Testing the Converse of the Pythagorean Theorem

The Pythagorean Theorem states that if a triangle is a right triangle, then \( a^2 + b^2 = c^2 \).

The **converse** of the Pythagorean Theorem states that if \( a^2 + b^2 = c^2 \), then the triangle is a right triangle.

Decide whether the converse of the Pythagorean Theorem is true.

A. Verify that the following sets of lengths make the equation \( a^2 + b^2 = c^2 \) true. Record your results in the table.

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<th>Is ( a^2 + b^2 = c^2 ) true?</th>
<th>Makes a right triangle?</th>
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B. For each set of lengths in the table, cut strips of grid paper with a width of one square and lengths that correspond to the values of \( a \), \( b \), and \( c \).

C. For each set of lengths, use the strips of grid paper to try to form a right triangle. An example using the first set of lengths is shown. Record your findings in the table.

Reflect

1. **Draw Conclusions** Based on your observations, explain whether you think the converse of the Pythagorean Theorem is true.
Identifying a Right Triangle

The converse of the Pythagorean Theorem gives you a way to tell if a triangle is a right triangle when you know the side lengths.

**EXAMPLE 1**

Tell whether each triangle with the given side lengths is a right triangle.

**A** 9 inches, 40 inches, and 41 inches

Let $a = 9$, $b = 40$, and $c = 41$.

$$a^2 + b^2 = c^2$$

$$9^2 + 40^2 \overset{?}{=} 41^2$$  \hspace{1cm} \text{Substitute into the formula.}

$$81 + 1600 \overset{?}{=} 1681$$  \hspace{1cm} \text{Simplify.}

$$1681 = 1681$$  \hspace{1cm} \text{Add.}

Since $9^2 + 40^2 = 41^2$, the triangle is a right triangle by the converse of the Pythagorean Theorem.

**B** 8 meters, 10 meters, and 12 meters

Let $a = 8$, $b = 10$, and $c = 12$.

$$a^2 + b^2 = c^2$$

$$8^2 + 10^2 \overset{?}{=} 12^2$$  \hspace{1cm} \text{Substitute into the formula.}

$$64 + 100 \overset{?}{=} 144$$  \hspace{1cm} \text{Simplify.}

$$164 \neq 144$$  \hspace{1cm} \text{Add.}

Since $8^2 + 10^2 \neq 12^2$, the triangle is not a right triangle by the converse of the Pythagorean Theorem.

**YOUR TURN**

Tell whether each triangle with the given side lengths is a right triangle.

2. 14 cm, 23 cm, and 25 cm

3. 16 in., 30 in., and 34 in.

4. 27 ft, 36 ft, 45 ft

5. 11 mm, 18 mm, 21 mm
Using the Converse of the Pythagorean Theorem

You can use the converse of the Pythagorean Theorem to solve real-world problems.

**EXAMPLE 2**

Katya is buying edging for a triangular flower garden she plans to build in her backyard. If the lengths of the three pieces of edging that she purchases are 13 feet, 10 feet, and 7 feet, will the flower garden be in the shape of a right triangle?

Use the converse of the Pythagorean Theorem. Remember to use the longest length for $c$.

Let $a = 7$, $b = 10$, and $c = 13$.

$$a^2 + b^2 = c^2$$

$$7^2 + 10^2 \neq 13^2$$

Substitute into the formula.

$$49 + 100 \neq 169$$

Simplify.

$$149 \neq 169$$

Add.

Since $7^2 + 10^2 \neq 13^2$, the garden will not be in the shape of a right triangle.

**YOUR TURN**

6. A blueprint for a new triangular playground shows that the sides measure 480 ft, 140 ft, and 500 ft. Is the playground in the shape of a right triangle? Explain.

7. A triangular piece of glass has sides that measure 18 in., 19 in., and 25 in. Is the piece of glass in the shape of a right triangle? Explain.

8. A corner of a fenced yard forms a right angle. Can you place a 12 foot long board across the corner to form a right triangle for which the leg lengths are whole numbers? Explain.
1. Lashandra used grid paper to construct the triangle shown. (Explore Activity)
   a. What are the lengths of the sides of Lashandra’s triangle?
      _______ units, _______ units, _______ units
   b. Use the converse of the Pythagorean Theorem to determine whether the triangle is a right triangle.

      \[ a^2 + b^2 = c^2 \]
      \[ \square^2 + \square^2 \neq \square^2 \]
      \[ \square^2 + \square^2 \neq \square^2 \]
      \[ \square^2 + \square^2 \neq \square^2 \]

      The triangle that Lashandra constructed is / is not a right triangle.

2. A triangle has side lengths 9 cm, 12 cm, and 16 cm. Tell whether the triangle is a right triangle. (Example 1)
   Let \( a = \) _______, \( b = \) _______, and \( c = \) _______.

      \[ a^2 + b^2 = c^2 \]
      \[ \square^2 + \square^2 \neq \square^2 \]
      \[ \square^2 + \square^2 \neq \square^2 \]
      \[ \square^2 + \square^2 \neq \square^2 \]

      By the converse of the Pythagorean Theorem, the triangle is / is not a right triangle.

3. The marketing team at a new electronics company is designing a logo that contains a circle and a triangle. On one design, the triangle’s side lengths are 2.5 in., 6 in., and 6.5 in. Is the triangle a right triangle? Explain. (Example 2)

4. How can you use the converse of the Pythagorean Theorem to tell if a triangle is a right triangle?
Tell whether each triangle with the given side lengths is a right triangle.

5. 11 cm, 60 cm, 61 cm

6. 5 ft, 12 ft, 15 ft

7. 9 in., 15 in., 17 in.

8. 15 m, 36 m, 39 m

9. 20 mm, 30 mm, 40 mm

10. 20 cm, 48 cm, 52 cm

11. 18.5 ft, 6 ft, 17.5 ft

12. 2 mi, 1.5 mi, 2.5 mi

13. 35 in., 45 in., 55 in.

14. 25 cm, 14 cm, 23 cm

15. The emblem on a college banner consists of the face of a tiger inside a triangle. The lengths of the sides of the triangle are 13 cm, 14 cm, and 15 cm. Is the triangle a right triangle? Explain.

16. Kerry has a large triangular piece of fabric that she wants to attach to the ceiling in her bedroom. The sides of the piece of fabric measure 4.8 ft, 6.4 ft, and 8 ft. Is the fabric in the shape of a right triangle? Explain.

17. A mosaic consists of triangular tiles. The smallest tiles have side lengths 6 cm, 10 cm, and 12 cm. Are these tiles in the shape of right triangles? Explain.

18. History In ancient Egypt, surveyors made right angles by stretching a rope with evenly spaced knots as shown. Explain why the rope forms a right angle.
19. **Justify Reasoning** Yoshi has two identical triangular boards as shown. Can he use these two boards to form a rectangle? Explain.

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20. **Critique Reasoning** Shoshanna says that a triangle with side lengths 17 m, 8 m, and 15 m is not a right triangle because $17^2 + 8^2 = 353$, $15^2 = 225$, and $353 \neq 225$. Is she correct? Explain.

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21. **Make a Conjecture** Diondre says that he can take any right triangle and make a new right triangle just by doubling the side lengths. Is Diondre’s conjecture true? Test his conjecture using three different right triangles.

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22. **Draw Conclusions** A diagonal of a parallelogram measures 37 inches. The sides measure 35 inches and 1 foot. Is the parallelogram a rectangle? Explain your reasoning.

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23. **Represent Real-World Problems** A soccer coach is marking the lines for a soccer field on a large recreation field. The dimensions of the field are to be 90 yards by 48 yards. Describe a procedure she could use to confirm that the sides of the field meet at right angles.

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