-\frac{3}{4-x}=\frac{2x-2}{x^2-x-12}$

10. $\frac{10k}{k^2-9}=\frac{5}{3-k}-\frac{k-2}{k+3}$

11. $\frac{2}{x+4}+\frac{1}{2-x}+\frac{2x-1}{x^2+2x-8}=0$

12. A group of college students volunteer over spring break by painting houses. Working alone, Ira can paint a room in 7 hours. Paul can paint the same room in 6 hours. Write an equation that can be used to find how long it will take them working together to paint the room. Solve.

13. DVD's can be manufactured for \$0.35 each. The company has an initial development cost of \$13,500. If the first 100 DVD's will be given away for free, write a function for the average cost to manufacture a DVD. What is the average cost of 3000 DVD's? How many discs must be sold to bring the average cost to around \$10?

14. Car A travels 180 miles in the same amount of time that it takes Car B to go 120 miles. If one car is going 20 mph faster than the other, find the speed of both cars.

15. Alicia can row 6 miles downstream in the same time it takes her to row 4 miles upstream. She rows downstream 3 mph faster then she rows upstream. Find Alicia's rowing rate each way.