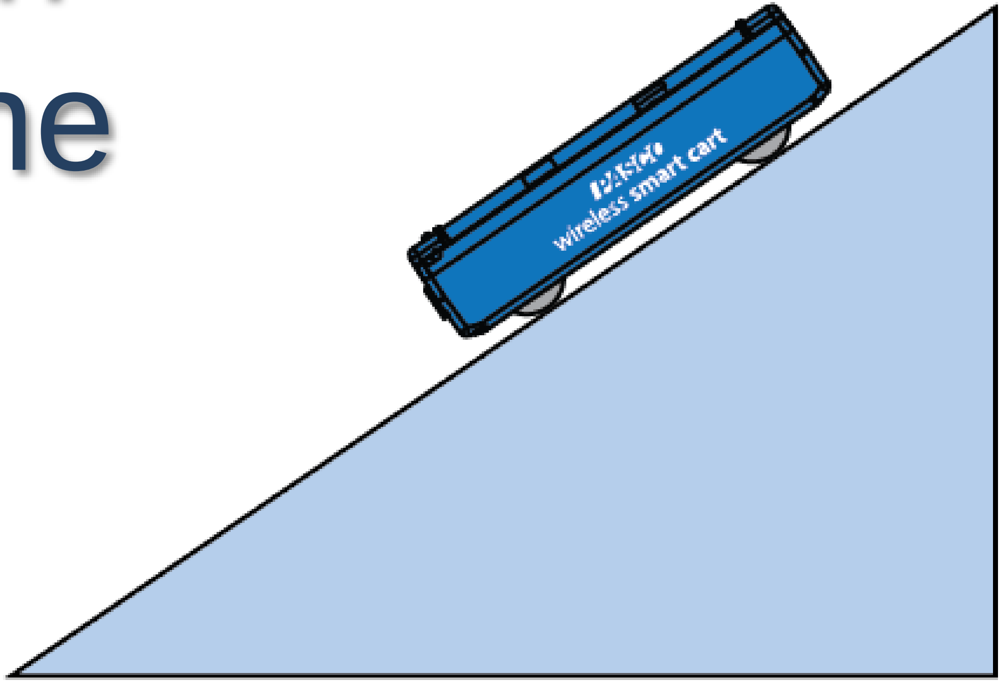
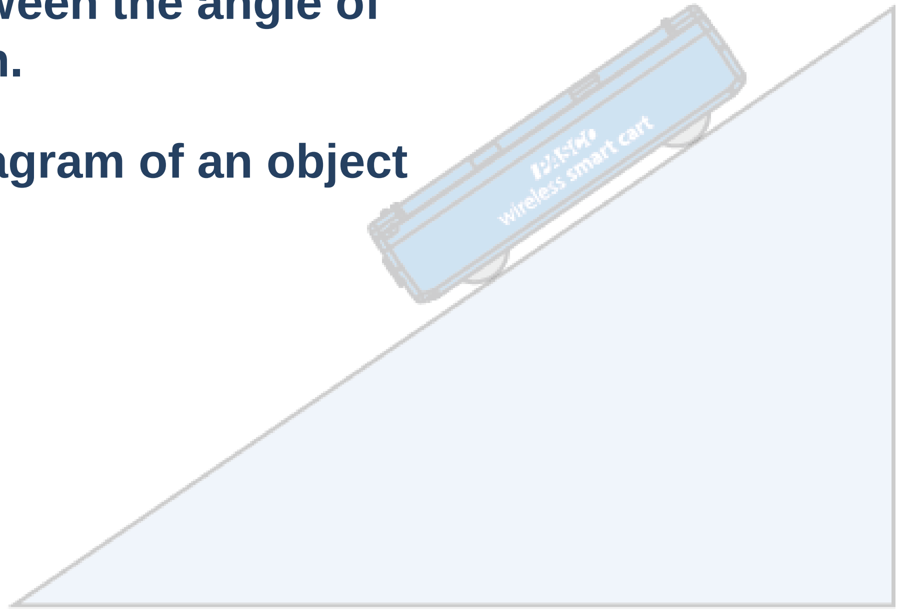
Motion on an inclined plane



Objectives

- Calculate the motion of objects on inclined planes.
- Demonstrate the relationship between the angle of an inclined plane and acceleration.
- Draw and interpret a free-body diagram of an object on an inclined plane.



Assessment

1. **Winston releases a cart on an inclined plane that is 3.2 m long and 1.8 m high. What is the acceleration of the cart?**

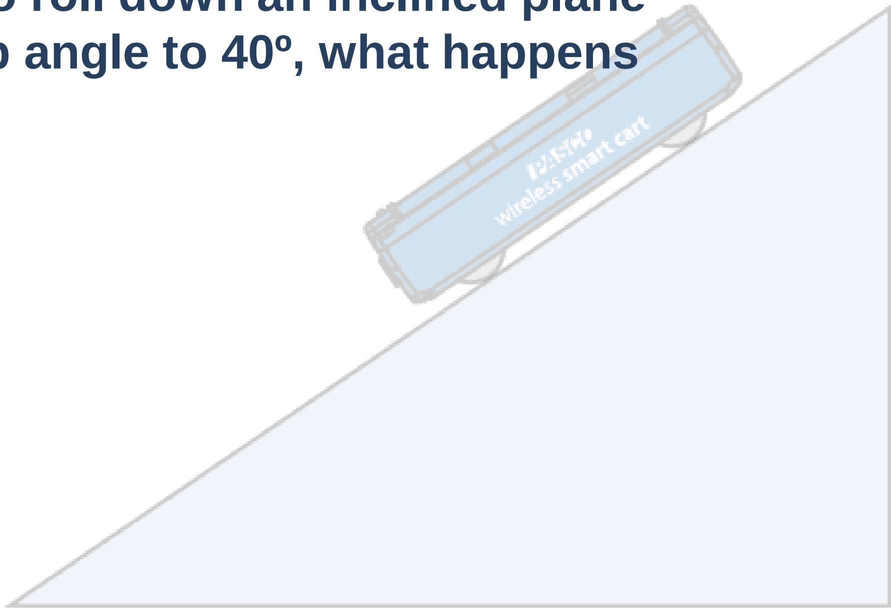
2. **Zoe measures the time for a ball to roll down an inclined plane set at 30º. If she changes the ramp angle to 40º, what happens to the time to reach the bottom?**

   A. **increases**

   B. **decreases**

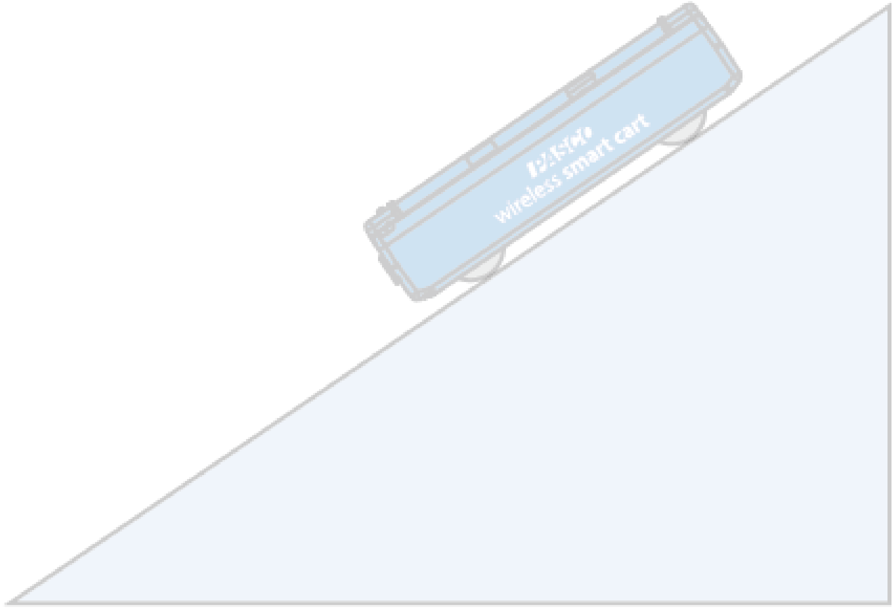
   C. **stays the same**

   D. **not enough information**



Physics terms

- **inclined plane**
- **ramp coordinates**



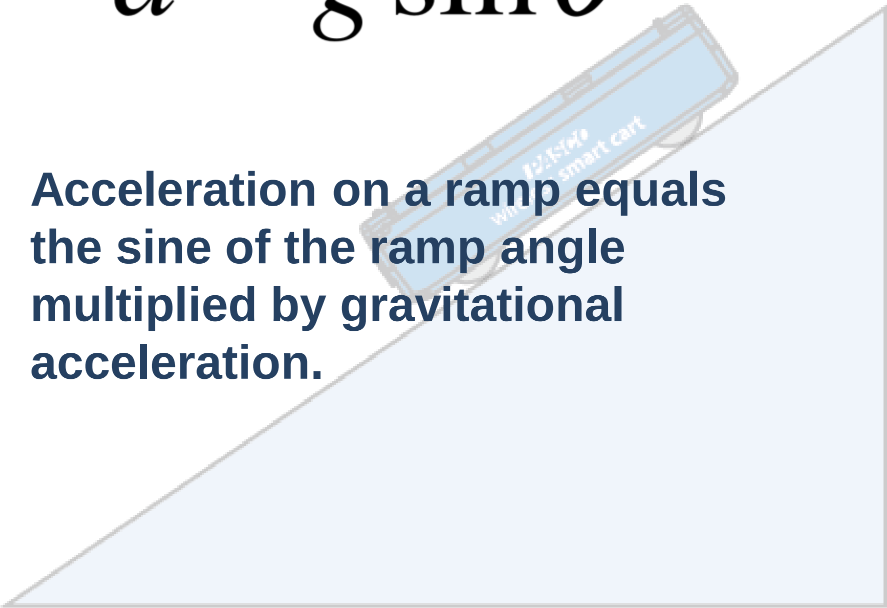
Equations

$$a_{ramp} = \left(\frac{h}{L}\right) g$$

Acceleration on a ramp equals the ratio of the height to the length of the ramp, multiplied by gravitational acceleration.

$$a = g \sin\theta$$

Acceleration on a ramp equals the sine of the ramp angle multiplied by gravitational acceleration.



Inclined planes

An *inclined plane* is a smooth surface that is tilted at an angle.

It is also called a ramp.

Here are some examples:

- sledding hills
- wheelchair ramps
- ramps in skateboard parks
- airplane emergency evacuation slides

Inclined planes

Have you ever rolled a marble down a ramp or sledded down a hill?

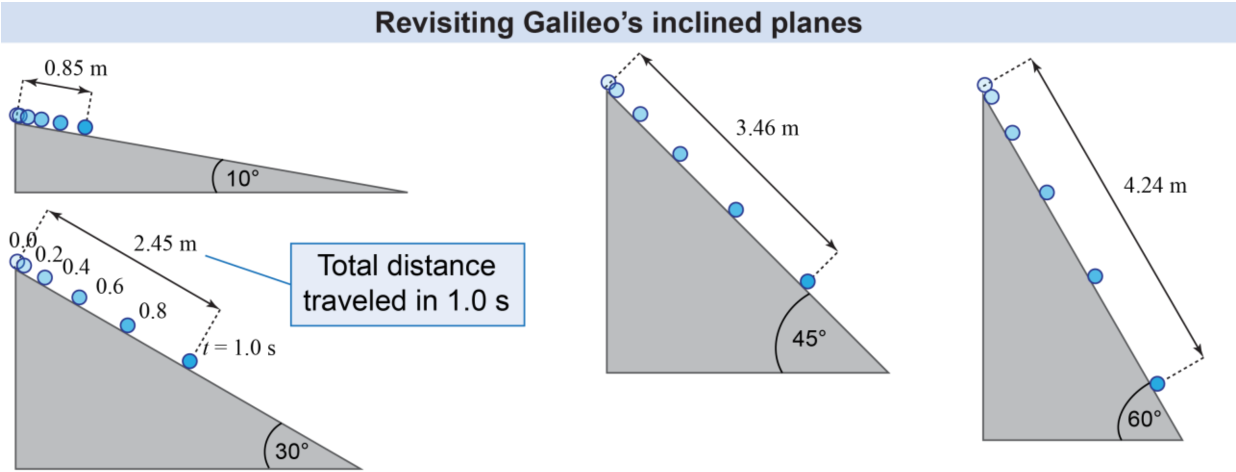
Do you think the motion accelerated, or did the velocity stay constant?

Discuss this with the person sitting next to you.

Inclined planes

Galileo used inclined planes to study motion.

Inclined planes slow down the motion, allowing Galileo to accurately measure how far an object moved in a given time.

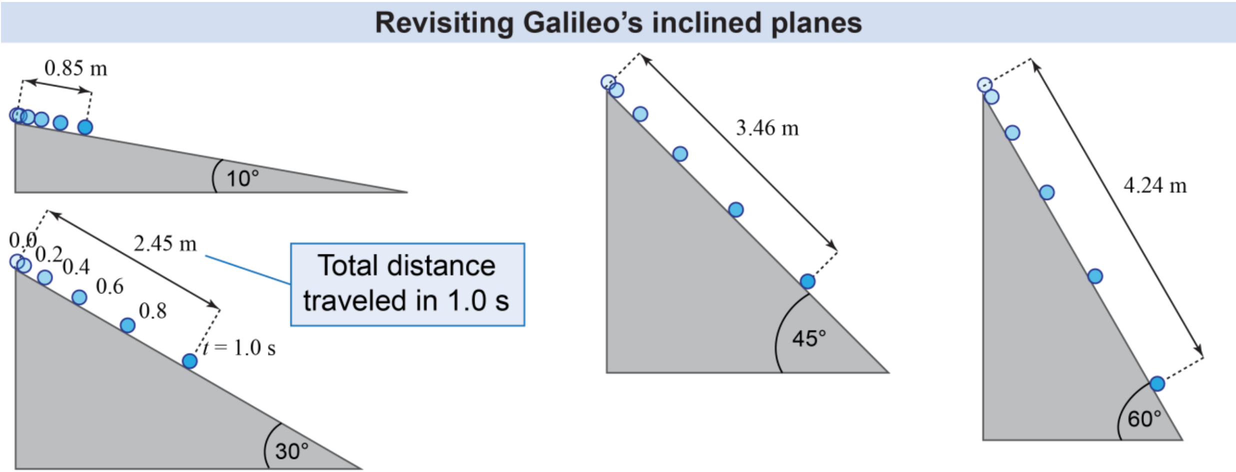


Inclined planes

As ramp angle increases, the distance traveled in one second increases.

This means the ball moves faster on the steeper ramps.

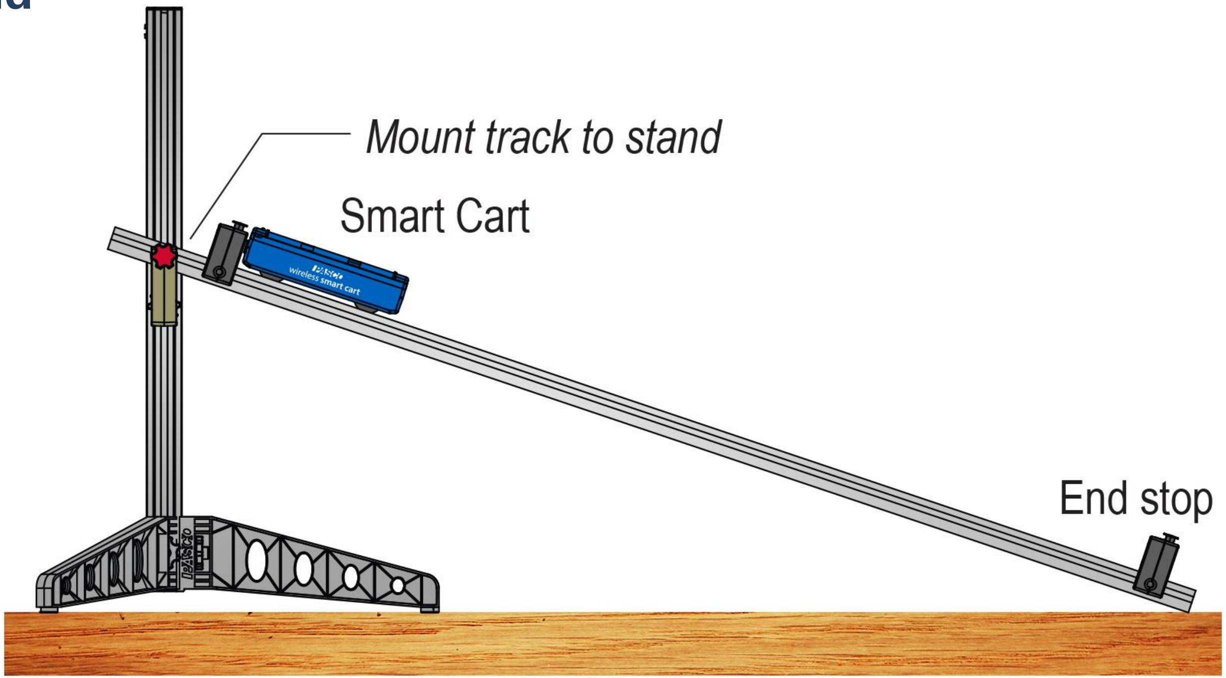
But exactly how does the motion depend on the ramp angle?



Investigation

How does the motion of a cart on a ramp depend on the ramp angle?

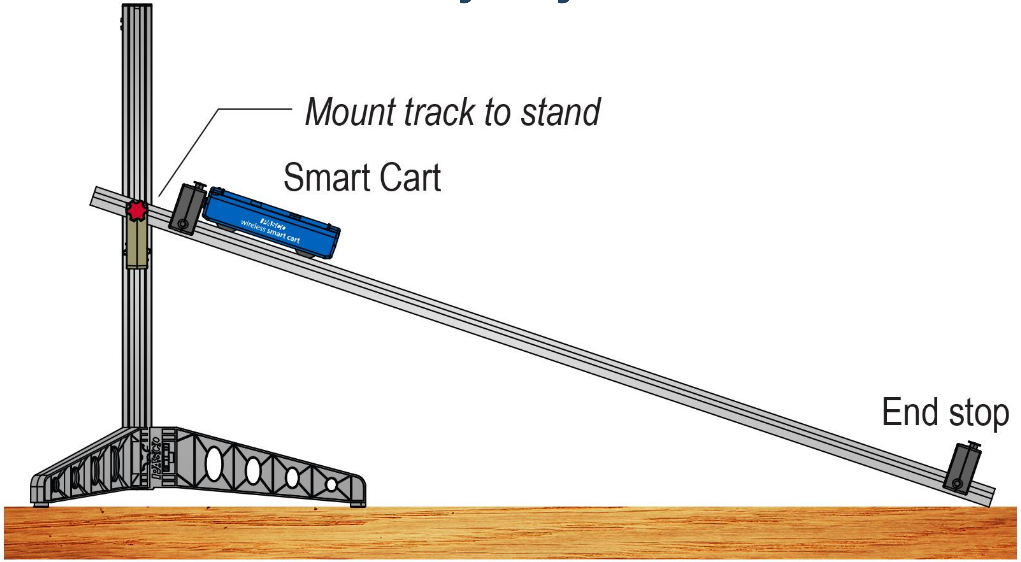
Investigation 6C on page 192.



Investigation

Part 1: Acceleration down a ramp

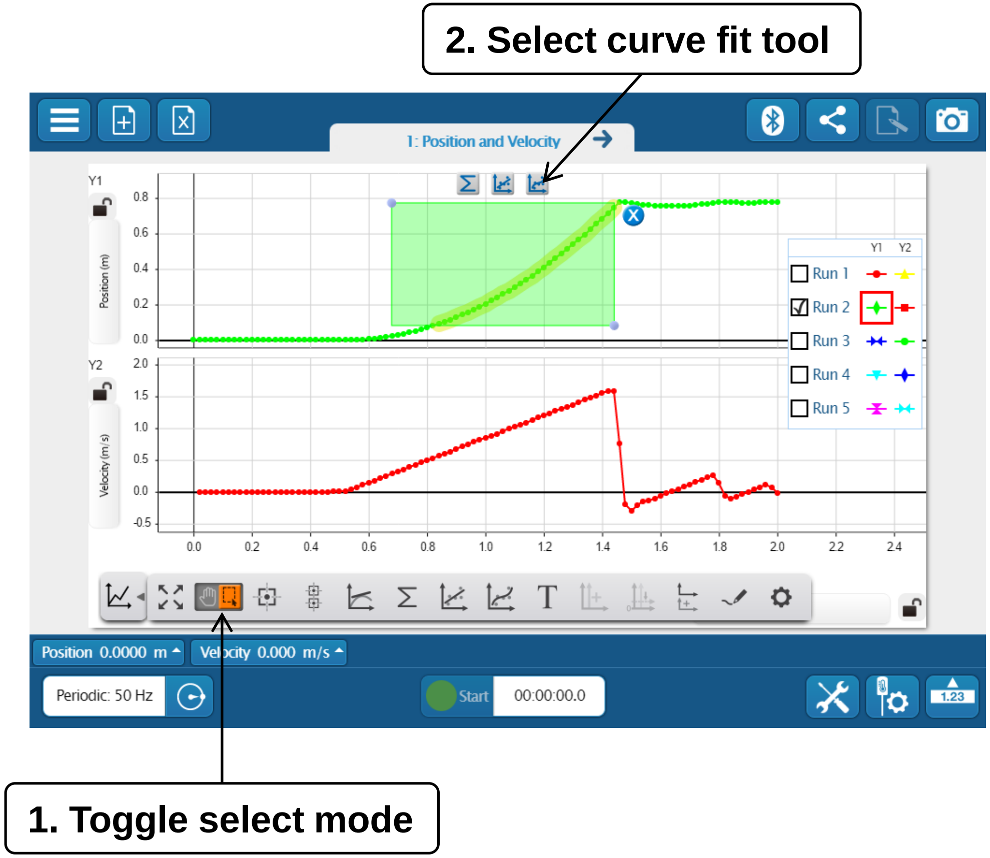
1. **Set up the track to act as an inclined plane with height around 30 cm.**
2. **Open the experiment file 06C_AccelerationOnAnInclinedPlane, and then power-on the Smart Cart and connect it wirelessly to your software.**
3. **Begin data collection and release the Smart Cart down the track. Data collection stops automatically.**



Investigation

Part 1: Acceleration down a ramp

4. Use the curve fit tool in SPARKvue to fit a curve to each graph as the cart moves down the track. Sketch the shape of the graphs.

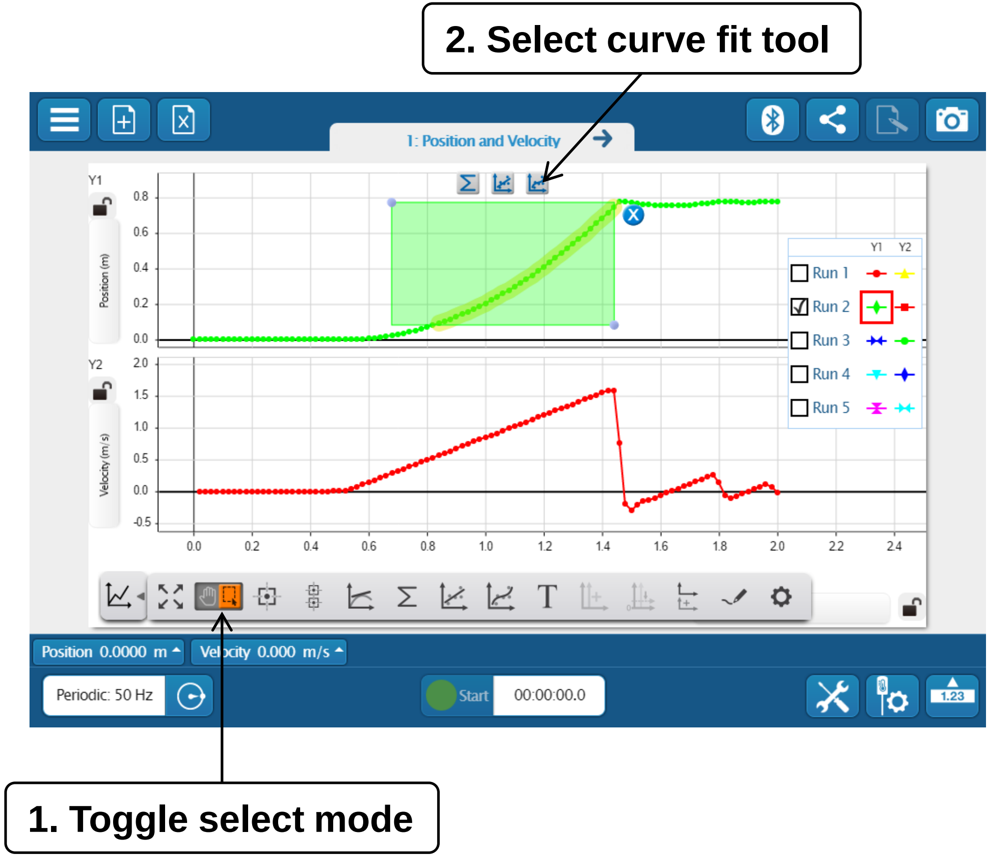


Investigation

Questions for Part 1

a. What motion equation are you fitting to the position vs. time graph? The velocity vs. time graph?

b. Explain the connection between the slope of the velocity vs. time graph and the acceleration.



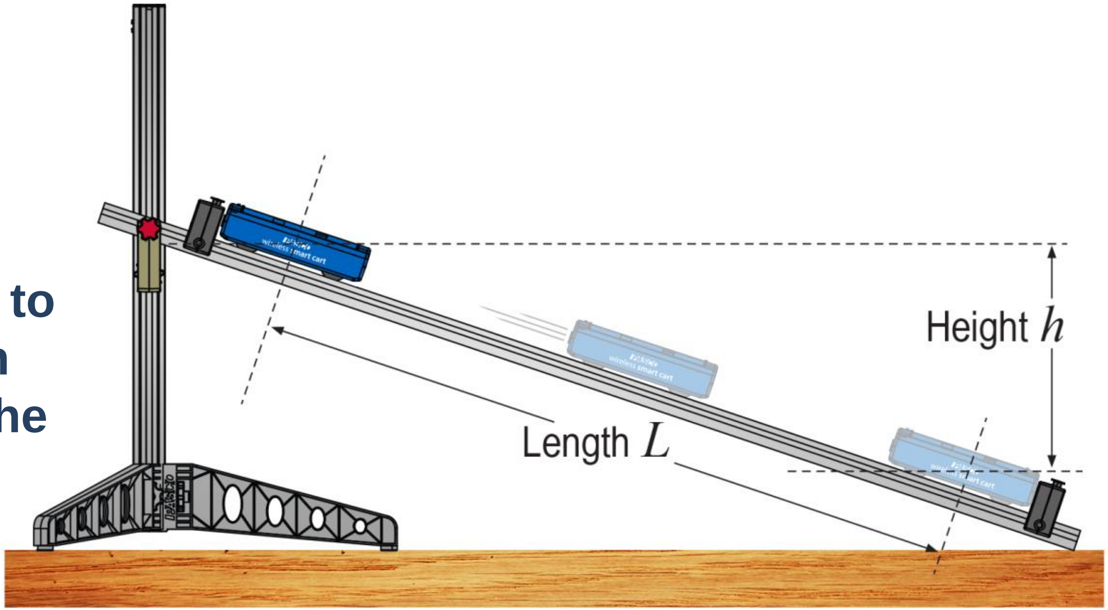
Investigation

Part 2: How does acceleration vary with the ramp's inclination?

1. Measure the height h and length L of the ramp.

2. Record data and release the Smart Cart.

3. Use the resulting graphs to measure the acceleration of the Smart Cart down the ramp.



Investigation

Part 2: How does acceleration vary with the ramp's inclination?

4. Repeat for four more values of *h*, lowering the height around 2-3 cm each time. Enter your values in the table.

5. Calculate the inclination of the ramp (*h*/*L*) for each run and enter the values in the table.

Table 1: Acceleration varies with ramp angle

Height (m)	Length (m)	Acceleration (m/s^2)	Inclination

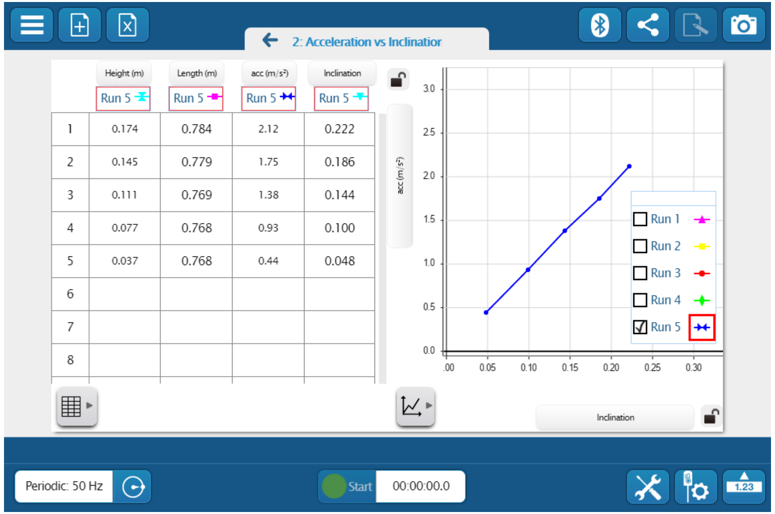
Investigation

Questions for Part 2

a. **How does the acceleration vary as the slope of the ramp increases?**

b. **Go to Page 2 of the SPARKvue file and graph acceleration versus inclination. Fit a line to the graph. What is the value of the slope of a line through your data (with units)?**

c. **Is this slope similar to any physical constant you have learned about in Physics? Explain.**

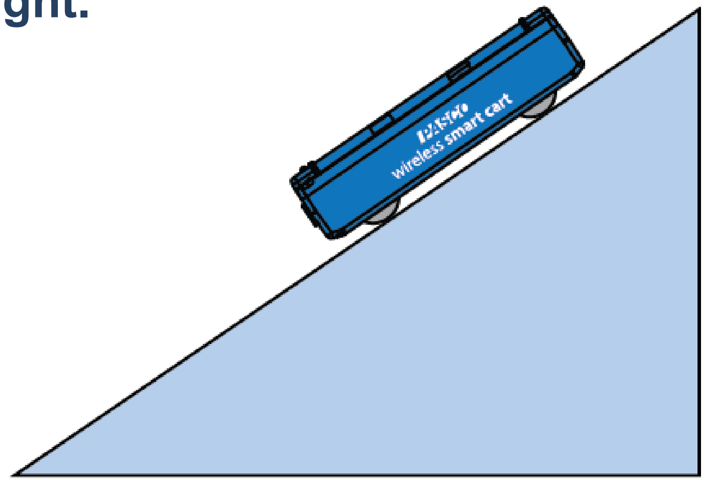


Understanding the acceleration

The *acceleration* of an object on a ramp depends on the ramp's length and height.

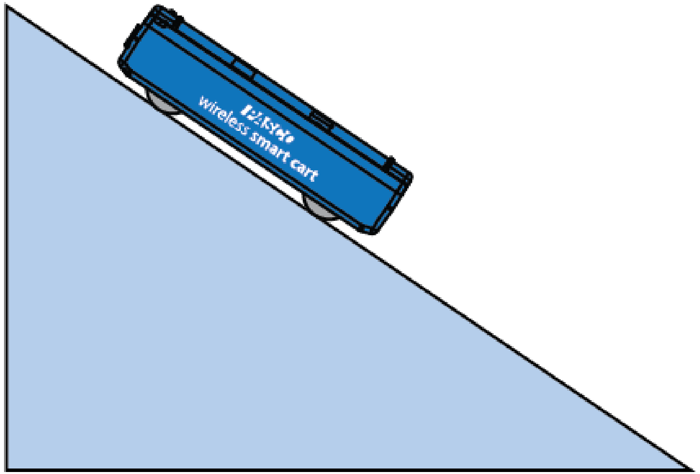
But why?

To really understand this, you need to think about the *forces* acting on the object.



Forces on a ramp

How would you draw a free-body diagram for a cart on a ramp?

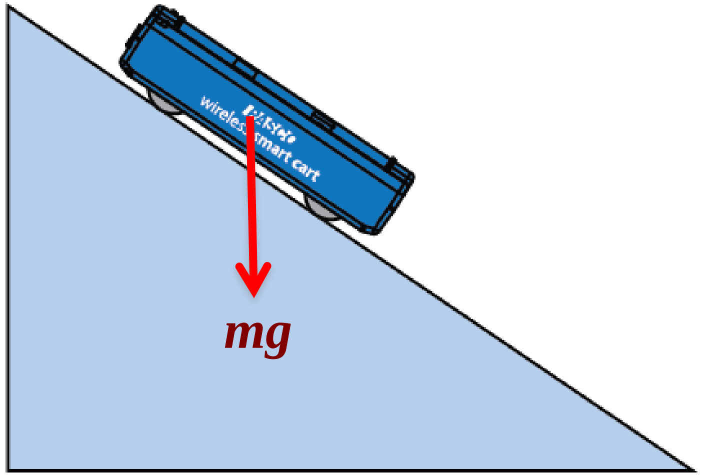


Forces on a ramp

How would you draw a free-body diagram for a cart on a ramp?

Two forces act on the cart.

- **One is the gravitational force.**

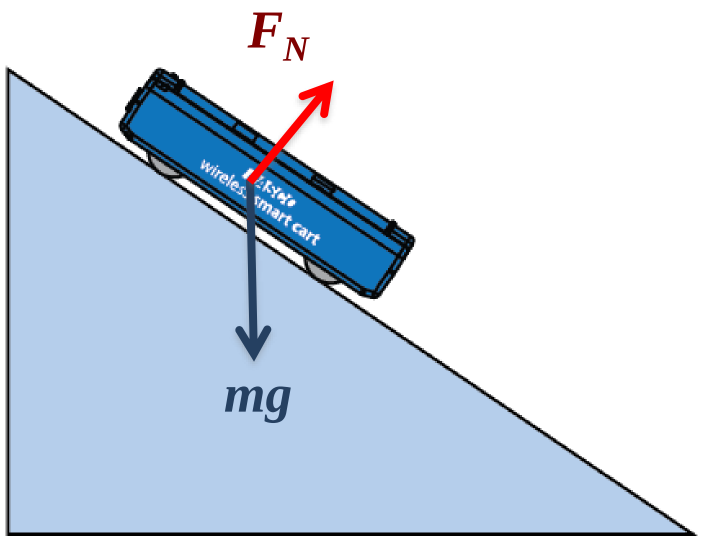


Forces on a ramp

How would you draw a free-body diagram for a cart on a ramp?

Two forces act on the cart.

- **One is the gravitational force.**
- **The other is the normal force, which acts perpendicular to the ramp.**

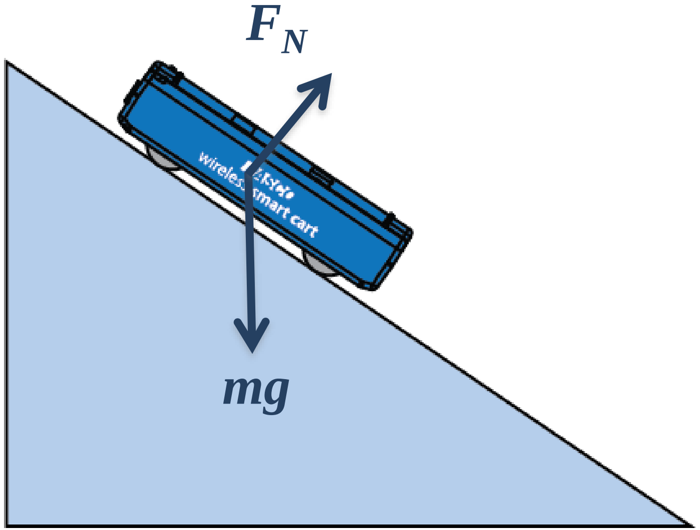


Forces on a ramp

These forces do not completely cancel each other out.

If they did, the cart would not accelerate.

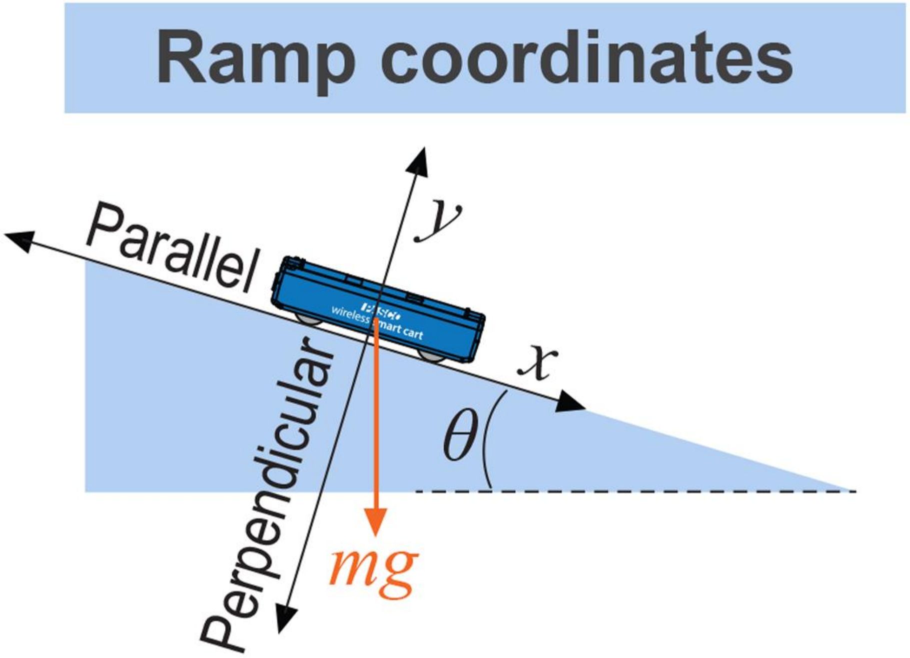
There must be a net force.
To find F_{net}, it helps to tilt the coordinate axis system.



Keeping it simple

Tilting the coordinate system for a ramp makes things simpler.

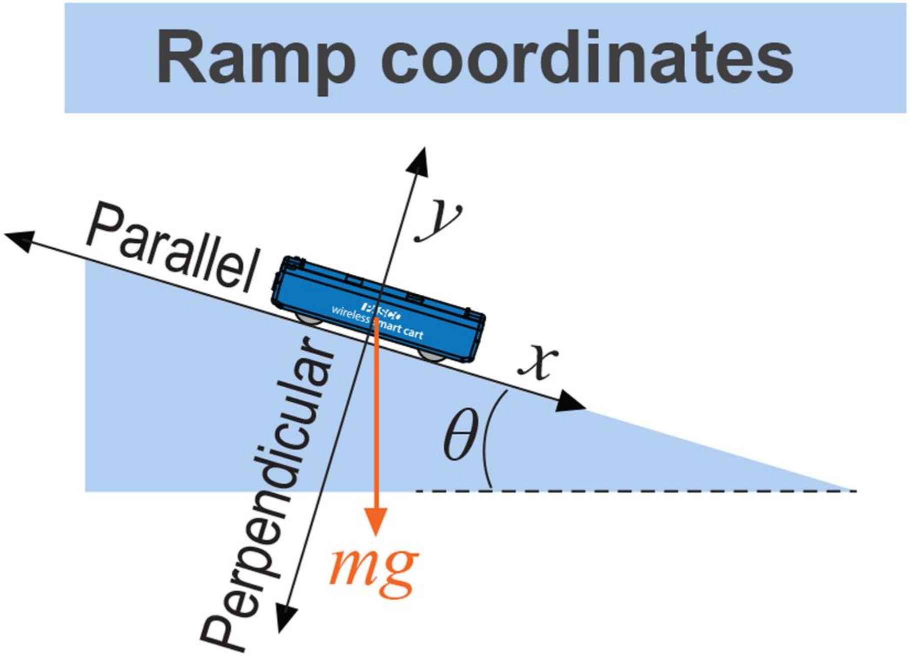
It turns complex 2-D motion into simple 1-D motion.



Coordinate system on a ramp

Tilting the coordinate system gives you 1-D motion along the *x*-axis.

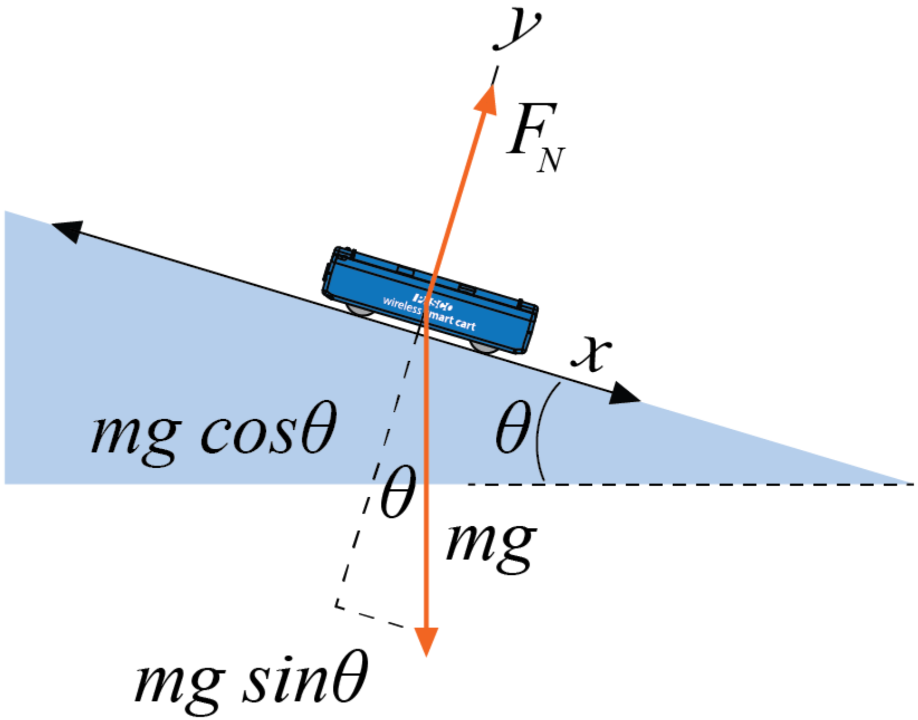
- **The *x-axis is parallel* to the ramp, and matches the direction the object travels.**
- **The *y-axis is perpendicular* to the ramp. There is no motion along the *y*-axis.**



Components of gravity

The force of gravity has *x*- and *y*- components.

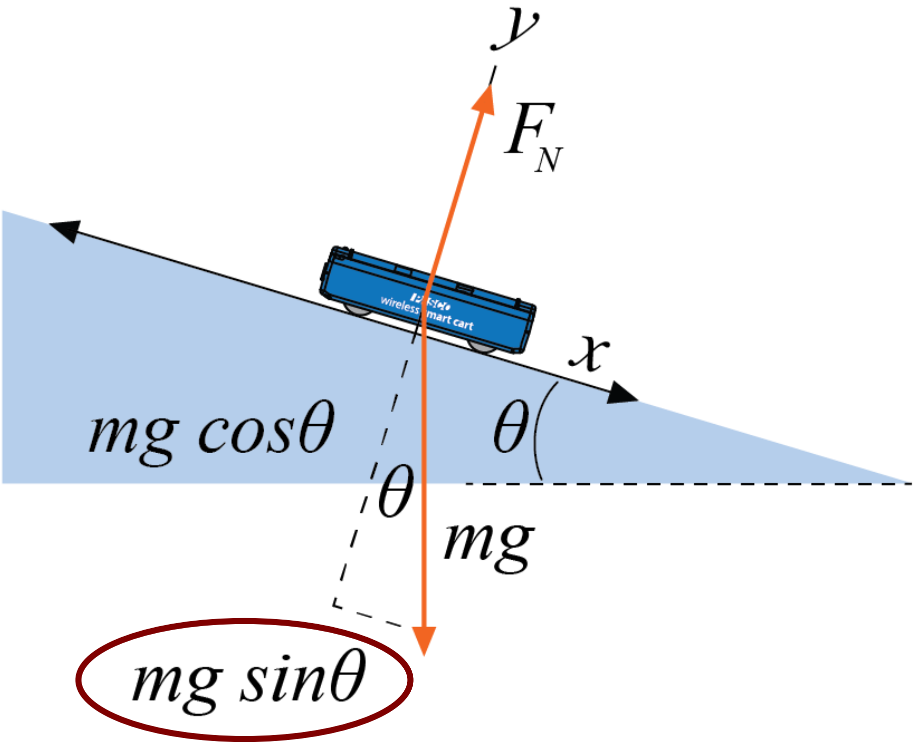
- **On the *x-axis* there is only the *x*-component of gravity: $mg \sin \theta$.**
- **On the *y-axis* the forces cancel each other out: $F_N = mg \cos \theta$.**



Net force on a ramp

- On the *x-axis* there is only the *x*-component of gravity: $mg \sin\theta$.

This is the net force on the cart.



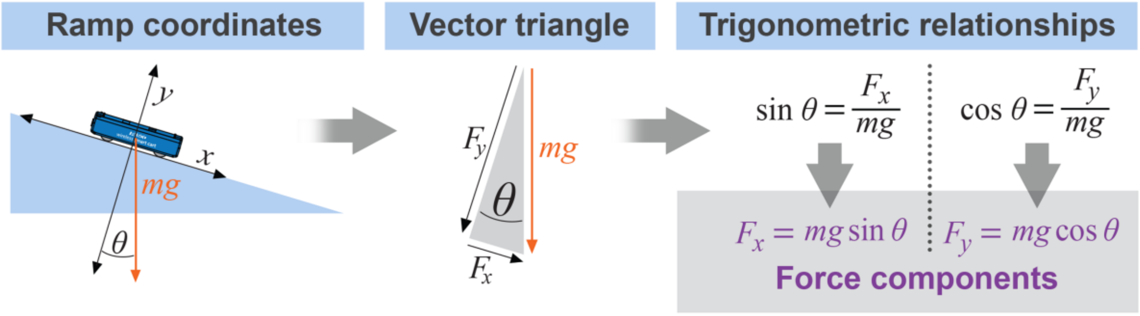
Acceleration on a ramp

The acceleration on a ramp can now be found from Newton's second law:

$$F_{net} = ma$$

$$ma = mg\sin\theta$$

$$a = g\sin\theta$$



Ramp angle and acceleration

On the right is a table of the acceleration for different values of ramp angle, *θ*.

The acceleration at 0º is 0. What does this physically represent?

Acceleration vs. angle (no friction)

Angle	sin θ	Acceleration (m/s^2)
0°	0.000	0
5°	0.087	0.85
10°	0.174	1.70
20°	0.342	3.35
30°	0.500	4.90
45°	0.707	6.93
60°	0.866	8.49
90°	1.000	9.80

Ramp angle and acceleration

On the right is a table of the acceleration for different values of ramp angle, ***θ***.

The acceleration at 0º is 0. What does this physically represent?

A ramp at 0º is flat. Gravity is not pulling the object down the ramp, so there is no acceleration.

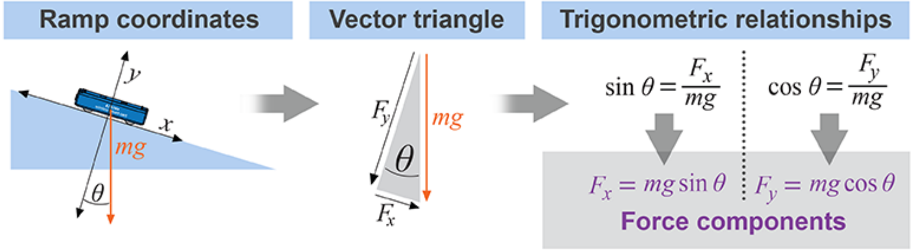
Acceleration vs. angle (no friction)		
Angle	sin θ	Acceleration (m/s^2)
0°	0.000	0
5°	0.087	0.85
10°	0.174	1.70
20°	0.342	3.35
30°	0.500	4.90
45°	0.707	6.93
60°	0.866	8.49
90°	1.000	9.80

Exploring the ideas

Click this interactive calculator on page 194.

Acceleration along the ramp

The car's motion depends on the net force in the x-direction (along the ramp). Because the ramp is at an angle, the weight force is not vertical in ramp coordinates but is tilted at the ramp angle. A component of the car's weight equal to $mg \sin\theta$ lies parallel to the ramp surface. This component causes the car to accelerate down the ramp at $a_x = g \sin\theta$.



Interactive equation

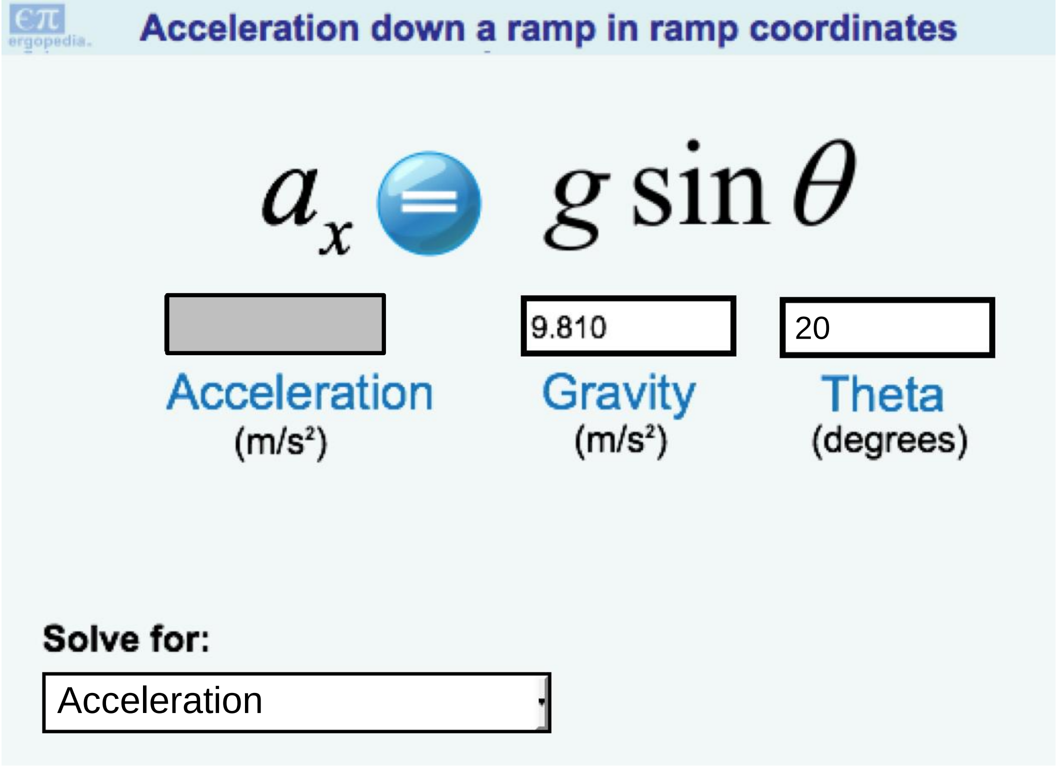
$$(6.12) \quad a_x = g \sin\theta$$

a_x = acceleration along ramp (m/s^2)
g = acceleration of gravity = 9.8 m/s^2
θ = ramp inclination angle (degrees)

Acceleration in ramp coordinates

Engaging with the concepts

A ramp is placed at an angle of 20º. What is the acceleration of an object released from the top?



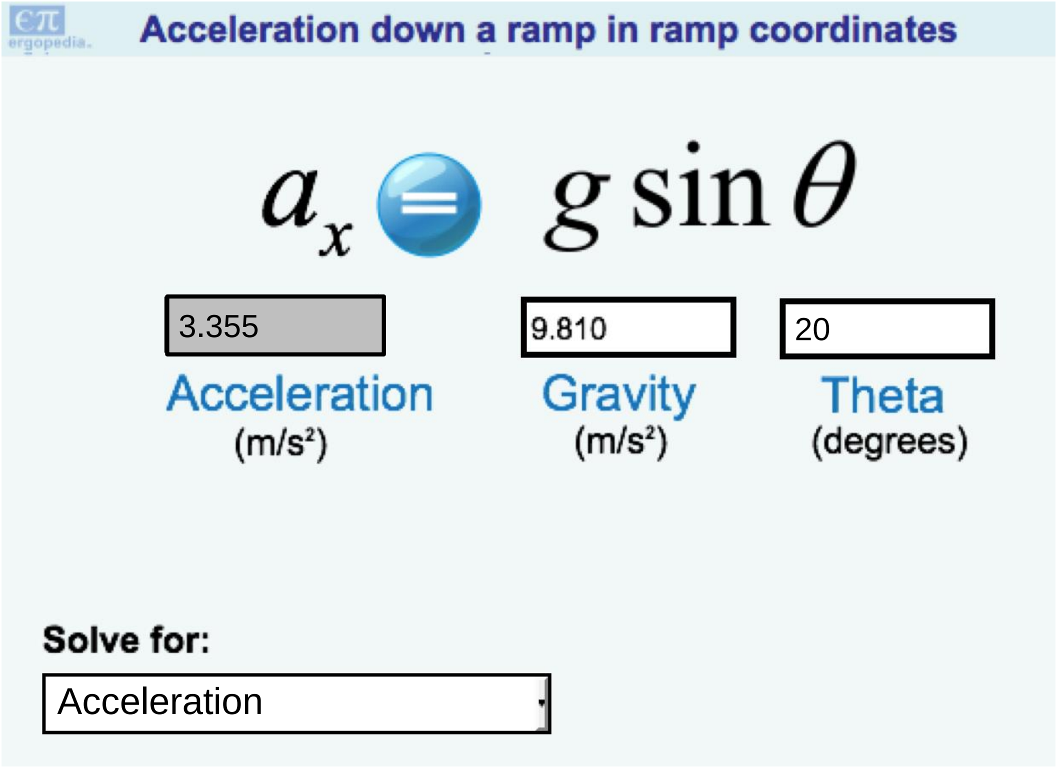
Engaging with the concepts

A ramp is placed at an angle of 20º. What is the acceleration of an object released from the top?

$a = g \sin \theta$

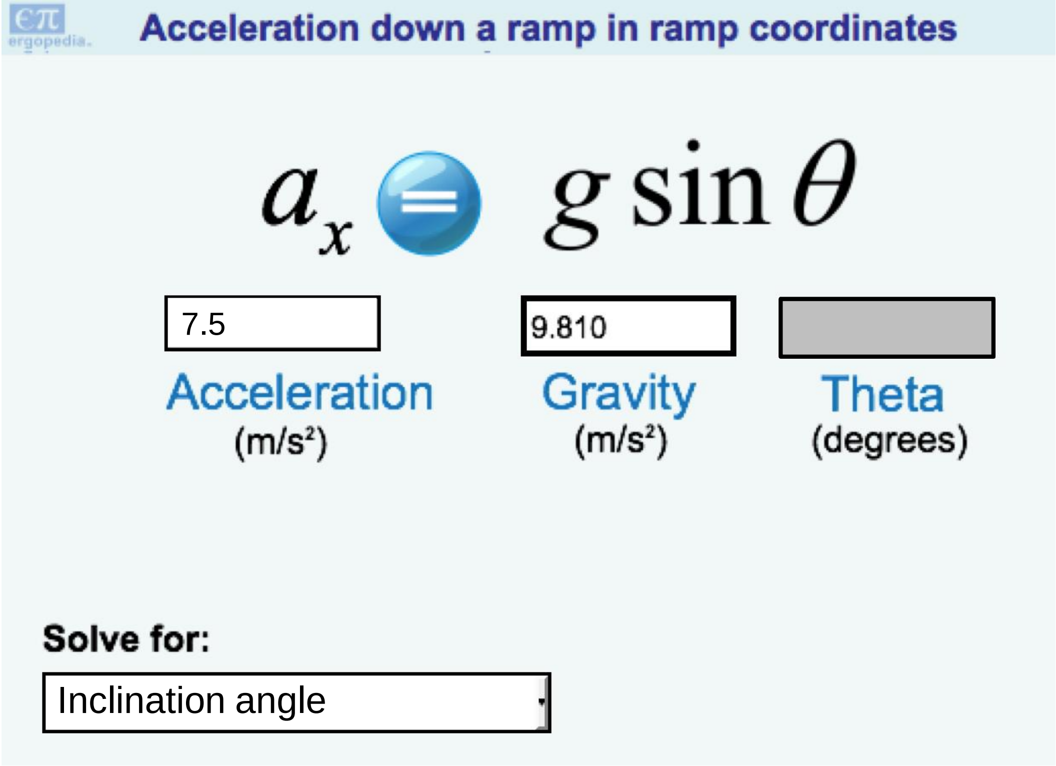
$a = 9.81 \text{ m/s}^2 \sin 20°$

$a = 3.4 \text{ m/s}^2$



Engaging with the concepts

Kojo wants to create a ramp that accelerates objects at 7.5 m/s^2. At what angle should he set his ramp?



Engaging with the concepts

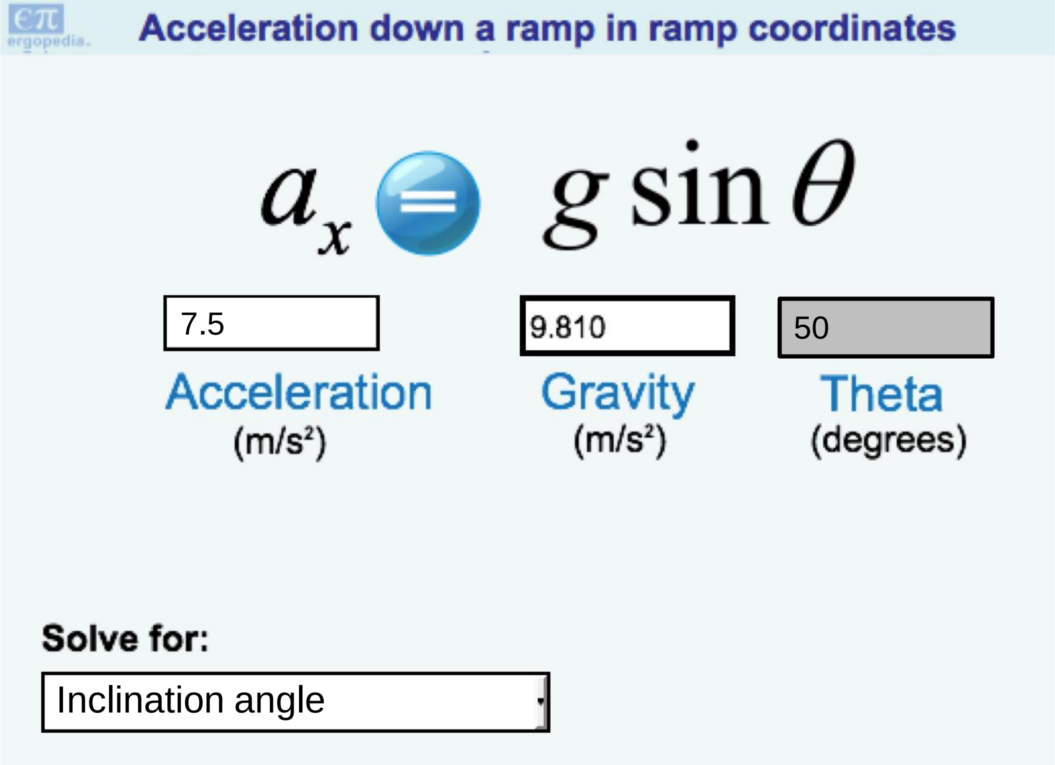
Kojo wants to create a ramp that accelerates objects at 7.5 m/s². At what angle should he set his ramp?

$$a = g \sin \theta$$

$$\sin \theta = 7.5\ \text{m/s}^2 / 9.81\ \text{m/s}^2$$

$$\sin \theta = 0.77$$

$$\theta = 50^\circ$$

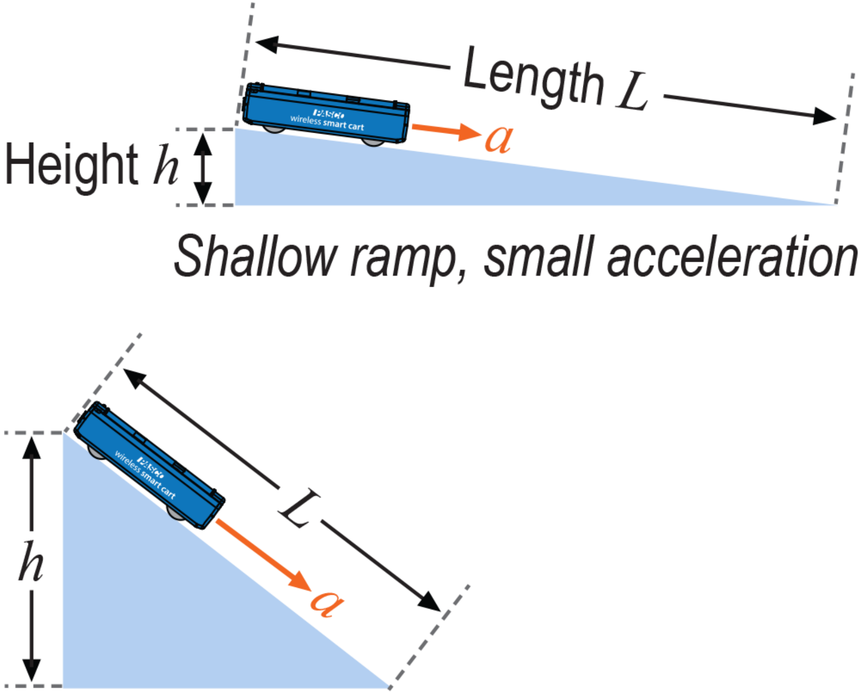


Inclined planes

As you saw in the investigation, the acceleration on an inclined plane <u>increases</u> as the angle increases.

How can you predict the acceleration from the ramp's height and length?

Acceleration down a ramp



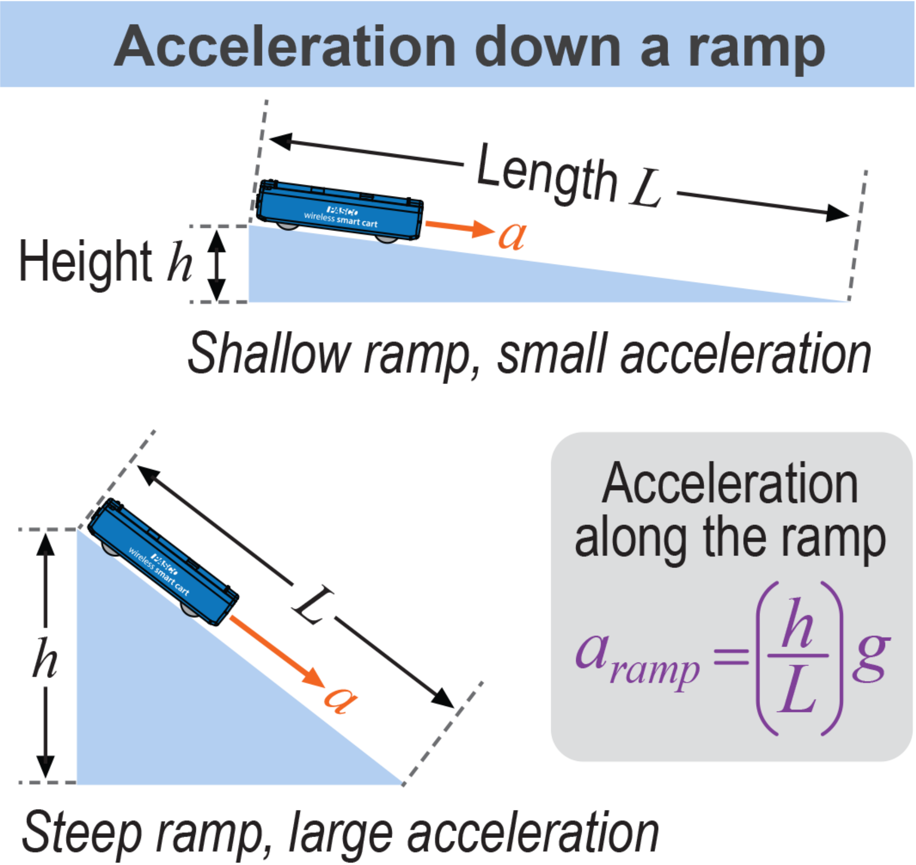
Shallow ramp, small acceleration

Steep ramp, large acceleration

Inclined planes

The relationship between acceleration, height, and length for motion on an inclined plane is:

$$a_{ramp} = \left(\frac{h}{L}\right)g$$



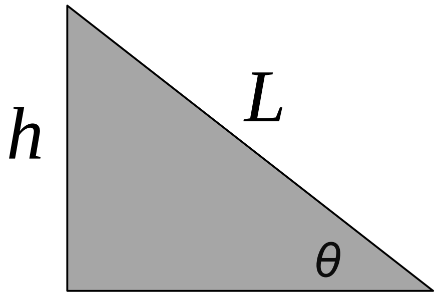
Inclined planes

These two ways to calculate the acceleration on a ramp are equivalent.

$$a_{ramp} = \left(\frac{h}{L}\right) g$$

$$a_{ramp} = g \sin\theta$$

$$\sin \boldsymbol{\theta} = \frac{opposite}{hypotenuse} = \frac{h}{L}$$



Exploring the ideas

Click the interactive calculator on page 193.

Interactive equation

Inclined planes and ramps

You may have ridden a bicycle on a hill and noticed the substantial acceleration going down and the extra force required to go up. A hill is a natural example of an **inclined plane**, which is a flat but sloped surface also called a *ramp*. Because of the angle, a fraction of the object's weight acts downward, along the ramp. The fraction increases as the angle increases and explains why you accelerate faster down a steep hill than a shallow one. The relationship between acceleration and ramp steepness is given in equation (6.11).

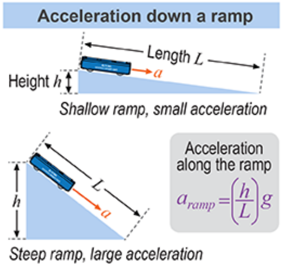
Acceleration down a ramp

$$(6.11)\quad a_{ramp} = \left(\frac{h}{L}\right) g$$

a_{ramp} = acceleration down the ramp (m/s^2)
h = vertical height of the ramp (m)
L = distance along the ramp (m)
g = acceleration due to gravity (m/s^2)

Acceleration varies with the steepness of a ramp

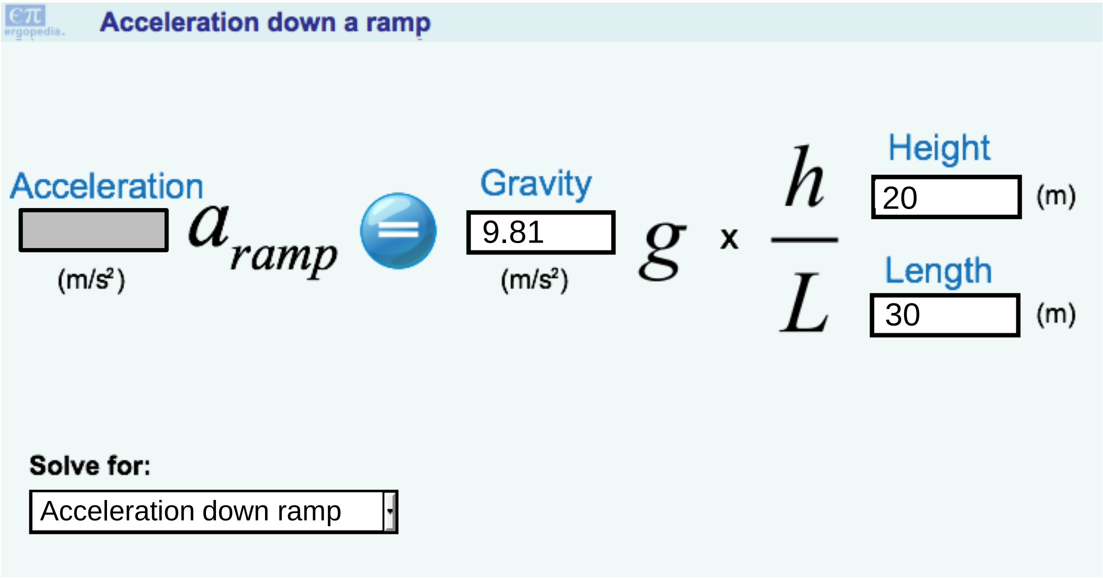
If you ever drove over a high mountain pass, you probably saw signs on the descent warning truck drivers about high speeds. It may seem obvious, but trucks (and cars) can rapidly accelerate to high speeds on steep downhills if their drivers are not careful. The underlying phenomenon is found in the equation above: The acceleration due to gravity increases as the slope of the inclined plane (or ramp) increases. The slope of the hill is given by the ratio (h/L) in equation (6.11).



Engaging with the concepts

Anushka has a frictionless ramp that is 30 m long and 20 m high.

What is the acceleration of an object on the ramp?



Use the calculator to solve!

Engaging with the concepts

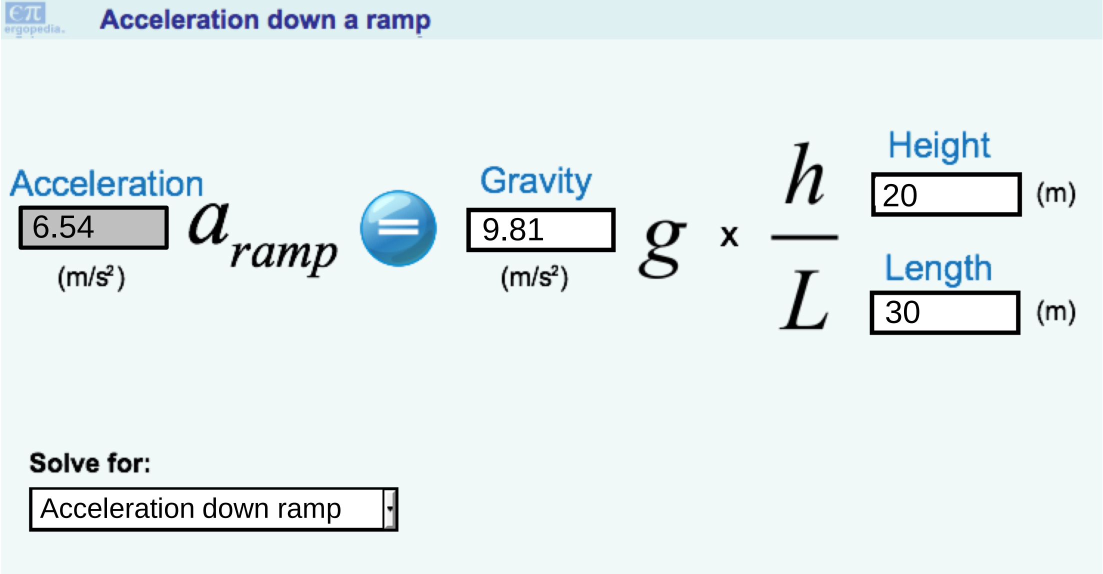
Anushka has a frictionless ramp that is 30 m long and 20 m high.

What is the acceleration of an object on the ramp?

$a = g\ (h/L)$

$a = 9.81\ \text{m/s}^2\ \ (20\ \text{m} / 30\ \text{m})$

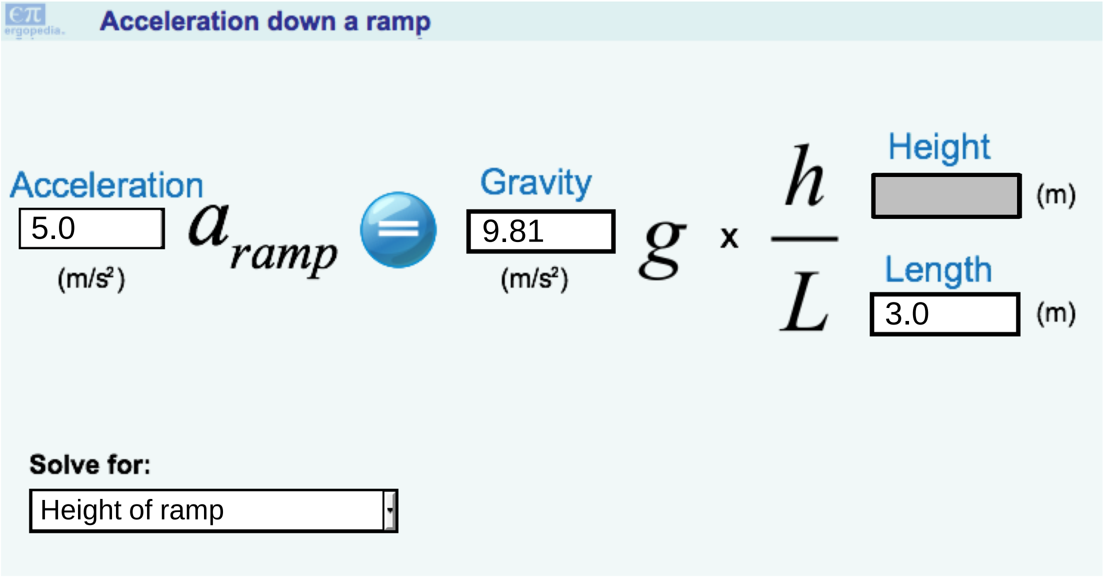
$a = 6.54\ \text{m/s}^2$



Engaging with the concepts

Derek wants to give his car an acceleration of 5.0 m/s².

He has a ramp that is 3.0 m long. How high should he make the ramp?



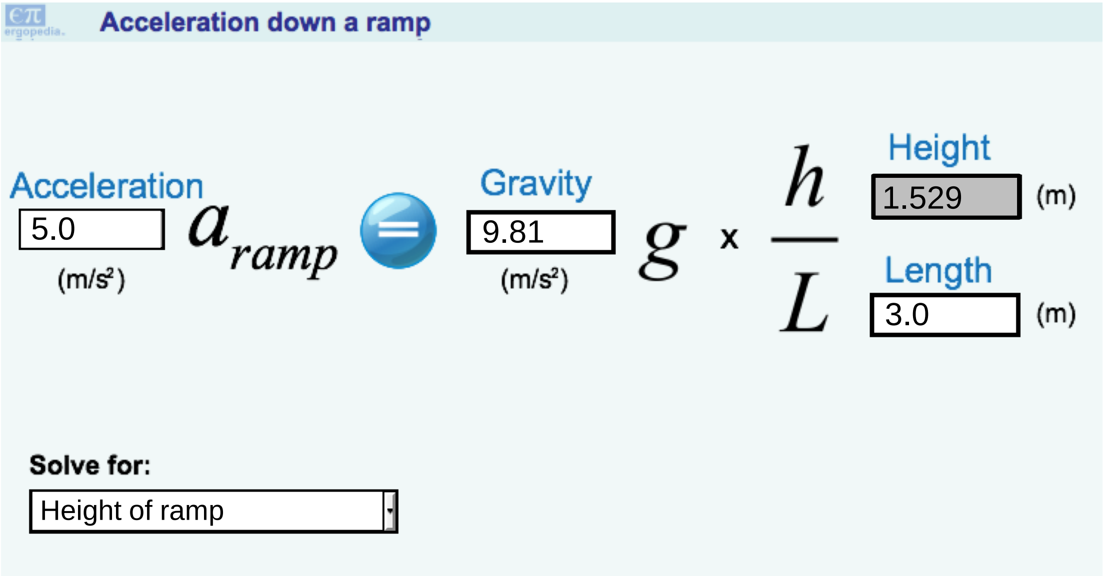
Use the calculator to solve!

Engaging with the concepts

Derek wants to give his car an acceleration of 5.0 m/s².

He has a ramp that is 3.0 m long. How high should he make the ramp?

***h* = 1.5 m**

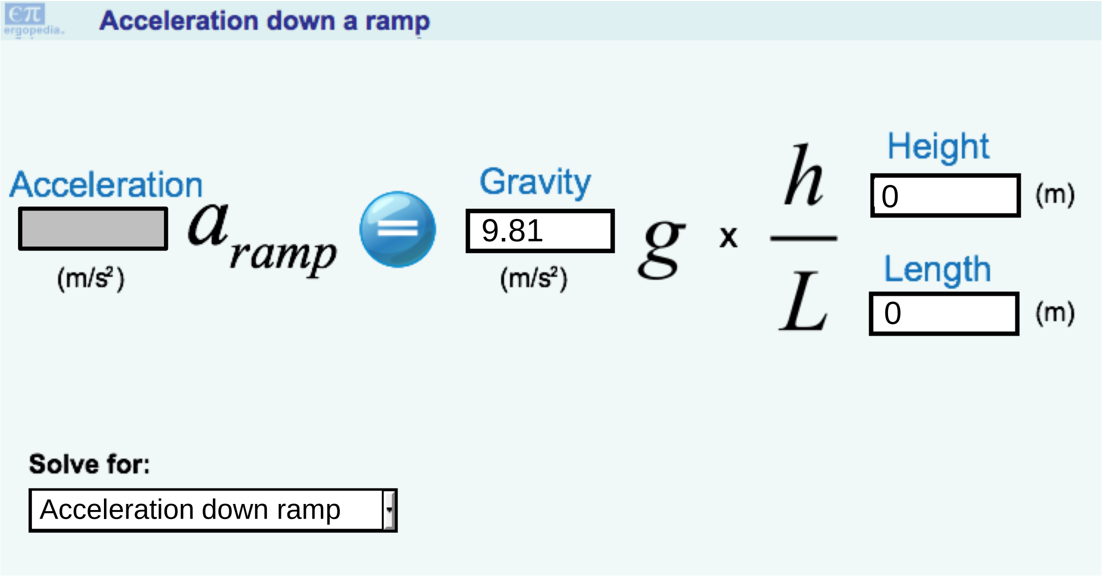


$$a = g\left(\frac{h}{L}\right) \Rightarrow h = \frac{a}{g}L = \frac{5.0\ \text{m/s}^2}{9.81\ \text{m/s}^2}(3.0\ \text{m}) = 1.5\ \text{m}$$

Engaging with the concepts

What is the greatest possible acceleration for an object on a ramp?

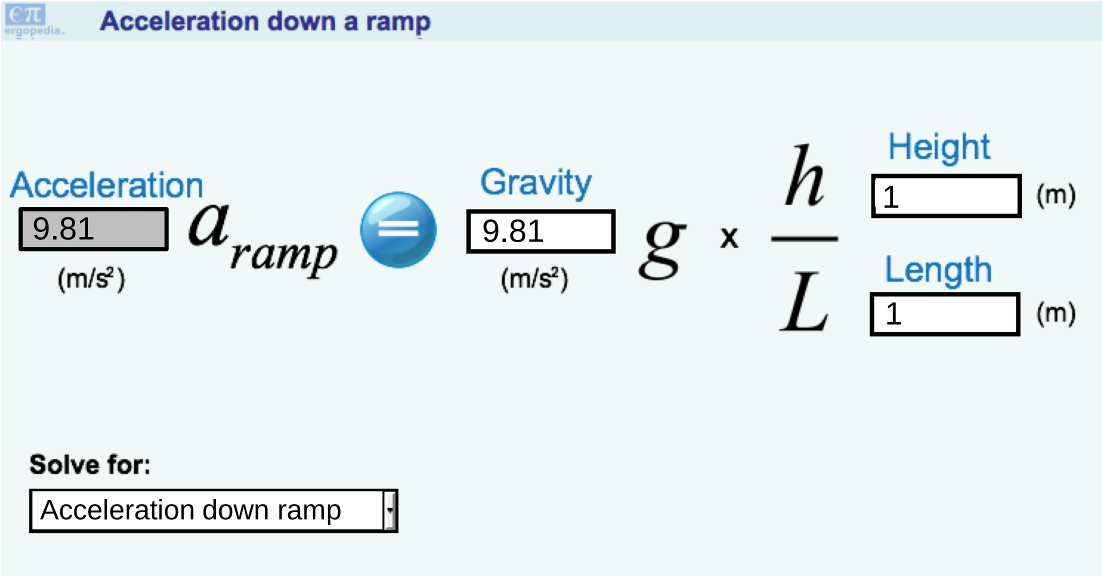
Why?



Engaging with the concepts

What is the greatest possible acceleration for an object on a ramp?

Why?



a_{ramp} can never be greater than g = 9.8 m/s^2

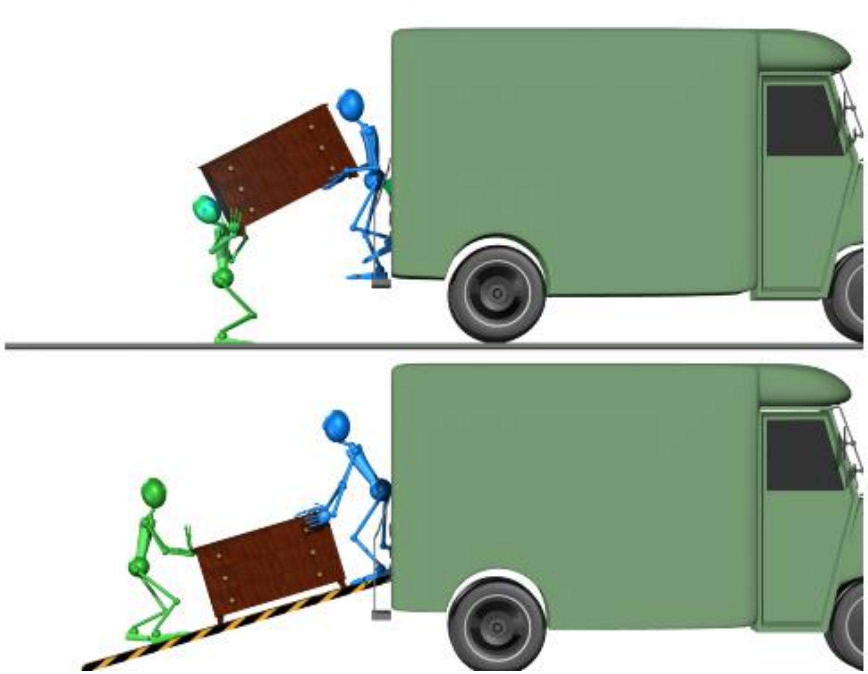
This is because the acceleration on the ramp is caused by gravity.

Inclined planes

So far, you have investigated objects going *down* inclined planes.

Is there anything special about objects going *up* inclined planes?

Which of these jobs looks easier?

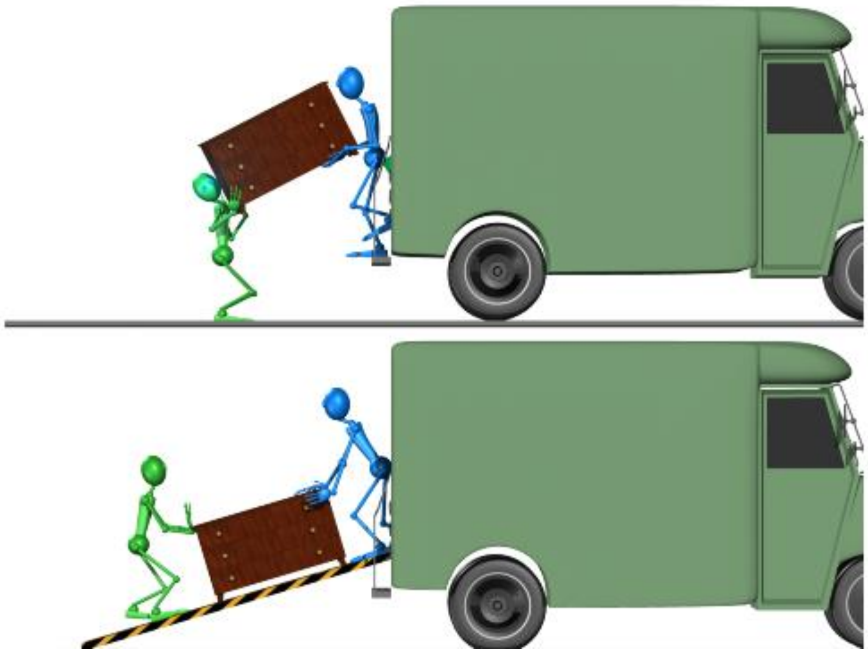


Inclined planes

Inclined planes reduce the amount of force needed to raise an object.

The component of gravity pushing an object down the ramp is rather small compared to mg.

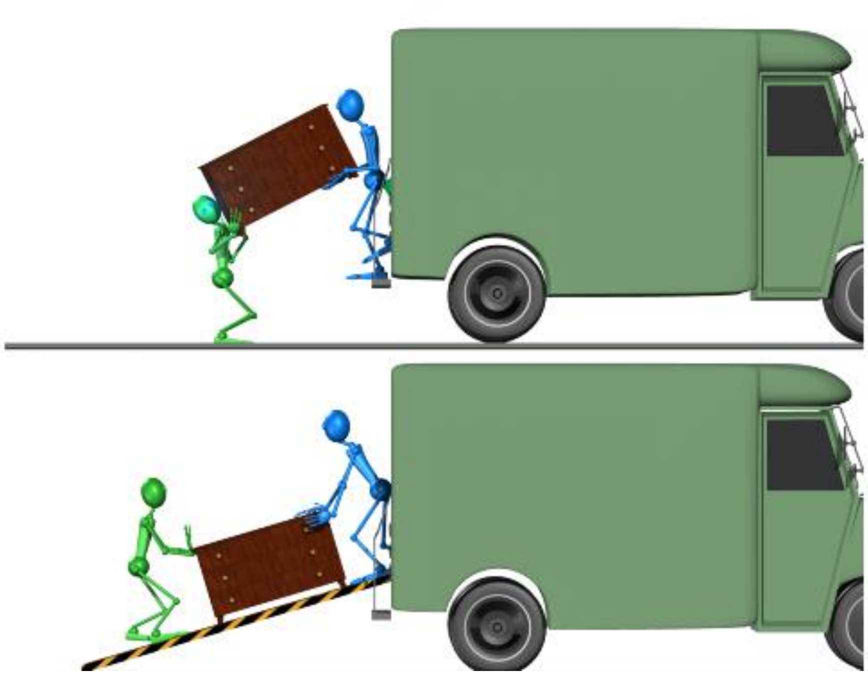
This makes it easier to push something up a ramp than to lift it, assuming there isn't much friction.



Inclined planes

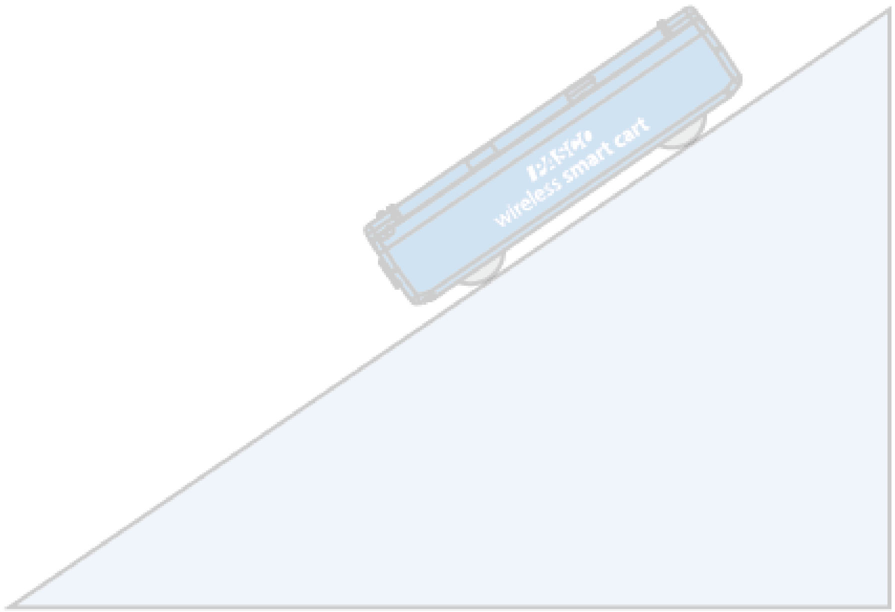
Using a ramp to lift heavy objects is an example of a simple machine.

You’ll learn more about simple machines in a later lesson.



Assessment

1. Winston releases a cart on an inclined plane that is 3.2 m long and 1.8 m high. What is the acceleration of the cart?

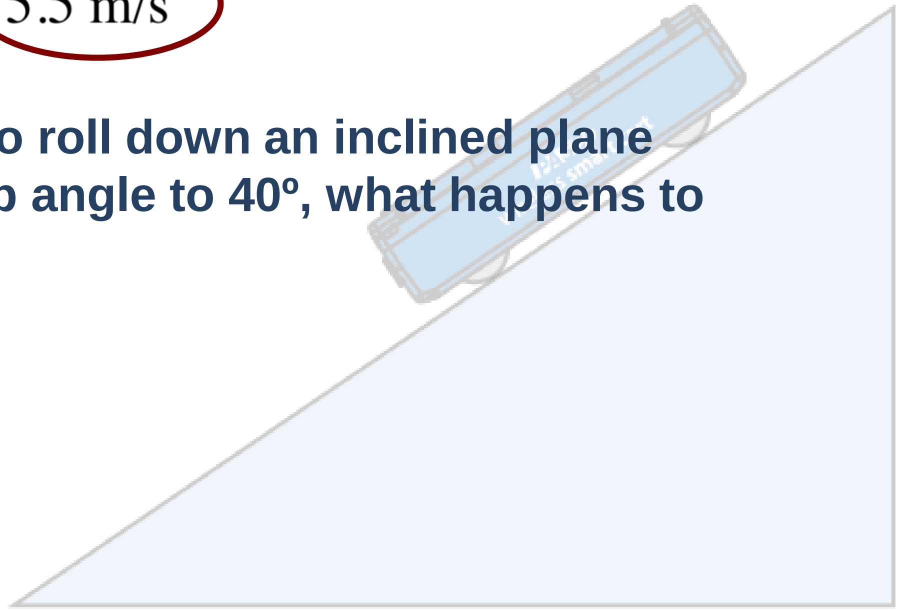


Assessment

1. Winston releases a cart on an inclined plane that is 3.2 m long and 1.8 m high. What is the acceleration of the cart?

$$a = \left(\frac{h}{L}\right)g = \left(\frac{1.8\ \text{m}}{3.2\ \text{m}}\right)9.8\ \text{m/s}^2 = 5.5\ \text{m/s}^2$$

2. Zoe measures the time for a ball to roll down an inclined plane set at 30º. If she changes the ramp angle to 40º, what happens to the time to reach the bottom?
   A. increases
   B. decreases
   C. stays the same
   D. not enough information

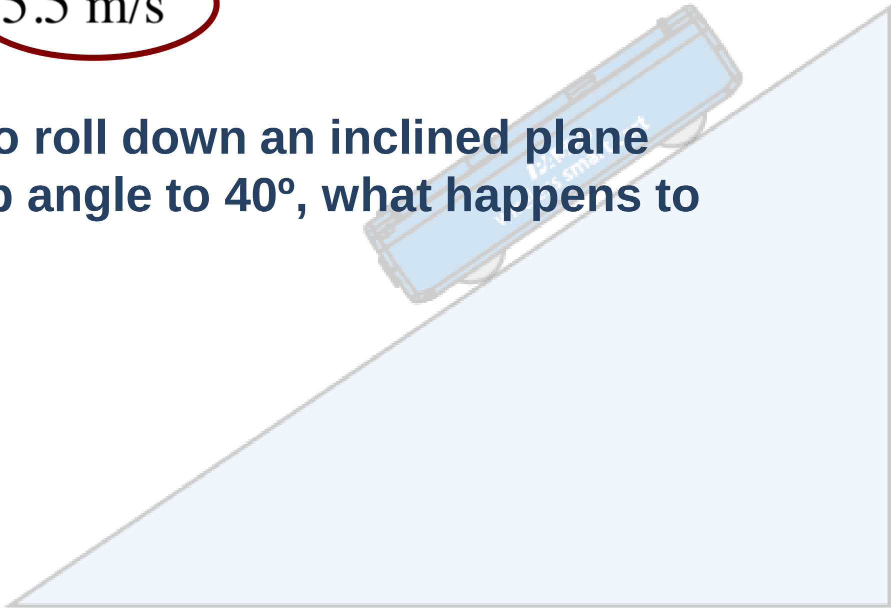


Assessment

1. **Winston releases a cart on an inclined plane that is 3.2 m long and 1.8 m high. What is the acceleration of the cart?**

$$a = \left(\frac{h}{L}\right)g = \left(\frac{1.8\text{ m}}{3.2\text{ m}}\right)9.8\text{ m/s}^2 = 5.5\text{ m/s}^2$$

2. **Zoe measures the time for a ball to roll down an inclined plane set at 30º. If she changes the ramp angle to 40º, what happens to the time to reach the bottom?**
   - A. increases
   - B. decreases
   - C. stays the same
   - D. not enough information



Investigation (Alternate)

Investigation 6D provides an interactive simulation for experimenting with the relationship between ramp angle and acceleration.

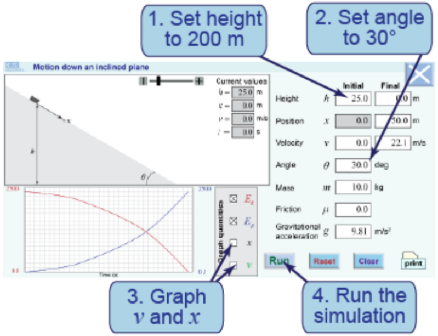
The simulation of a cart on a ramp is on page 195.

Part 1: Position–time and velocity–time graphs

1. Set the initial height h_0 to 200 m.
2. Set the inclination angle θ to 30°.
3. Check the boxes to graph both position and velocity versus time.
4. Run the simulation.

a. *Describe the shapes of the position–time and velocity–time graphs.*

b. *Using the equations of motion, explain why each graph has that shape.*



What are the shapes of the position–time and velocity–time graphs?

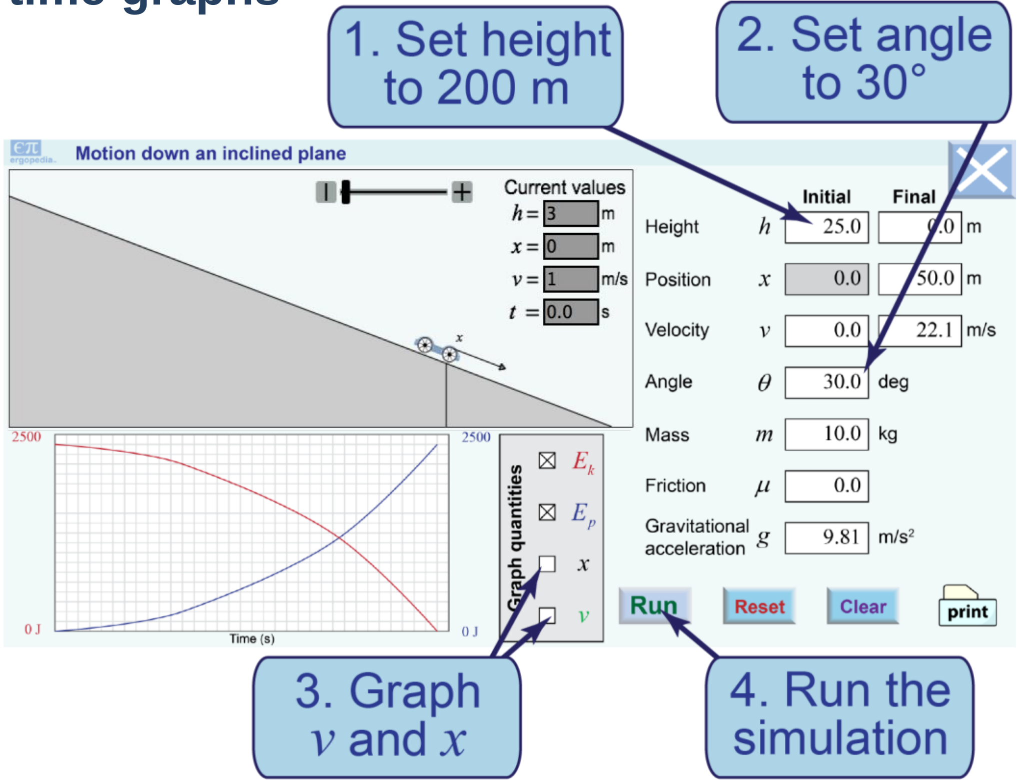
Interactive simulation

In this interactive simulation, you will study motion along an inclined plane. The block slides faster or slower down the plane depending on its steepness. You can plot the position and/or velocity as a function of time *in a rotated reference frame along the plane*.

Investigation (Alternate)

Part 1: Position-time and velocity-time graphs

1. **Set the initial height h_0 to 200 m.**
2. **Set the inclination angle θ to 30° .**
3. **Check the boxes to graph both position and velocity versus time.**
4. **Run the simulation.**

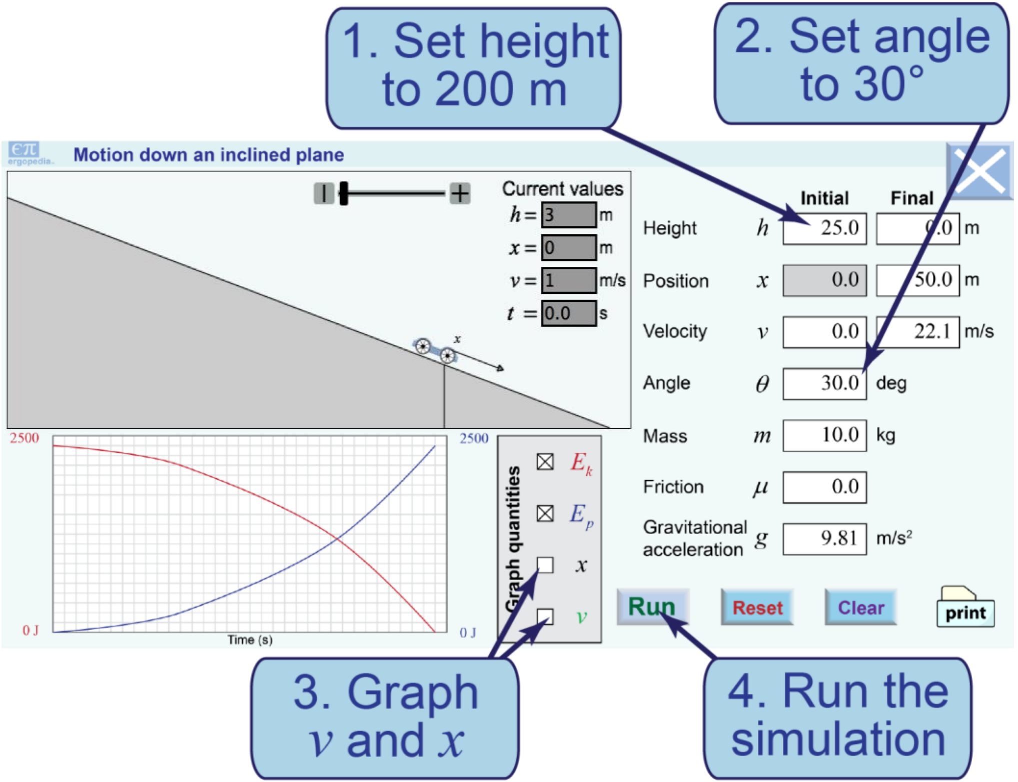


Investigation (Alternate)

Questions for Part 1

a. Describe the shapes of the position-time and velocity-time graphs.

b. Using the equations of motion, explain why each graph has that shape.

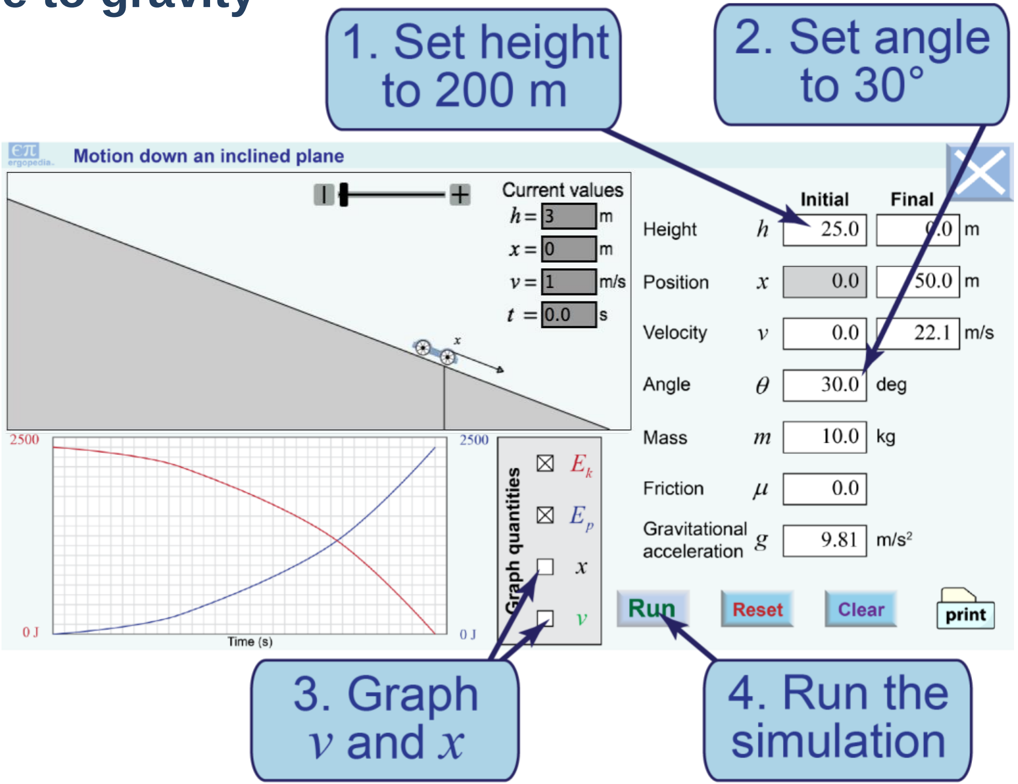


Investigation (Alternate)

Part 2: Measuring acceleration due to gravity

1. **Devise a procedure to measure *g* using the simulation, a table, a graph, and $a = (h/x)\ g$.**

2. **Your procedure should use at least five different values of θ.**



Investigation (Alternate)

Part 2: Measuring acceleration due to gravity

3. Your procedure should include graphing the data, drawing a trend line through the data points, measuring the slope, and using the slope as part of calculating *g*.

Table 1: Acceleration varies with ramp angle

Angle, θ	height, h (m)	distance, x (m)	velocity, v (m/s)	acceleration, a (m/s^2)	g (m/s^2)

Investigation (Alternate)

Questions for Part 2

a. For your written report, write down your procedure.

b. What is the value of the slope of a line through your data?

c. What is the physical meaning of the graph's slope?