

LESSON 10.2 Algebraic Representations of Dilations

 **FL** 8.G.1.3

Describe the effect of dilations, ... on two-dimensional figures using coordinates.



ESSENTIAL QUESTION

How can you describe the effect of a dilation on coordinates using an algebraic representation?

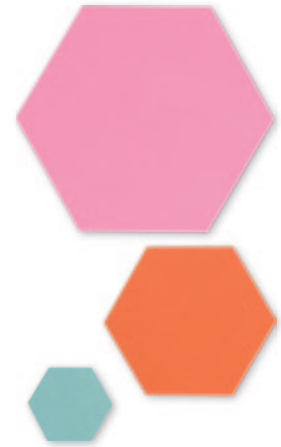
EXPLORE ACTIVITY 1

 **FL** 8.G.1.3

Graphing Enlargements

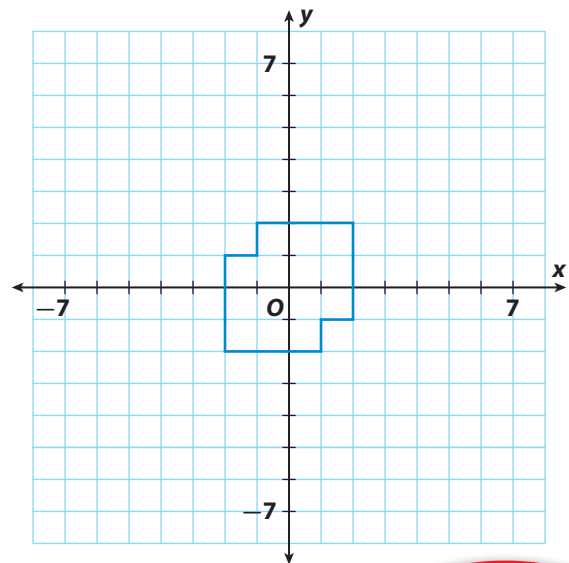
When a dilation in the coordinate plane has the origin as the center of dilation, you can find points on the dilated image by multiplying the x - and y -coordinates of the original figure by the scale factor. For scale factor k , the algebraic representation of the dilation is $(x, y) \rightarrow (kx, ky)$. For enlargements, $k > 1$.

The figure shown on the grid is the preimage. The center of dilation is the origin.



- A** List the coordinates of the vertices of the preimage in the first column of the table.

Preimage (x, y)	Image ($3x, 3y$)
(2, 2)	(6, 6)



- B** What is the scale factor for the dilation? _____
- C** Apply the dilation to the preimage and write the coordinates of the vertices of the image in the second column of the table.
- D** Sketch the image after the dilation on the coordinate grid.

Math Talk

 **Mathematical Practices**

What effect would the dilation $(x, y) \rightarrow (4x, 4y)$ have on the radius of a circle?

EXPLORE ACTIVITY 1 (cont'd)

Reflect

- How does the dilation affect the length of line segments?

- How does the dilation affect angle measures?

EXPLORE ACTIVITY 2



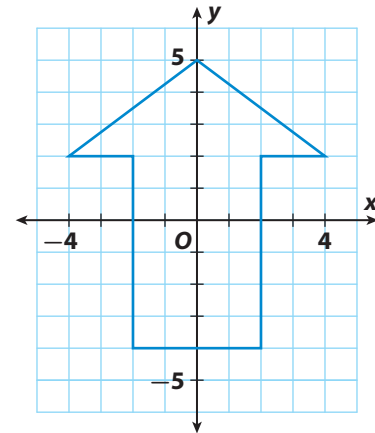
Graphing Reductions

For scale factors between 0 and 1, the image is smaller than the preimage. This is called a reduction.

The arrow shown is the preimage. The center of dilation is the origin.

- List the coordinates of the vertices of the preimage in the first column of the table.
- What is the scale factor for the dilation? _____
- Apply the dilation to the preimage and write the coordinates of the vertices of the image in the second column of the table.
- Sketch the image after the dilation on the coordinate grid.

Preimage (x, y)	Image ($\frac{1}{2}x, \frac{1}{2}y$)



Reflect

- How does the dilation affect the length of line segments?

- How would a dilation with scale factor 1 affect the preimage?

Center of Dilation Outside the Image

The center of dilation can be inside or outside the original image and the dilated image. The center of dilation can be anywhere on the coordinate plane as long as the lines that connect each pair of corresponding vertices between the original and dilated image intersect at the center of dilation.



EXAMPLE 1 FL 8.G.1.3

Graph the image of $\triangle ABC$ after a dilation with the origin as its center and a scale factor of 3. What are the vertices of the image?

STEP 1 Multiply each coordinate of the vertices of $\triangle ABC$ by 3 to find the vertices of the dilated image.

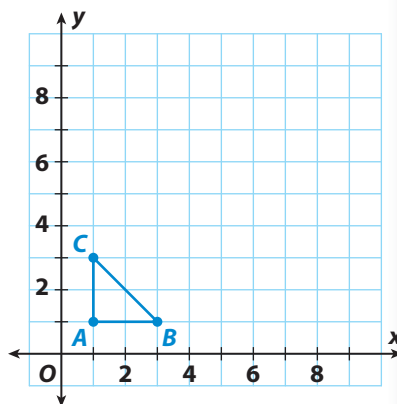
$$\triangle ABC (x, y) \rightarrow (3x, 3y) \triangle A'B'C'$$

$$A(1, 1) \rightarrow A'(1 \cdot 3, 1 \cdot 3) \rightarrow A'(3, 3)$$

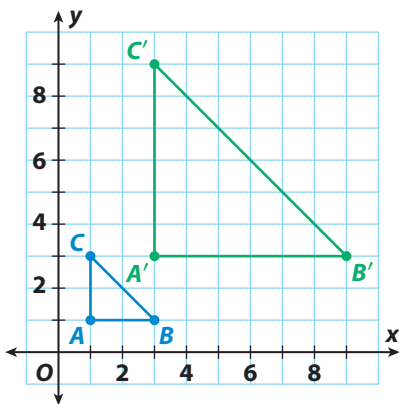
$$B(3, 1) \rightarrow B'(3 \cdot 3, 1 \cdot 3) \rightarrow B'(9, 3)$$

$$C(1, 3) \rightarrow C'(1 \cdot 3, 3 \cdot 3) \rightarrow C'(3, 9)$$

The vertices of the dilated image are $A'(3, 3)$, $B'(9, 3)$, and $C'(3, 9)$.



STEP 2 Graph the dilated image.

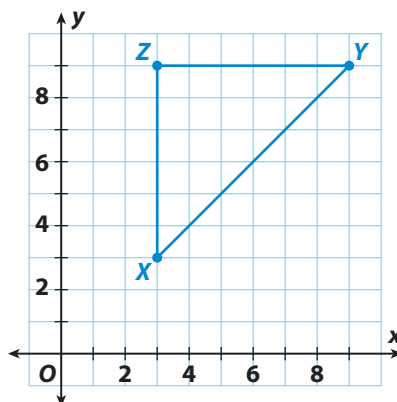


Math Talk
Mathematical Practices

Describe how you can check graphically that you have drawn the image triangle correctly.

YOUR TURN

5. Graph the image of $\triangle XYZ$ after a dilation with a scale factor of $\frac{1}{3}$ and the origin as its center. Then write an algebraic rule to describe the dilation.

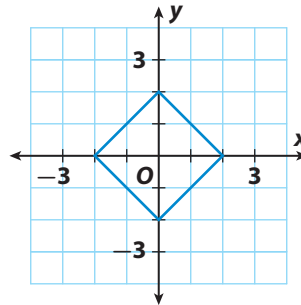


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Guided Practice

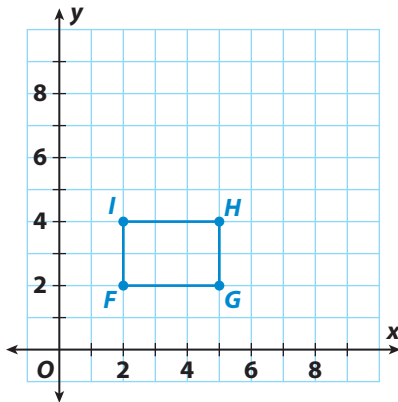
1. The grid shows a diamond-shaped preimage. Write the coordinates of the vertices of the preimage in the first column of the table. Then apply the dilation $(x, y) \rightarrow (\frac{3}{2}x, \frac{3}{2}y)$ and write the coordinates of the vertices of the image in the second column. Sketch the image of the figure after the dilation. (Explore Activities 1 and 2)

Preimage	Image
(2, 0)	(3, 0)

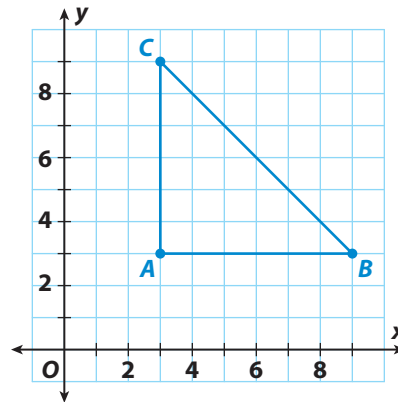


Graph the image of each figure after a dilation with the origin as its center and the given scale factor. Then write an algebraic rule to describe the dilation. (Example 1)

2. scale factor of 1.5



3. scale factor of $\frac{1}{3}$






ESSENTIAL QUESTION CHECK-IN

4. A dilation of $(x, y) \rightarrow (kx, ky)$ when $0 < k < 1$ has what effect on the figure? What is the effect on the figure when $k > 1$?

10.2 Independent Practice

 **FL** 8.G.1.3

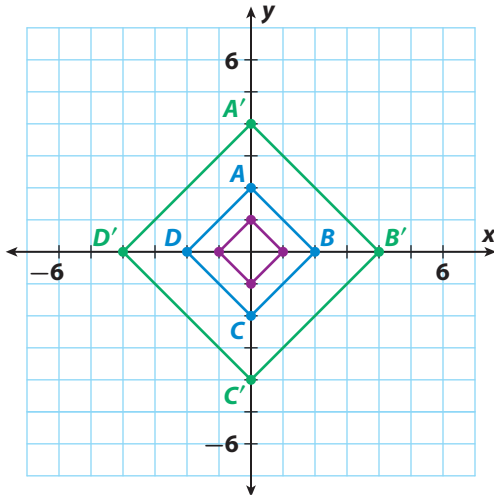


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5. The blue square is the preimage. Write two algebraic representations, one for the dilation to the green square and one for the dilation to the purple square.



6. **Critical Thinking** A triangle has vertices $A(-5, -4)$, $B(2, 6)$, and $C(4, -3)$. The center of dilation is the origin and $(x, y) \rightarrow (3x, 3y)$. What are the vertices of the dilated image?

7. **Critical Thinking** $M'N'O'P'$ has vertices at $M'(3, 4)$, $N'(6, 4)$, $O'(6, 7)$, and $P'(3, 7)$. The center of dilation is the origin. $MNOP$ has vertices at $M(4.5, 6)$, $N(9, 6)$, $O'(9, 10.5)$, and $P'(4.5, 10.5)$. What is the algebraic representation of this dilation?

8. **Critical Thinking** A dilation with center $(0,0)$ and scale factor k is applied to a polygon. What dilation can you apply to the image to return it to the original preimage?

9. **Represent Real-World Problems** The blueprints for a new house are scaled so that $\frac{1}{4}$ inch equals 1 foot. The blueprint is the preimage and the house is the dilated image. The blueprints are plotted on a coordinate plane.

a. What is the scale factor in terms of inches to inches?

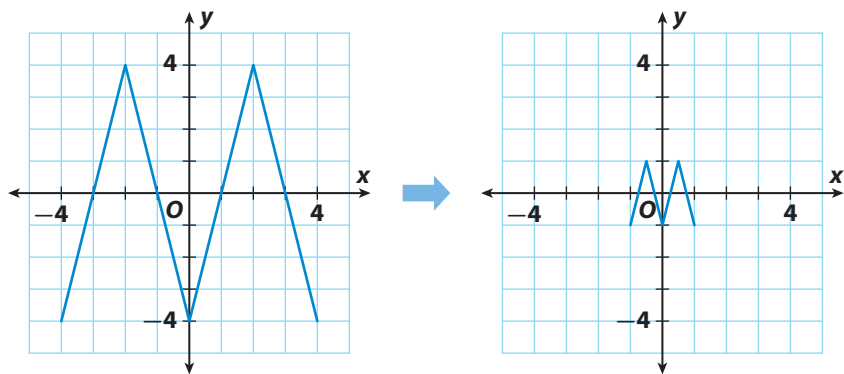
b. One inch on the blueprint represents how many inches in the actual house? How many feet?

c. Write the algebraic representation of the dilation from the blueprint to the house.

d. A rectangular room has coordinates $Q(2, 2)$, $R(7, 2)$, $S(7, 5)$, and $T(2, 5)$ on the blueprint. The homeowner wants this room to be 25% larger. What are the coordinates of the new room?

e. What are the dimensions of the new room, in inches, on the blueprint? What will the dimensions of the new room be, in feet, in the new house?

10. Write the algebraic representation of the dilation shown.



H.O.T. FOCUS ON HIGHER ORDER THINKING

11. **Critique Reasoning** The set for a school play needs a replica of a historic building painted on a backdrop that is 20 feet long and 16 feet high. The actual building measures 400 feet long and 320 feet high. A stage crewmember writes $(x, y) \rightarrow (\frac{1}{12}x, \frac{1}{12}y)$ to represent the dilation. Is the crewmember's calculation correct if the painted replica is to cover the entire backdrop? Explain.

12. **Communicate Mathematical Ideas** Explain what each of these algebraic transformations does to a figure.

a. $(x, y) \rightarrow (y, -x)$ _____

b. $(x, y) \rightarrow (-x, -y)$ _____

c. $(x, y) \rightarrow (x, 2y)$ _____

d. $(x, y) \rightarrow (\frac{2}{3}x, y)$ _____

e. $(x, y) \rightarrow (0.5x, 1.5y)$ _____

13. **Communicate Mathematical Ideas** Triangle ABC has coordinates $A(1, 5)$, $B(-2, 1)$, and $C(-2, 4)$. Sketch triangle ABC and $A'B'C'$ for the dilation $(x, y) \rightarrow (-2x, -2y)$. What is the effect of a negative scale factor?

Work Area