

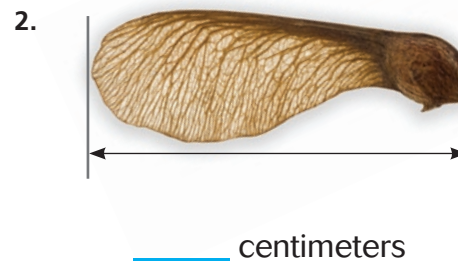
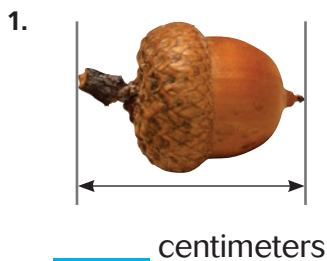
Show What You Know



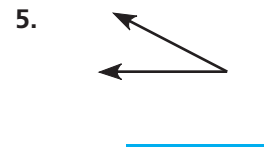
Check your understanding of important skills.

Name _____

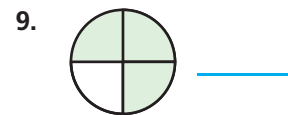
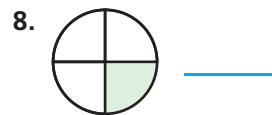
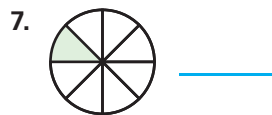
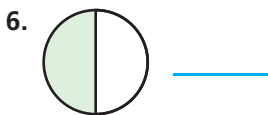
- **Use a Metric Ruler** Use a centimeter ruler to measure.
Find the length in centimeters.



- **Classify Angles** Classify the angle. Write *acute*, *right*, or *obtuse*.



- **Parts of a Whole** Write a fraction for each shaded part.

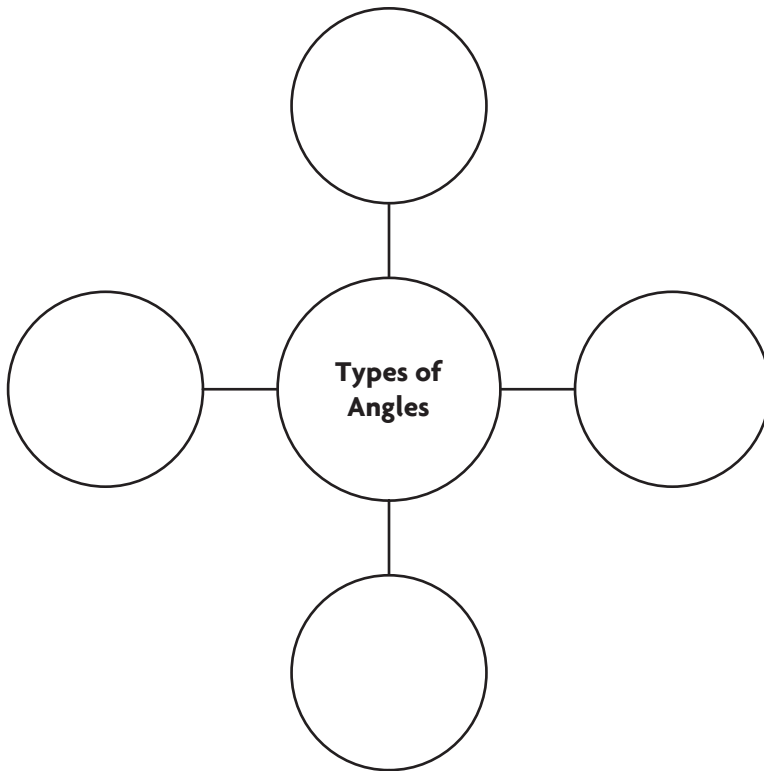


The Sunshine Skyway Bridge crosses over Tampa Bay, Florida. Bridges and other building structures can model geometric figures. Be a Math Detective and investigate the bridge. Describe the geometric figures you see. Then classify the labeled angles and triangle.

Vocabulary Builder

► Visualize It

Complete the Bubble Map using review words.



Review Words

acute
circle
obtuse
ray
right
straight
vertex

Preview Words

clockwise
counterclockwise
degree ($^{\circ}$)
protractor

► Understand Vocabulary

Draw a line to match each word with its definition.

- | | |
|--------------------------|--|
| 1. protractor | • In the same direction in which the hands of a clock move |
| 2. degree ($^{\circ}$) | • In the opposite direction in which the hands of a clock move |
| 3. clockwise | • A tool for measuring the size of an angle |
| 4. counterclockwise | • The unit used for measuring angles |

Name _____

Angles and Fractional Parts of a Circle

Essential Question How can you relate angles and fractional parts of a circle?

Investigate

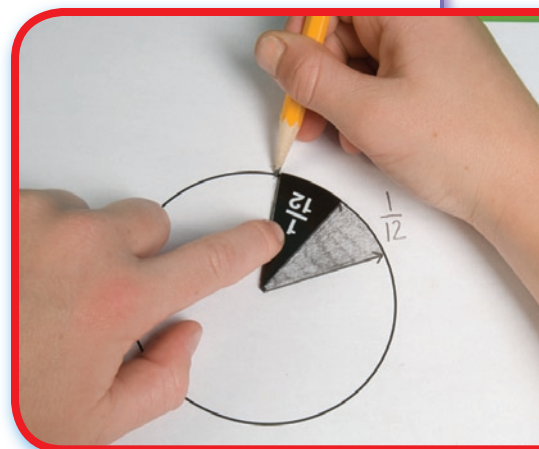
Materials ■ fraction circles

- A.** Place a $\frac{1}{12}$ piece on the circle. Place the tip of the fraction piece on the center of the circle. Trace the fraction piece.

What figure is formed by the fraction piece? _____

What parts of the fraction piece represent the rays of the angle? _____

On what part of the circle is the vertex of the angle? _____



- B.** Shade the angle formed by the $\frac{1}{12}$ piece. Label it $\frac{1}{12}$.

- C.** Place the $\frac{1}{12}$ piece back on the shaded angle. Turn it counterclockwise.
Counterclockwise is the direction opposite from the way the hands move on a clock.

Trace the fraction piece in its new position. How many twelfths have you

traced in all? _____ Label $\frac{2}{12}$.

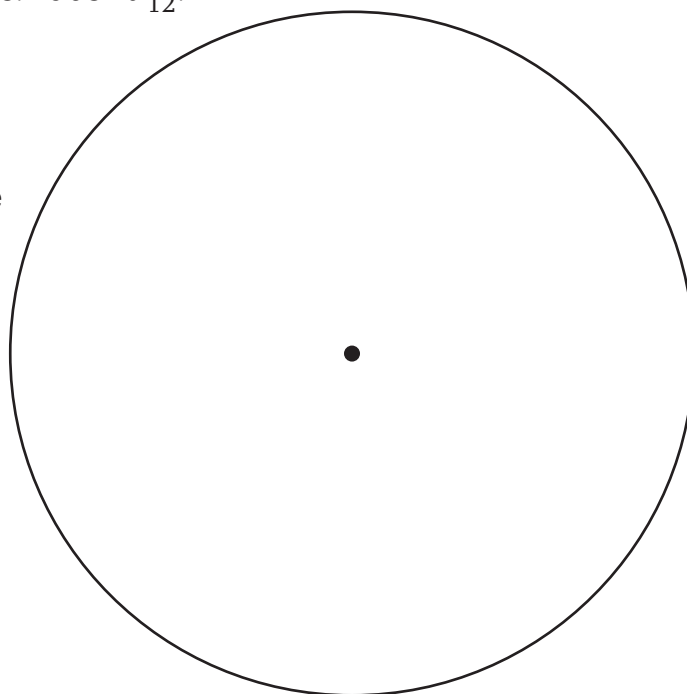
- D.** Turn the fraction piece counterclockwise again and trace it. Label the total number of twelfths.

Continue until you reach the shaded angle.

What figure is formed by turning and tracing the fraction piece? _____

How many times did you need to turn the $\frac{1}{12}$ piece to make a circle? _____

How many angles come together in the center of the circle? _____



Draw Conclusions

1. **Compare** the size of the angle formed by a $\frac{1}{4}$ piece and the size of the angle formed by a $\frac{1}{12}$ piece. Use a $\frac{1}{4}$ piece and your model on page 417 to help.

2. **H.O.T. Synthesize** Describe the relationship between the size of the fraction piece and the number of turns it takes to make a circle.

Make Connections

You can relate fractions and angles to the hands of a clock.

Let the hands of the clock represent the rays of an angle. Each 5-minute mark represents a $\frac{1}{12}$ turn **clockwise**.



15 minutes elapse.

The minute hand makes a

_____ turn clockwise.



30 minutes elapse.

The minute hand makes a

_____ turn clockwise.



45 minutes elapse.

The minute hand makes a

_____ turn clockwise.



60 minutes elapse.

The minute hand makes a

_____ turn clockwise.

Math Talk **MATHEMATICAL PRACTICES** Explain how an angle formed in a circle using a $\frac{1}{4}$ fraction piece is like a $\frac{1}{4}$ turn and 15 minutes elapsing on a clock.

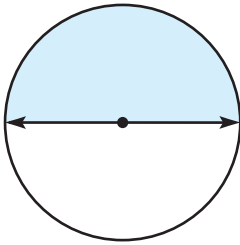
Name _____

Share and Show

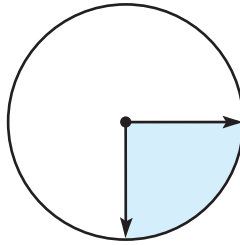


Tell what fraction of the circle the shaded angle represents.

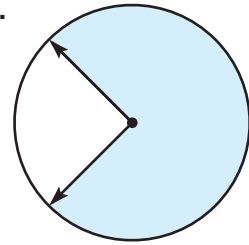
1.



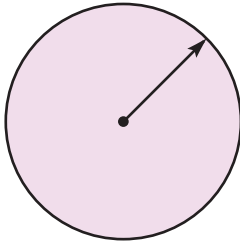
2.



3.

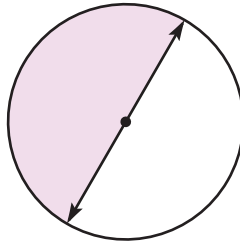


4.

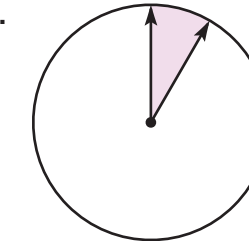




5.

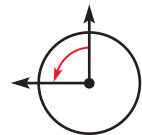


6.

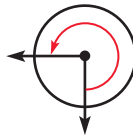


Tell whether the angle on the circle shows a $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or 1 full turn clockwise or counterclockwise.

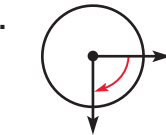
7.



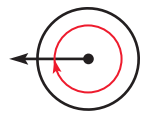
8.



9.



10.

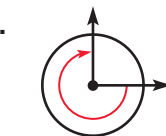


11.





12.



13. **H.O.T.** Susan watched the game from 1 P.M. to 1:30 P.M. **Describe** the turn the minute hand made.

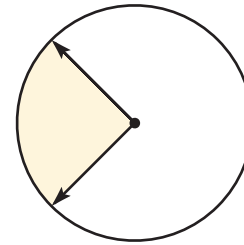


14. **Write Math** Compare the angles in Exercises 1 and 5. Does the position of the angle affect the size of the angle? **Explain.**

Problem Solving

H.O.T. Sense or Nonsense?

15. Whose statement makes sense? Whose statement is nonsense? **Explain** your reasoning.



The shaded angle represents $\frac{1}{4}$ of the circle.

The shaded angle represents $\frac{3}{8}$ of the circle.



Carla's Statement

Adam's Statement

- For the statement that is nonsense, write a statement that makes sense.

- What is another way to describe the size of the angle? **Explain.**

Name _____

Degrees

Essential Question How are degrees related to fractional parts of a circle?

CONNECT You can use what you know about angles and fractional parts of a circle to understand angle measurement. Angles are measured in units called **degrees**. Think of a circle divided into 360 equal parts. An angle that turns through $\frac{1}{360}$ of the circle measures 1 degree.

Math Idea

The symbol for degrees is $^\circ$.

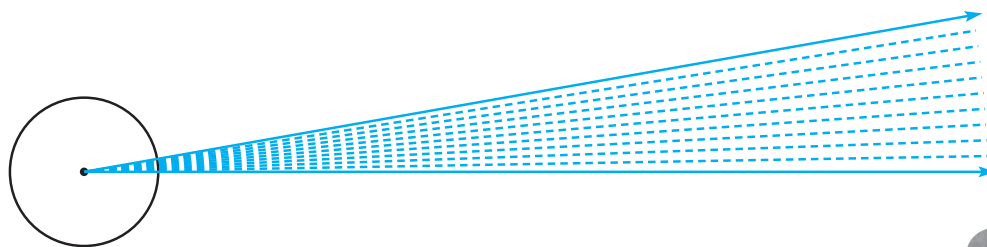


UNLOCK the Problem REAL WORLD

The angle between two spokes on the bicycle wheel turns through $\frac{10}{360}$ of a circle. What is the measure of the angle between the spokes?

- What part of an angle does a spoke represent?

Example 1 Use fractional parts to find the angle measure.



Each $\frac{1}{360}$ turn measures _____ degree.

Ten $\frac{1}{360}$ turns measure _____ degrees.

So, the measure of the angle between the spokes is _____.

Math Talk

How many degrees is the measure of an angle that turns through 1 whole circle? **Explain.**

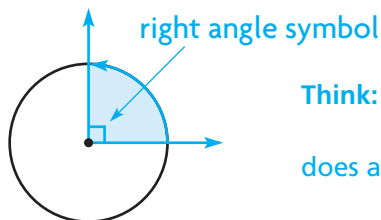
MATHEMATICAL PRACTICES



▲ The Penny Farthing bicycle was built in the 1800s.



Example 2 Find the measure of a right angle.



Think: Through what fraction of a circle

does a right angle turn? _____

STEP 1 Write $\frac{1}{4}$ as an equivalent fraction with 360 in the denominator.

$$\frac{1}{4} = \frac{\boxed{}}{360}$$

Think: $4 \times 9 = 36$, so $4 \times \underline{} = 360$.

Remember

To write an equivalent fraction, multiply the numerator and denominator by the same factor.

STEP 2 Write $\frac{90}{360}$ in degrees.

An angle that turns through $\frac{1}{360}$ of a circle measures _____.

An angle that turns through $\frac{90}{360}$ of a circle measures _____.

So, a right angle measures _____.

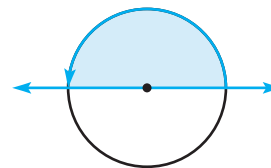
Try This! Find the measure of a straight angle.

Through what fraction of a circle does a straight angle turn? _____

Write $\frac{1}{2}$ as an equivalent fraction with 360 in the denominator.

$$\frac{1}{2} = \frac{\boxed{}}{360}$$

Think: $2 \times 18 = 36$, so $2 \times \underline{} = 360$.



So, a straight angle measures _____.

1. How can you describe the measure of an acute angle in degrees?

2. How can you describe the measure of an obtuse angle in degrees?

Name _____

Share and Show

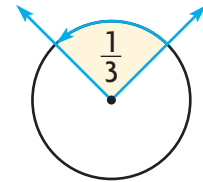


1. Find the measure of the angle.

Through what fraction of a circle does the angle turn? _____

$$\frac{1}{3} = \frac{\boxed{}}{360}$$

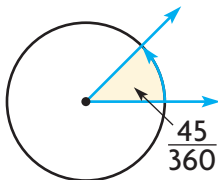
Think: $3 \times 12 = 36$, so $3 \times \underline{\hspace{1cm}} = 360$.



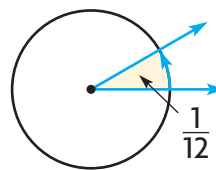
So, the measure of the angle is _____.

Tell the measure of the angle in degrees.

2.



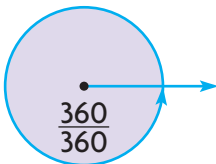
3.



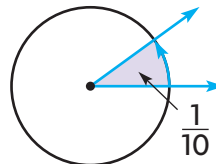
On Your Own

Tell the measure of the angle in degrees.

4.



5.



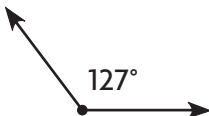
Math Talk

MATHEMATICAL PRACTICES

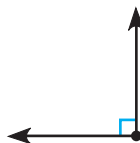
If an angle measures 60° , through what fraction of a circle does it turn? **Explain.**

Classify the angle. Write *acute*, *obtuse*, *right*, or *straight*.

6.



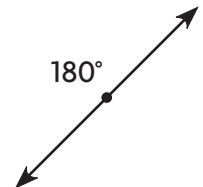
7.



8.

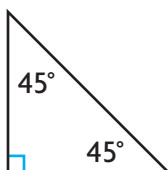


9.

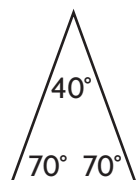


Classify the triangle. Write *acute*, *obtuse*, or *right*.

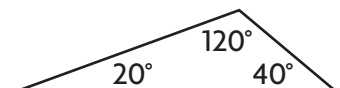
10.



11.



12.



UNLOCK the Problem REAL WORLD

13. Ava started reading at 3:30 P.M. She stopped for a snack at 4:15 P.M. During this time, through what fraction of a circle did the minute hand turn? How many degrees did the minute hand turn?



a. What are you asked to find? _____

b. What information can you use to find the fraction of a circle through which the minute hand turned? _____

c. How can you use the fraction of a circle through which the minute hand turned to find how many degrees it turned? _____

d. Show the steps to solve the problem.

STEP 1 $\frac{3 \times \boxed{}}{4 \times \boxed{}} = \frac{?}{360}$

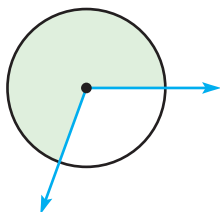
STEP 2 $\frac{3 \times 90}{4 \times 90} = \frac{\boxed{}}{360}$

e. Complete the sentences.

From 3:30 P.M. to 4:15 P.M., the minute hand made a _____ turn clockwise.

The minute hand turned _____ degrees.

14. **H.O.T.** **Write Math** Is this angle measure obtuse? **Explain.**



15. **Test Prep** How many degrees are in an angle that turns through $\frac{2}{4}$ of a circle?

- (A) 90°
- (B) 180°
- (C) 270°
- (D) 360°

Name _____

Measure and Draw Angles

Essential Question How can you use a protractor to measure and draw angles?



Emma wants to make a clay sculpture of her daughter as she appears in the photo from her dance recital. How can she measure $\angle DCE$, or the angle formed by her daughter's arms?

A **protractor** is a tool for measuring the size of an angle.

Activity Measure $\angle DCE$ using a protractor.

Materials ■ protractor

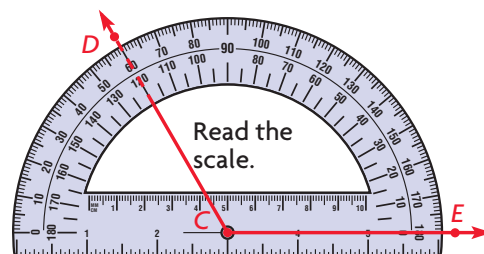
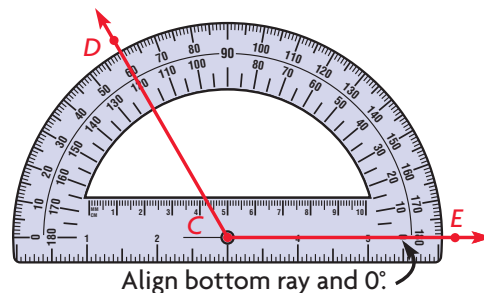
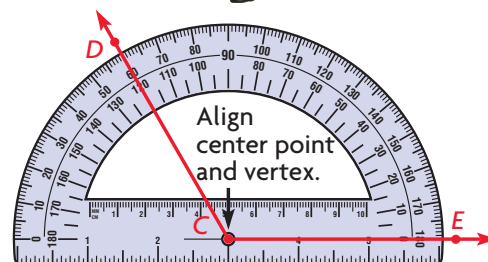
STEP 1 Place the center point of the protractor on vertex C of the angle.

STEP 2 Align the 0° mark on the scale of the protractor with ray CE.

STEP 3 Find where ray CD intersects the same scale. Read the angle measure on that scale. Extend the ray if you need to.

The $m\angle DCE =$ _____. **Read the $m\angle DCE$ as the "measure of angle DCE".**

So, the angle formed by Emma's daughter's arms is _____.



Math Talk

MATHEMATICAL PRACTICES

Can you line up either ray of the angle with the protractor when measuring? **Explain.**

Draw Angles You can also use a protractor to draw an angle of a given measure.

Activity Draw $\angle KLM$ with a measure of 82° .

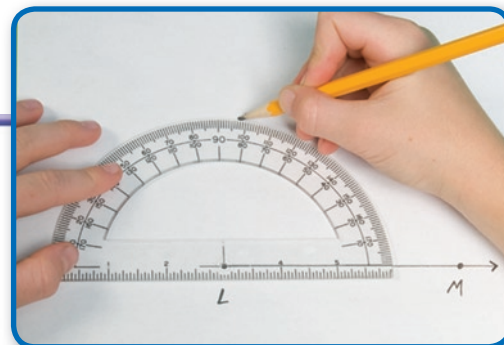
Materials ■ protractor

STEP 1 Use the straight edge of the protractor to draw and label ray LM .

STEP 2 Place the center point of the protractor on point L . Align ray LM with the 0° mark on the protractor.

STEP 3 Using the same scale, mark a point at 82° . Label the point K .

STEP 4 Use the straight edge of the protractor to draw ray LK .



Share and Show

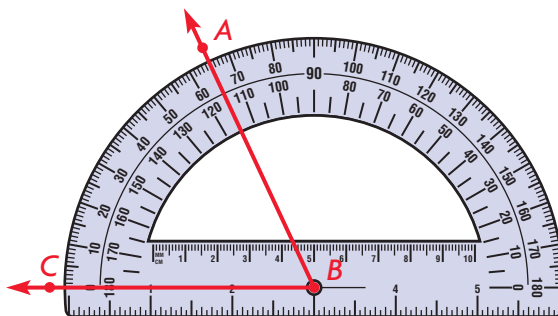
1. Measure $\angle ABC$.

Place the center of the protractor on point _____.

Align ray BC with _____.

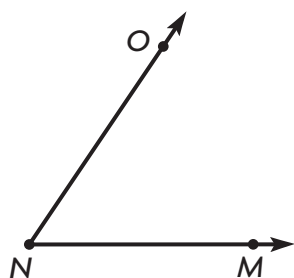
Read where _____ intersects the same scale.

So, the $m\angle ABC$ is _____.



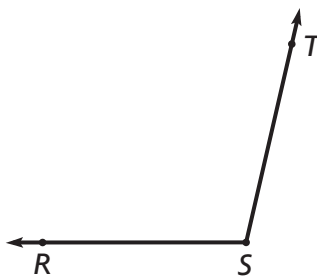
Use a protractor to find the angle measure.

- 2.



$m\angle ONM =$ _____

- 3.



$m\angle TSR =$ _____



ERROR Alert

Be sure to use the correct scale on the protractor. Ask yourself: Is the measure reasonable?

Use a protractor to draw the angle.

4. 170°

5. 78°

Math Talk

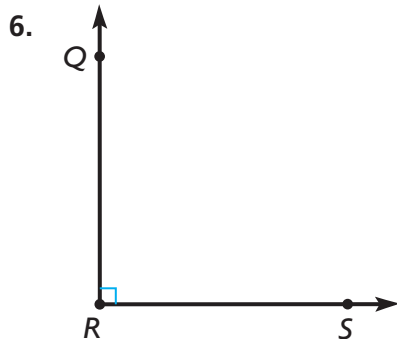
MATHEMATICAL PRACTICES

Describe how drawing and measuring angles are similar.

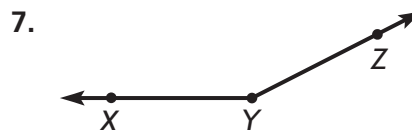
Name _____

On Your Own

Use a protractor to find the angle measure.



$m\angle QRS =$ _____



$m\angle XYZ =$ _____

Use a protractor to draw the angle.

8. 115°

9. 67°

Draw an example of each. Label the angle with its measure.

10. an acute angle

11. an obtuse angle

12. a straight angle

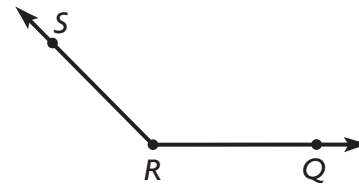
13. a right angle

14.   Draw an angle with a measure of 0° .
Describe your drawing.

Problem Solving **REAL WORLD**

15. Mrs. Murphy is building a wheelchair ramp outside her business. The angle of the ramp should be 5° . Draw a picture in the space to the right to show a model of the ramp.
16. **H.O.T.** **What's the Error?** Tracy measured an angle as 50° that was actually 130° . **Explain** her error.

17. **Test Prep** What is the measure of $\angle QRS$?
- (A) 35° (B) 45° (C) 135° (D) 155°



Connect to Science

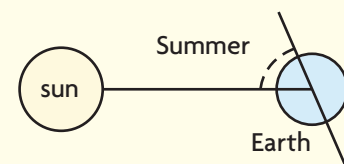
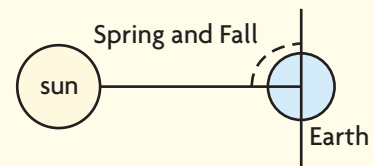
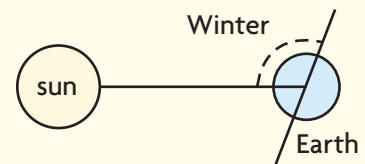
Earth's Axis

Earth revolves around the sun yearly. The Northern Hemisphere is the half of Earth that is north of the equator. The seasons of the year are due to the tilt of Earth's axis.

Use the diagrams and a protractor for 18–20.

18. In the Northern Hemisphere, Earth's axis is tilted away from the sun on the first day of winter, which is often on December 21. What is the measure of the marked angle on the first day of winter, the shortest day of the year?
19. Earth's axis is not tilted away from or toward the sun on the first days of spring and fall, which are often on March 20 and September 22. What is the measure of the marked angle on the first day of spring or fall?
20. In the Northern Hemisphere, Earth's axis is tilted toward the sun on the first day of summer, which is often on June 21. What is the measure of the marked angle on the first day of summer, the longest day of the year?

Northern Hemisphere





Mid-Chapter Checkpoint

► Vocabulary

Choose the best term from the box.

- The unit used to measure an angle is called
a _____. (p. 421)
- _____ is the opposite of the
direction in which the hands of a clock move. (p. 417)
- A _____ is a tool for measuring the size
of an angle. (p. 425)

Vocabulary

clockwise

counterclockwise

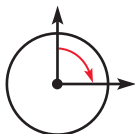
degree ($^{\circ}$)

protractor

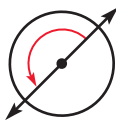
► Concepts and Skills

Tell whether the angle on the circle shows a $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or 1 full turn clockwise or counterclockwise.

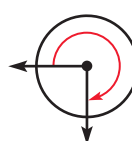
4.



5.



6.

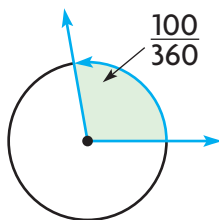


7.

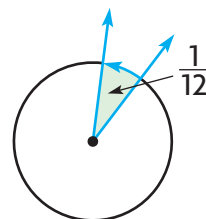


Tell the measure of the angle in degrees.

8.



9.



Use a protractor to draw the angle.

10. 75°

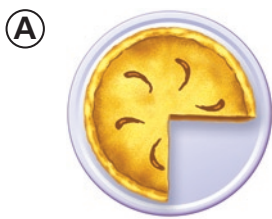
11. 127°

Fill in the bubble completely to show your answer.

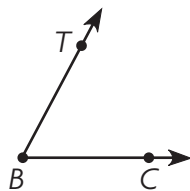
12. Phillip watched a beach volleyball game from 1:45 P.M. to 2:00 P.M. How many degrees did the minute hand turn?



- (A) 90°
 (B) 180°
 (C) 270°
 (D) 360°
13. Which piece of pie forms a 180° angle?



14. Which best describes the $m\angle CBT$? Use a protractor to help you.



- (A) acute; 58°
 (B) acute; 62°
 (C) obtuse; 62°
 (D) obtuse; 118°

Name _____

Join and Separate Angles

Essential Question How can you determine the measure of an angle separated into parts?

Investigate

Materials ■ construction paper ■ scissors ■ protractor

A. Use construction paper. Draw an angle that measures exactly 70° . Label it $\angle ABC$.

B. Cut out $\angle ABC$.

C. Separate $\angle ABC$ by cutting it into two parts. Begin cutting at the vertex and cut between the rays.

What figures did you form? _____

D. Use a protractor to measure the two angles you formed.

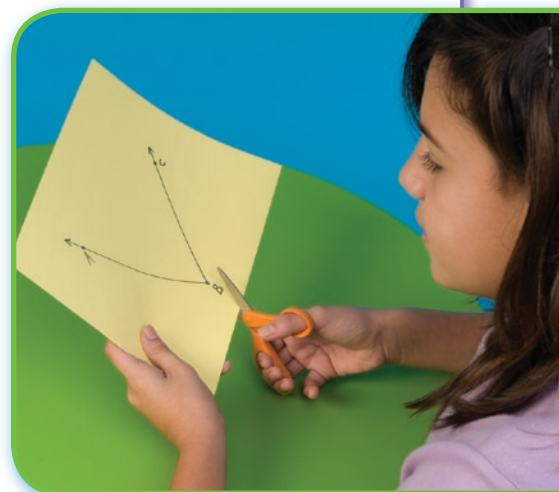
Record the measures. _____

E. Find the sum of the angles you formed.

$$\underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

part + part = whole

F. Join the two angles. Compare the $m\angle ABC$ to the sum of the measures of its parts. **Explain** how they compare.



Math Idea

You can think of $\angle ABC$ as the whole and the two angles you formed as the parts of the whole.

Draw Conclusions.....

1. **What if** you cut $\angle ABC$ into two different angles? What can you conclude about the sum of the measures of these two angles? **Explain.**

2. **H.O.T.** Seth cut $\angle ABC$ into 3 parts. Draw a model that shows two different ways he could have separated his angle.

3. **Generalize** Write a sentence that compares the measure of an angle to the sum of its parts.

Make Connections.....

Materials ■ protractor

You can write the measure of the angles shown in a circle as a sum.

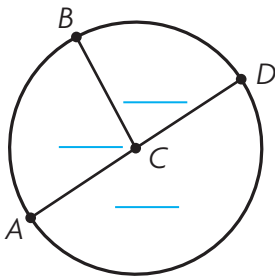
- STEP 1** Use a protractor to find the measure of each angle.
-
- STEP 2** Label each angle with its measure.
-
- STEP 3** Write the sum of the angle measures as an equation.

+

+

=

part + part + part = whole



MATHEMATICAL PRACTICES

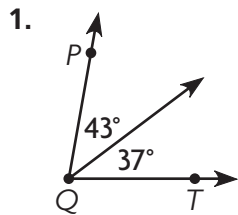
Math Talk Describe the angles shown in the circle above using the words *whole* and *part*.

Name _____

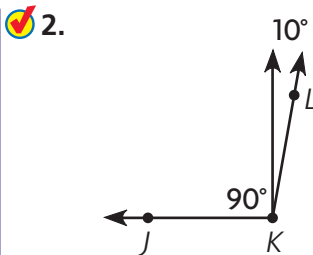
Share and Show



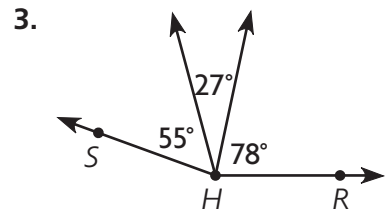
Add to find the measure of the angle. Write an equation to record your work.



$m\angle PQT = \underline{\hspace{2cm}}$

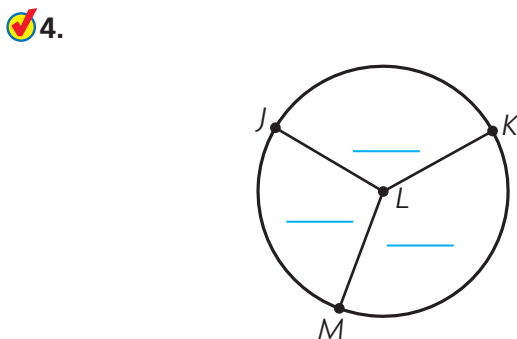


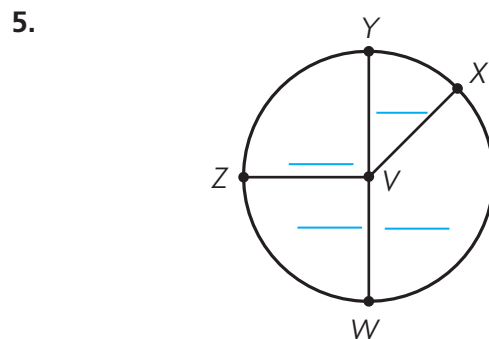
$m\angle JKL = \underline{\hspace{2cm}}$



$m\angle RSH = \underline{\hspace{2cm}}$

Use a protractor to find the measure of each angle. Label each angle with its measure. Write the sum of the angle measures as an equation.





6. **Write Math** Look back at Exercise 1. Suppose you joined an angle measuring 10° to $\angle PQT$. **Describe** and draw the new angle. Include all three parts in your drawing and description.

UNLOCK the Problem REAL WORLD

7. Stephanie, Kay, and Shane each ate an equal-sized piece of a pizza. The measure of the angle of each piece was 45° . When the pieces were together, what is the measure of the angle they formed?

(A) 90° (B) 135° (C) 180° (D) 225°

a. What are you asked to find? _____

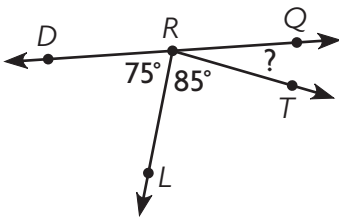
b. What information do you need to use? _____

c. Tell how you can use addition to solve the problem. _____

d. Fill in the bubble for the correct answer choice above.

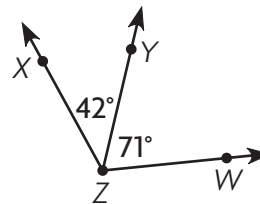


8. **H.O.T.** What is the $m\angle QRT$?



(A) 20°
(B) 150°
(C) 160°
(D) 180°

9. Which equation can you use to find the $m\angle XZW$?



(A) $71^\circ - 42^\circ = \square$
(B) $71^\circ + 42^\circ = \square$
(C) $71^\circ \times 42^\circ = \square$
(D) $180^\circ - 113^\circ = \square$

Name _____

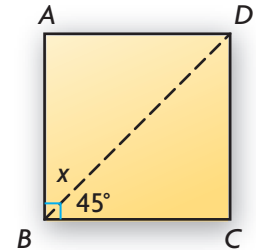
Problem Solving • Unknown Angle Measures

Essential Question How can you use the strategy *draw a diagram* to solve angle measurement problems?



Mr. Tran is cutting a piece of kitchen tile as shown at the right. He needs tiles with 45° angles to make a design. After the cut, what is the angle measure of the part left over? Can Mr. Tran use both pieces in the design?

Use the graphic organizer below to solve the problem.



Read the Problem

What do I need to find?

I need to find

What information do I need to use?

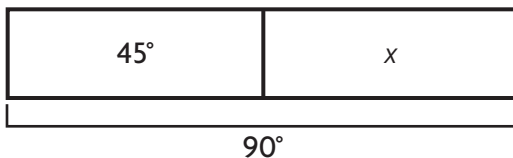
I can use the measures of the angles I know.

How will I use the information?

I can draw a bar model and use the information to

Solve the Problem

I can draw a bar model to represent the problem. Then I can write an equation to solve the problem.



$$m\angle ABD + m\angle CBD = m\angle ABC$$

$$x + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

The $m\angle ABD = \underline{\hspace{2cm}}$.

Since both tiles measure $\underline{\hspace{2cm}}$, Mr. Tran can use both pieces in the design.

Math Talk

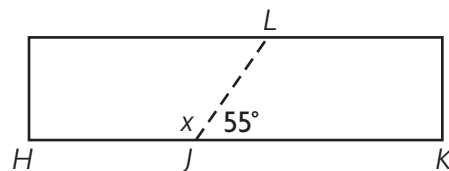
MATHEMATICAL PRACTICES

What other equation can you write to solve the problem? **Explain.**



Try Another Problem

Marisol is building a frame for a sandbox, but the boards she has are too short. She must join two boards together to build a side as shown. At what angle did she cut the first board?



Read the Problem

What do I need to find?

What information do I need to use?

How will I use the information?

Solve the Problem

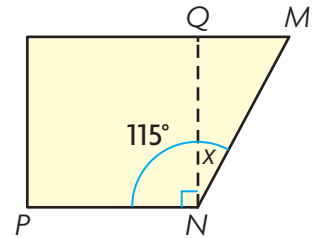
- **Explain** how you can check the answer to the problem.

Name _____

Share and Show

1. Laura cuts a square out of scrap paper as shown. What is the angle measure of the piece left over?

First, draw a bar model to represent the problem.



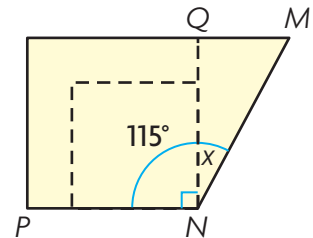
Next, write the equation you need to solve.

Last, find the angle measure of the piece left over.

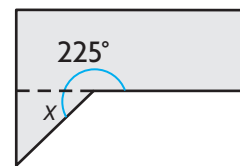
The $m\angle MNQ = \underline{\hspace{2cm}}$.

So, the angle measure of the piece left over is $\underline{\hspace{2cm}}$.

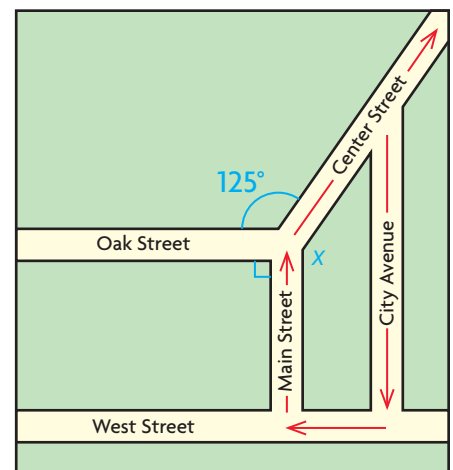
2.  **What if** Laura cut a smaller square as shown? Would the $m\angle MNQ$ be different? **Explain.**



3.  Jackie trimmed a piece of scrap metal to make a straight edge as shown. What is the measure of the piece she trimmed off?



4.  The map shows Marco's paper route. When Marco turns right onto Center Street from Main Street, what degree turn does he make? **Hint:** Draw a dashed line to extend Oak Street to form a 180° angle.



On Your Own

Choose a STRATEGY

Act It Out
Draw a Diagram
Find a Pattern
Make a Table or List
Solve a Simpler Problem

5. **Write Math** Two angles form a straight angle. One angle measures 89° . What is the measure of the other angle? **Explain.**

6. **H.O.T. Pose a Problem** Look back at Problem 5. Write a similar problem about two angles that form a right angle.

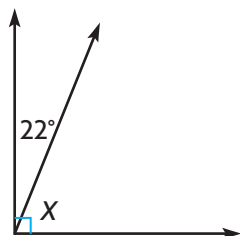
7. Sam paid \$20 for two t-shirts. The price of each t-shirt was a multiple of 5. What are the possible prices of the t-shirts?

8. Zayna has 3 boxes with 15 art books in each box. She has 2 bags with 11 math books in each bag. If she gives 30 books away, how many art and math books does she have now?

9. **H.O.T. What's the Question?** It measures greater than 0° and less than 90° .

10. **Test Prep** What is the unknown angle measure?

- (A) 22°
(B) 68°
(C) 90°
(D) 158°



SHOW YOUR WORK

Name _____



Chapter Review/Test

► Vocabulary

Choose the best term from the box.

- The size of an angle can be measured using a tool called a _____. (p. 425)
- _____ is the direction in which the hands of a clock move. (p. 418)

Vocabulary

clockwise

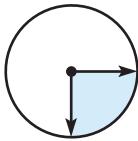
counterclockwise

protractor

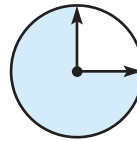
► Concepts and Skills

Tell what fraction of the circle the shaded angle represents.

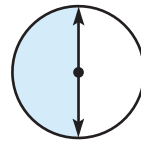
3.



4.



5.

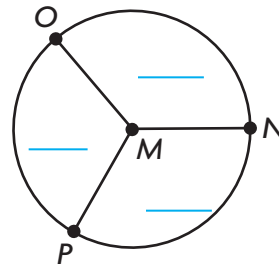


Use a protractor to draw the angle.

6. 68°

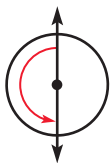
7. 145°

- Use a protractor to find the measure of each angle. Label each angle with its measure. Write the sum of the angle measures as an equation.

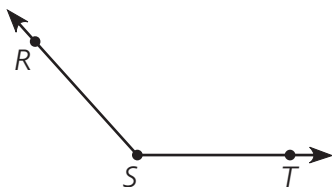


Fill in the bubble completely to show your answer.

9. Which describes the turn the angle on the circle shows?



- (A) 90° clockwise
(B) 90° counterclockwise
(C) 180° clockwise
(D) 180° counterclockwise
10. Which best describes the $m\angle RST$? Use a protractor to help you.



- (A) acute; 48°
(B) obtuse; 48°
(C) obtuse; 132°
(D) obtuse; 148°
11. The pocket watch was invented in 1524. The time is 6 P.M. After 1 hour, how many degrees does the minute hand turn?

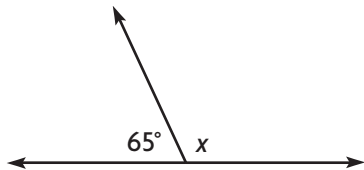


- (A) 45°
(B) 90°
(C) 180°
(D) 360°

Name _____

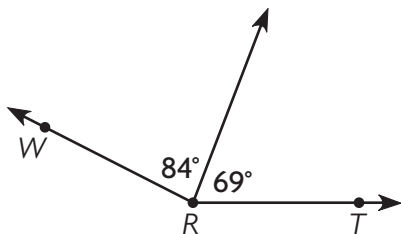
Fill in the bubble completely to show your answer.

12. What is the unknown angle measure?



- (A) 25°
(B) 115°
(C) 125°
(D) 180°

13. Which equation can you use to find the $m\angle WRT$?



- (A) $84^\circ + 69^\circ = \square$
(B) $84^\circ - 69^\circ = \square$
(C) $84^\circ \times 69^\circ = \square$
(D) $180^\circ - 153^\circ = \square$

14. If an angle measures 100° , through what fraction of a circle does the angle turn?

- (A) $\frac{1}{100}$
(B) $\frac{1}{4}$
(C) $\frac{100}{360}$
(D) $\frac{1}{2}$

► Constructed Response

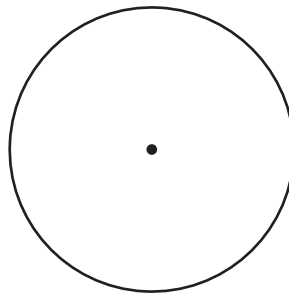
15. How many right angles are there in an angle that turns through 360° ? **Explain** how you know.

16. Soccer practice began at 2:30 P.M. and stopped at 3:00 P.M. because of rain. During this time, through what fraction of a circle did the minute hand turn? How many degrees did the minute hand turn? **Explain.**

► Performance Task

17. Charlotte divided a whole pizza into 4 pieces. One piece formed a straight angle. One piece formed a right angle. Two pieces formed acute angles with the same degree measure.

- A** Draw angles to represent the 4 pieces.



- B** Label each angle with its degree measure.
- C** Label each angle as a fraction of a circle.
- D** Write an equation that represents the degree measure of the whole pizza as the sum of the measures of its parts.
