

ATM

Kothe & Arnett

Week 3 & 4

April 13th – April 24th

Base e and Natural Logarithms

10.8

Vocabulary

natural logarithm: a logarithm with base e

natural base: the number e , which is found using

$$\left(1 + \frac{1}{n}\right)^n$$

natural base exponential function: an exponential function with base e

natural logarithmic function: the inverse of the natural base exponential function

History



The number e is a famous irrational number, and is one of the most important numbers in mathematics. The first few digits are

2.7182818284590452353602874713527...

It is often called **Euler's number** after Leonhard Euler. e is the base of the natural logarithms (invented by John Napier).

Ex 1

Use a calculator to estimate $e^{0.5}$ to four decimal places.

1.6487

Ex 2

Use a calculator to estimate e^{-8} to four decimal places.

0.0003

Ex 3

Use a calculator to estimate $\ln 3$ to four decimal places.

$$1.0986$$

Ex 4

Use a calculator to estimate $\ln \frac{1}{4}$ to four decimal places.

$$-1.3863$$

Writing Equivalent Expressions

Ex 5

Write an equivalent logarithmic equation.

$$e^x = 23 \quad \log_e 23 = x$$

$$\ln 23 = x$$

Ex 6

Write an equivalent logarithmic equation.

$$e^x = 6 \quad \ln 6 = x$$

Writing Equivalent Expressions

Ex 7

Write an equivalent exponential equation.

$$\ln x \approx 1.2528 \quad e^{1.2528} = x$$

Ex 8

Write an equivalent exponential equation.

$$\ln x = 2.25 \quad e^{2.25} = x$$

Inverse Properties

$$e^{\ln x} = x \quad \ln e^x = x$$

Same as normal logs

Writing Equivalent Expressions

Ex 9

Evaluate $e^{\ln 21}$

$$21$$

Ex 10

Evaluate $e^{\ln(x+3)}$

$$x + 3$$

Ex 11

Evaluate $\ln e^{x^2-1}$

$$x^2 - 1$$

Ex 12

Evaluate $\ln e^7$

$$7$$

Solving Equations

Ex 14

Solve $3e^{2x} + 4 = 10$

$$3e^{2x} = 6$$

$$e^{2x} = 2$$

$$\ln e^{2x} = \ln 2$$

$$-2x = \ln 2$$

$$-2x \approx 0.6931$$

$$x \approx -0.3466$$

Solving Equations

Ex 13

Solve $e^x = 10$

$$\ln e^x = \ln 10$$

$$x = \ln 10$$

$$x \approx 2.3026$$

Solving Equations

Ex 15

Solve $2e^{-2x} + 5 = 15$

$$2e^{-2x} = 10$$

$$e^{-2x} = 5$$

$$\ln e^{-2x} = \ln 5$$

$$-2x = \ln 5$$

$$-2x \approx 1.6094$$

$$x \approx -0.8047$$

Solving Equations

Ex 16

Solve $\ln 3x = 0.5$

$$e^{\ln 3x} = e^{0.5}$$

$$3x = e^{0.5}$$

$$3x \approx 1.6487$$

$$x \approx 0.5496$$

Solving Equations

Ex 17

Solve $\ln 2x = 0.7$

$$e^{\ln 2x} = e^{0.7}$$

$$2x = e^{0.7}$$

$$2x \approx 2.0138$$

$$x \approx 1.0069$$

Solving Equations

Ex 18

Solve $\ln(2x-3) = 2.5$

$$e^{\ln(2x-3)} = e^{2.5}$$

$$2x-3 = e^{2.5}$$

$$2x-3 \approx 12.1825$$

$$2x \approx 15.1825$$

$$x \approx 7.5912$$

Solving Equations

Ex 19

Solve $\ln(x-3) = 3$

$$e^{\ln(x-3)} = e^3$$

$$x-3 = e^3$$

$$x-3 \approx 20.0855$$

$$x \approx 23.0855$$

Oral Exercises

Give each equation in exponential form.

1. $\ln 4 = 1.39$

2. $\ln \frac{1}{4} = -1.39$

3. $\ln e = 1$

Give each equation in logarithmic form.

4. $e^2 = 7.39$

5. $e^{-2} = 0.14$

6. $e^{1/5} = 1.22$

Simplify.

7. $\ln \frac{1}{e}$

8. $\ln e^{12}$

9. $\ln \sqrt{e}$

Solve.

10. $\ln x = 5$

11. $\ln x = \frac{1}{3}$

12. $e^x = 3$

13. $x = \ln e^{3/4}$

14. $e^{\ln x} = 10$

15. $\ln x = -\frac{1}{2}$

16. Approximate to three decimal places the value of $\left(1 + \frac{1}{5000}\right)^{5000}$

Written Exercises

Write each equation in exponential form.

A 1. $\ln 8 = 2.08$

2. $\ln 100 = 4.61$

3. $\ln \frac{1}{3} = -1.10$

4. $\ln \frac{1}{e^2} = -2$

Write each equation in logarithmic form.

5. $e^3 = 20.1$

6. $e^7 = 1097$

7. $e^{1/2} = 1.65$

8. $\sqrt[3]{e} = 1.40$

Simplify. If the expression is undefined, say so.

9. $\ln e^2$

10. $\ln e^{10}$

11. $\ln \frac{1}{e^3}$

12. $\ln \sqrt[3]{e}$

13. $\ln 1$

14. $\ln 0$

15. $e^{\ln 5}$

16. $e^{\ln 0.5}$

Write as a single logarithm.

17. $\ln 3 + \ln 4$

18. $\ln 8 - \ln 2$

19. $2 \ln 3 - \ln 5$

20. $\ln 7 + \frac{1}{2} \ln 9$

21. $\frac{1}{3} \ln 8 + \ln 5 + 3$

22. $4 \ln 2 - \ln 3 - 1$

Solve for x . Leave answers in terms of e .

23. $\ln x = 3$

24. $\ln \frac{1}{x} = 2$

25. $\ln (x - 4) = -1$

26. $\ln |x| = 1$

27. $\ln x^2 = 9$

28. $\ln \sqrt{x} = 3$

Solve for x . Leave answers in terms of natural logarithms.

29. $e^x = 2$

30. $e^{-x} = 3$

31. $e^{2x} = 25$

32. $e^{3x} = 8$

33. $e^{x-2} = 2$

34. $\frac{1}{e^x} = 7$

Solve. Leave answers in terms of e or natural logarithms.

35. $\sqrt{e^x} = 3$

36. $e^{-2x} = 0.2$

37. $(e^x)^5 = 1000$

38. $3e^{2x} + 2 = 50$

39. $\ln (\ln x) = 0$

40. $|\ln x| = 1$

41. $\ln x + \ln (x + 3) = \ln 10$

42. $2 \ln x = \ln (x + 1)$

43. $e^{2x} - 7e^x + 12 = 0$

ATM Worksheet

10.8 Base e and Natural Logarithms

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Please choose and circle the best answer choice for each problem.

1. Evaluate: $e^{\ln \frac{1}{2}}$

a. $\sqrt{e}$

b. $\ln \sqrt{e}$

c. $\frac{1}{\sqrt{e}}$

d. $\frac{1}{2}$

2. Write as a single logarithm: $2\ln 6 - \ln 2 - \ln 9$

a. $\ln 2$

b. $\ln 1$

c. $\ln 25$

d. $\ln \frac{2}{3}$

3. Solve: $\ln 4x = 7$

a. $\left\{e^{\frac{4}{7}}\right\}$

b. $\{3e^7\}$

c. $\left\{e^{\frac{7}{4}}\right\}$

d. $\left\{\frac{e^7}{4}\right\}$

4. Natural logarithms use which value as a base?

a. 10

b. 2

c. e

d. $\ln$

5. Solve: $e^{5x} = 2$

a. $\left\{\frac{\ln 2}{5}\right\}$

b. $\{5\ln 2\}$

c. $\left\{\frac{\ln 5}{2}\right\}$

d. $\left\{\frac{2}{5}e\right\}$

6. Solve: $10 + 5e^{2x} = 17$

a. 1.4

b. 0.5666

c. 0.6592

d. 0.1682

7. Solve: $2 + 6e^{4x} = 19$

a. 0.2604

b. 0.4907

c. 0.5074

d. 2.8333

8. Write as a single logarithm: $3\ln 3 + 3\ln c$

a. $\ln(27 + c^3)$

b. $\ln 9c^3$

c. $\ln 27c$

d. $\ln 27c^3$

9. Write the equation in logarithmic form for $e^3 = 7$

a. $\ln 3 = 7$

b. $\log 3 = 7$

c. $\ln 7 = 3$

d. $\log 7 = 3$

10. Write the equation in exponential form for $\ln 16 = 4$?

a. $e^3 = 4$

b. $e^4 = 3$

c. $e^4 = 16$

d. $2^{16} = 4$

11. Write as a single logarithm: $3 \ln a - \frac{1}{2}(\ln b + \ln c^2)$

a. $\ln\left(\frac{3a}{0.5bc^2}\right)$

b. $\frac{3}{2}\ln\left(\frac{a}{bc^2}\right)$

c. $\ln\left(\frac{a^3}{bc}\right)$

d. $\ln\left(\frac{a^3}{c\sqrt{b}}\right)$

12. Solve: $\ln 2 + \ln x = 5$

a. 50,000

b. 74.2

c. 10

d. 3

13. Solve: $\ln(2x - 1) = 8$

a. 1489.979

b. 2979.958

c. 2981.458

d. 1490.979

14. Solve: $\ln(e^{\ln 7x}) = 9$

a. $\frac{7}{9}$

c. $\frac{e^9}{7}$

d. $\frac{7}{e^9}$

b. $\frac{9}{7}$

15. Solve: $\ln 12x - \ln 3 = \ln 7$

a. 2

b. $\frac{4}{7}$

c. $\frac{7}{4}$

d. $-\frac{7}{4}$

16. Solve: $\ln(3x) - \ln(5) = 5$

a. $\frac{5e^5}{3}$

b. $\ln(3x)$

c. $\ln(3x - 10)$

d. $e^3 = 4$

Bonus:

Simplify $\ln \frac{1}{e^2}$

a. 2

b. -2

c. $\frac{1}{2}$

d. $-\frac{1}{2}$

Name _____

Period _____

ATM

Quiz: 10.8 Base e and Natural Logarithms

March 29, 2019

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

10. _____

11. _____

12. _____

13. _____

14. _____

15. _____

16. _____

B. _____