NUNIT 6 Statistics



CAREERS IN MATH

Psychologist A psychologist investigates the physical, mental, emotional, and social aspects of human behavior. Psychologists use math to evaluate and interpret data about human activities and the human mind. They create and use mathematical models to predict behavior of humans, both individually and in groups.

If you are interested in a career in psychology, you should study the following mathematical subjects:

- Algebra
- Trigonometry
- Probability and Statistics
- Calculus

Research other careers that require the analysis of data and use of mathematical models.

Unit 6 Performance Task

At the end of the unit, check out how **psychologists** use math.

Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters within found words to answer the riddle at the bottom of the page.

Ε	Т	F	Ν	R	В	W	D	F	Μ	0	Ν	J	0	V
Ν	L	Ε	L	Η	L	D	С	С	W	0	Т	R	U	R
0	Y	Ρ	Т	R	E	Ν	D	L	I	Ν	Ε	Т	Т	W
I	Q	V	Μ	Μ	V	Т	В	Т	U	D	D	В	L	F
T	U	D	Κ	Α	Ε	В	Α	В	R	S	Q	I.	Т	W
(A)	R	Q	0	S	S	L	Т	S	Μ	Ν	Т	В	Ε	С
Ι	Y	S	Ζ	Υ	U	M	Ρ	Κ	D	G	U	Ε	R	Q
С	В	Т	S	Ρ	В	Τ	Α	В	Q	Ρ	В	Н	R	V
0	U	Μ	0	Μ	G	V	С	R	W	Q	G	Ζ	U	Ε
S	W	Ρ	R	Κ	V	Т	U	Ε	G	L	Μ	G	Ζ	S
S	Χ	L	Ε	Т	V	U	G	Q	С	I	J	I	G	С
Α	D	Е	0	Α	В	L	W	Α	Q	D	Ν	Ν	В	U
G	V	J	Q	V	С	Χ	Α	Υ	I	V	Μ	Α	Ζ	W
Ν	L	F	W	0	Μ	Α	Т	L	Т	U	Ζ	В	L	Μ
N	0	I	Т	Α	L	0	Ρ	R	Ε	Т	Ν	I	V	G

A description of how sets of data are related. (Lesson 14.1)

A set of closely related data. (Lesson 14.1)

A data point that is very different from the rest of the data in the set. (Lesson 14.1) A straight line that comes closest to the points on a scatter plot (2 words). (Lesson 14.2) Using a trend line to predict a value between data points you already know. (Lesson 14.2) A relative frequency found by dividing a row total or a column total by the grand total. (Lesson 15.2)

Q: Why doesn't Joe Average have any friends?

A: Because he's so _____ ___ ____!

Scatter Plots



ESSENTIAL QUESTION

How can you use scatter plots to solve real-world problems?



LESSON 14.1

Scatter Plots and Association

FL 8.SP.1.1

LESSON 14.2

Trend Lines and Predictions





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Math On the Spot

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Real-World Video

An anthropologist measures dinosaur bones. To estimate a dinosaur's height based on the length of a bone, he can make a scatter plot comparing bone length and height of several dinosaurs.

Animated Math

Interactively explore key concepts to see how math works.



Personal Math Trainer

Get immediate feedback and help as you work through practice sets.



Are Ready

Complete these exercises to review skills you will need for this module.



Online

Evaluate Expressions

EXAMPLE	Evaluate $4x + 3$ for $x = 5$.	
	4x + 3 = 4(5) + 3	Substitute the given value for x.
	= 20 + 3	Multiply.
	= 23	Add.

Evaluate each expression for the given value of x.

1.	6x - 5 for $x = 4$	2. $-2x + 7$ for $x = 2$	3. $5x - 6$ for $x = 3$
4.		5. $\frac{3}{4}x - 9$ for $x = -20$	6. $1.4x + 3.5$ for $x = -4$

Solve Two-Step Equations

EXAMPLE	5x + 3 = -7	
	-3 = -3	Subtract 3 from both sides.
	5x = -10	
	$\frac{5x}{5} = \frac{-10}{5}$	Divide both sides by 5.
	x = -2	•

Solve for *x*.

7. 3x + 4 = 10 **8.** 5x - 11 = 34 **9.** -2x + 5 = -9 **10.** 8x + 13 = -11

11. 4x - 7 = -27 **12.** $\frac{1}{2}x + 16 = 39$ **13.** $\frac{2}{3}x - 16 = 12$ **14.** 0.5x - 1.5 = -6.5

Reading Start-Up

Visualize Vocabulary

Use the 🗸 words to complete the right column of the chart.

Reviewing Slope				
Mathematical Representation	Review Word			
y = mx + b				
у				
т				
X				
Ь				

Understand Vocabulary

Match the term on the left to the correct expression on the right.

1. cluster

2. outlier

3. trend line

- **A.** A data point that is very different from the rest of the data in a set
- **B.** A straight line that comes closest to the points on a scatter plot.
- **C.** A set of closely grouped data.

Vocabulary

Review Words

bivariate data (datos bivariados) data (datos)

- linear equation (ecuación lineal)
- ✓ slope (*pendiente*)
- slope-intercept form of an equation (forma pendiente-intersección)
- x-coordinate (coordenada x)
- ✓ y-coordinate (coordenada y)
- ✓ y-intercept (intersección con el eje y)

Preview Words

cluster (agrupación) outlier (valor extremo) scatter plot (diagrama de dispersión) trend line (línea de tendencia)

Active Reading

Two-Panel Flip Chart Create a two-panel flip chart, to help you understand the concepts in this module. Label each flap with the title of one of the lessons in the module. As you study each lesson, write important ideas under the appropriate flap. Include any sample problems or equations that will help you remember the concepts later when you look back at your notes.



MODULE 14 Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

NFI 8.SP.1.1

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

What It Means to You

You will describe how the data in a scatter plot are related.

UNPACKING EXAMPLE 8.SP.1.1

The scatter plot shows Bob's height at various ages. Describe the type(s) of association between Bob's age and his height. Explain.

As Bob gets older, his height increases roughly along a straight line on the graph, so the association is positive and basically linear.



<mark>٦ ===</mark> 8.SP.1.2

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.



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What It Means to You

You will use a trend line to show the relationship between two quantities.

UNPACKING EXAMPLE 8.SP.1.2

Joyce is training for a 10K race. For each of her training runs, she recorded the distance she ran and the time she ran. She made a scatter plot of her data and drew a trend line. Use the trend line to predict how long it would take Joyce to run 4.5 miles.

Distance (mi)	Time (min)
4	38
2	25
1	7
2	16
3	26
5	55
2	20
4	45
3	31



For a distance of 4.5 miles, the trend line shows a time of 45 minutes. So, it will take Joyce about 45 minutes to run 4.5 miles.

Scatter Plots and Association



Construct and interpret scatter plots....Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

ESSENTIAL QUESTION

How can you construct and interpret scatter plots?

EL 8.SP.1.1



Making a Scatter Plot

Recall that a set of bivariate data involves two variables. Bivariate data are used to explore the relationship between two variables. You can graph bivariate data on a *scatter plot*. A **scatter plot** is a graph with points plotted to show the relationship between two sets of data.

The final question on a math test reads, "How many hours did you spend studying for this test?" The teacher records the number of hours each student studied and the grade the student received on the test.

A Make a prediction about the relationship between the number of hours spent studying and test grades.

Hours Spent Studying	Test Grade
0	75
0.5	80
1	80
1	85
1.5	85
1.5	95
2	90
3	100
4	90

B Make a scatter plot. Graph hours spent studying as the independent variable and test grades as the dependent variable.

Reflect

- 1. What trend do you see in the data?
- **2. Justify Reasoning** Do you think the grade associated with studying for 10 hours would follow this trend?



EXPLORE ACTIVITY 2



A **cluster** is a set of closely grouped data. Data may cluster around a point or along a line. An **outlier** is a data point that is very different from the rest of the data in the set.

FL 8.SP.1.1

A scientist gathers information about the eruptions of Old Faithful, a geyser in Yellowstone National Park. She uses the data to create a scatter plot. The data show the length of time between eruptions (interval) and how long the eruption lasts (duration).

A Describe any clusters you see in the scatter plot.

What do the clusters tell you about eruptions of Old Faithful?

Describe any outliers you see in the scatter plot.

Reflect

3. Suppose the geyser erupts for 2.2 minutes after a 75-minute interval. Would this point lie in one of the clusters? Would it be an outlier? Explain your answer.

4. Suppose the geyser erupts after an 80-minute interval. Give a range of possible duration times for which the point on the scatter plot would not be considered an outlier. Explain your reasoning.







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Determining Association

Association describes how sets of data are related. A *positive* association means that both data sets increase together. A negative association means that as one data set increases, the other decreases. No association means that there is no relationship between the two data sets.





Negative association

No association

20

16

12

8

4

0

4

8 Price (\$)

Buyers

8.SP.1.1

12

Data that show a positive or negative association and lie basically along a line exhibit a linear association. Data that show a positive or negative association but do not lie basically along a line exhibit a *nonlinear* association.

Real **EXAMPLE 1**

Susan asked 20 people if they would buy a new product she developed at each of several prices. The scatter plot shows how many of the 20 said "yes" at a given price. Describe the association between price and the number of buyers.

As price increases, the number of buyers decreases. So, there is a negative association. Because the data points do not lie along a line, the association is nonlinear.

Reflect

5. What If? Based on the association shown in the scatter plot, what might happen if Susan increased the price to \$14?



6. The plot shows the reading level and height for 16 students in a district. Describe the association and give a possible reason for it.





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Guided Practice



Age (years)	6	8	10	12	14
Height (inches)	45	50	55	61	63

1. Make a scatter plot of Bob's data. (Explore Activity 1)



2. Describe the association between Bob's age and his height. Explain the association. (Example 1)

3. The scatter plot shows the basketball shooting results for 14 players. Describe any clusters you see in the scatter plot. Identify any outliers. (Explore Activity 2)





ESSENTIAL QUESTION CHECK-IN

4. Explain how you can make a scatter plot from a set of bivariate data.

Class



14.1 Independent Practice 8.5P.1.1

Sports Use the scatter plot for 5–8.



5. Describe the association between the year and the distance jumped for the years 1960 to 1988.

6. Describe the association between the year and the distance jumped for the years after 1988.

7. For the entire scatter plot, is the association between the year and the distance jumped linear or nonlinear?

8. Identify the outlier and interpret its meaning.

9. Communicate Mathematical Ideas Compare a scatter plot that shows no association to one that shows negative association. For 10–11, describe a set of real-world bivariate data that the given scatter plot could represent. Define the variable represented on each axis.



FOCUS ON HIGHER ORDER THINKING

- **12. Multiple Representations** Describe what you might see in a table of bivariate data that would lead you to conclude that the scatter plot of the data would show a cluster.
- **13.** Justify Reasoning Is it possible for a scatter plot to have a positive or negative association that is not linear? Explain.

14. Critical Thinking To try to increase profits, a theater owner increases the price of a ticket by \$25 every month. Describe what a scatter plot might look like if *x* represents the number of months and *y* represents the profits. Explain your reasoning.

Work Area

 \mathbf{OT}



2. Do you think you can use a scatter plot that shows no association to make a prediction? Explain your answer.



Reflect

- 3. What type(s) of association does the scatter plot show?
- 4. What is the meaning of the slope in this situation?
- 5. What is the meaning of the *y*-intercept in this situation?

YOUR TURN

6. The scatter plot and trend line show the relationship between the number of rainy days in a month and the number of umbrellas sold each month. Write an equation for the trend line.

EXPLORE ACTIVITY 2

FL 8.SP.1.3

10

8

6

4

2

0

4

2

6

Number of rainy days

8

10

Number of umbrellas

Making Predictions

When you use a trend line or its equation to predict a value between data points that you already know, you *interpolate* the predicted value. When you make a prediction that is outside the data that you know, you *extrapolate* the predicted value.

Use the equation of the trend line in Example 1 to predict how many pages would be in a book with 26 chapters.

Is this prediction an example of interpolation or extrapolation? y = Write the equation for your trend line. y = Substitute the number of chapters for x. y = Simplify.

I predict that a book with 26 chapters will have ______ pages.

EXPLORE ACTIVITY 2 (cont'd)

Reflect

- **7.** Make a Prediction Predict how many pages would be in a book with 14 chapters. Is this prediction an example of interpolation or extrapolation?
- 8. Do you think that extrapolation or interpolation is more accurate? Explain.

Guided Practice

Angela recorded the price of different weights of several bulk grains. She made a scatter plot of her data. Use the scatter plot for 1–4.

- 1. Draw a trend line for the scatter plot. (Explore Activity 1)
- 2. How do you know whether your trend line is a good fit for the data? (Explore Activity 1)



3. Write an equation for your trend line. (Example 1)

4. Use the equation for your trend line to interpolate the price of 7 ounces and extrapolate the price of 50 ounces.

(Explore Activity 2)

ESSENTIAL QUESTION CHECK-IN

5. A trend line passes through two points on a scatter plot. How can you use the trend line to make a prediction between or outside the given data points?

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Class_

Date_

14.2 Independent Practice

ELD 8.SP.1.1, 8.SP.1.2, 8.SP.1.3

Use the data in the table for Exercises 6–10.

Apparent Temperature Due to Wind at 15 $^\circ$ F						
Wind speed (mi/h)	10	20	30	40	50	60
Wind chill (°F)	2.7	-2.3	-5.5	-7.9	-9.8	-11.4

- 6. Make a scatter plot of the data and draw a trend line.
- 7. What type of association does the trend line show?
- 8. Write an equation for your trend line.
- **9.** Make a Prediction Use the trend line to predict the wind chill at these wind speeds.
 - **a.** 36 mi/h ______ **b.** 100 mi/h ______
- **10.** What is the meaning of the slope of the line?

Use the data in the table for Exercises 11-14.

Apparent Temperature Due to Humidity at a Room Temperature of 72 °F						
Humidity (%)	0	20	40	60	80	100
Apparent temperature (°F)	64	67	70	72	74	76

- **11.** Make a scatter plot of the data and draw a trend line.
- **12.** Write an equation for your trend line.



13. Make a Prediction Use the trend line to predict the apparent

temperature at 70% humidity._____

14. What is the meaning of the *y*-intercept of the line?



Work Area



FOCUS ON HIGHER ORDER THINKING

15. Communicate Mathematical Ideas Is it possible to draw a trend line on a scatter plot that shows no association? Explain.

16. Critique Reasoning Sam drew a trend line that had about the same number of data points above it as below it, but did not pass through any data points. He then picked two data points to write the equation for the line. Is this a correct way to write the equation? Explain.

17. Marlene wanted to find a relationship between the areas and populations of counties in Texas. She plotted *x* (area in square miles) and *y* (population) for two counties on a scatter plot:

Kent County (903, 808) Edwards County (2118, 2002)

She concluded that the population of Texas counties is approximately equal to their area in square miles and drew a trend line through her points.

a. Critique Reasoning Do you agree with Marlene's method of creating a scatter plot and a trend line? Explain why or why not.

b. Counterexamples Harris County has an area of 1778 square miles and a population of about 4.3 million people. Dallas County has an area of 908 square miles and a population of about 2.5 million people. What does this data show about Marlene's conjecture that the population of Texas counties is approximately equal to their area?

MODULE QUIZ

Ready to Go On?

14.1 Scatter Plots and Association

An auto store is having a sale on motor oil. The chart shows the price per quart as the number of quarts purchased increases. Use the data for Exs. 1–2.

Number of quarts	1	2	3	4	5	6
Price per quart (\$)	2	1.50	1.25	1.10	1	0.95

- **1.** Use the given data to make a scatter plot.
- **2.** Describe the association you see between the number of quarts purchased and the price per quart. Explain.



14.2 Trend Lines and Predictions

The scatter plot below shows data comparing wind speed and wind chill for an air temperature of 20 $^{\circ}$ F. Use the scatter plot for Exs. 3–5.



6. How can you use scatter plots to solve real-world problems?





MODULE 14 MIXED REVIEW Assessment Readiness



Selected Response

1. Which scatter plot could have a trend line whose equation is y = 3x + 10?



- **2.** What type of association would you expect between a person's age and hair length?
 - (A) linear (C) none
 - (B) negative (D) positive
- **3.** Which is **not** shown on the scatter plot?
 - (A) cluster
 - (B) negative association
 - \bigcirc outlier
 - **D** positive association



4. A restaurant claims to have served 352,000,000 hamburgers. What is this number in scientific notation?

A 3.52 × 10 ⁶	(C) 35.2×10^7
B 3.52 × 10 ⁸	D 352 × 10 ⁶

5. Which equation describes the relationship between *x* and *y* in the table?

x	-8	-4	0	4	8
у	2	1	0	-1	-2

(A) y = -4x (C) y = 4x(B) $y = -\frac{1}{4}x$ (D) $y = \frac{1}{4}x$

Mini-Task

6. Use the data in the table.

Temp (°F)	97	94	87	92	100	90
Pool visitors	370	315	205	135	365	240

a. Make a scatterplot of the data.



- **b.** Which data point is an outlier?
- **c.** Predict the number of visitors on a day when the high temperature is 102 °F.

Two-Way Tables



ESSENTIAL QUESTION

How can you use two-way frequency tables to solve real-world problems?





Real-World Video

Two-way tables can help identify and compare probabilities for non-numerical data, such as the probability that girls will like one of two sports teams more than boys will.

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Are / Ready?

Complete these exercises to review skills you will need for this module.



Personal Math Trainer

Simplify Fractions

EXAMPLE	Simplify $\frac{18}{30}$. $\frac{1, 2, 3, 6, 9, 18}{1, 2, 3, 5, 6, 10, 30}$ $\frac{18 \div 6}{30 \div 6} = \frac{3}{5}$	List all the factors of the numerator and denominator. Find the greatest common factor (GCF). Divide the numerator and denominator by the GCF.

Write each fraction in simplest form.

1.	<u>25</u> 30 ———	2. $\frac{2}{3}$	<u>27</u> 36 ———	3.	<u>14</u> <u>16</u>	4.	<u>15</u> 45 ———
5.	$\frac{27}{63}$	6. $\frac{4}{7}$	<u>45</u> 75 ———	7.	8/27	8.	<u>16</u> 28 ———

Fractions, Decimals, Percents

EXAMPLEWrite $\frac{13}{20}$ as a
decimal and a
percent.0.65
20)13.00
100Write the fraction as a division problem.
Write a decimal point and zeros in the
dividend.
Place a decimal point in the quotient. $-\frac{120}{100}$
0.65 = 65%Write the decimal as a percent.

Write each fraction as a decimal and a percent.



Find the Percent of a Number

 EXAMPLE
 6.5% of 24 = ? Write the percent as a decimal.

 6.5% = 0.065 24 Multiply.

 $\frac{\times 0.065}{1.56}$ Multiply.

 Find each percent of a number.

 15. 4% of 40
 16. 7% of 300
 17. 4.3% of 1,200

 18. 2.9% of 780
 19. 1.6% of 75.20
 20. 3.56% of 3,200

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Visualize Vocabulary

Use the 🗸 words to complete the chart.



Understand Vocabulary

Complete the sentences using preview words.

- 1. The ______ is the number of times an event occurs.
- **2.** A ______ shows the frequencies of data that is categorized two ways.
- **3.** ______ is the ratio of the number of times an event occurs to the total number of events.

Vocabulary

Review Words

- ✓ association (asociación)
- cluster (grupo)
 data (datos)
- ✓ interpolation (interpolación) extrapolation (extrapo
 - lación)
 - outlier (parte aislada) scatter plot (gráfico de dispersión)
- trend line (la línea de tendencia)

Preview Words

conditional relative frequency (frecuencia relativa condicional) frequency (frecuencia) joint relative frequency (frecuencia relativa conjunta) marginal relative frequency (frecuencia relativa

(frecuencia relativa marginal)

relative frequency (frecuencia relativa)

two-way table (tabla de doble entrada)

two-way relative frequency table (*tabla de frecuencia relativa de doble entrada*)

Active Reading

Tri-Fold Before beginning the module, create a tri-fold to help you learn the concepts and vocabulary in this module. Fold the paper into three sections. Label the columns "What I Know," "What I Want to Know," and "What I Learned." Complete the first two columns before you read. After studying the module, complete the third column.



Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

8.SP.1.4

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. ...

Key Vocabulary

two-way table *(tabla de doble entrada)* A table that displays two-variable data by organizing it into rows and columns.

What It Means to You

You will use two-way tables to find relative frequencies.

UNPACKING EXAMPLE 8.SP.1.4

Soojinn counted the vehicles in the school parking lot and recorded the data in the two-way table shown.

	During School Day	After School Day	Total
Cars	36	14	50
Trucks	19	6	25
Total	55	20	75

What percent of the vehicles parked after school were trucks?

 $\frac{\text{trucks after school}}{\text{total vehicles after school}} = \frac{6}{20} = 0.3, \text{ or } 30\%$

30% of the vehicles in the school parking lot after school were trucks.

NFL 8.SP.1.4

... Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Key Vocabulary

conditional relative

frequency (frecuencia relativa condicional) The ratio of a joint relative frequency to a related marginal relative frequency in a two-way table.



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What It Means to You

You will use two-way tables to find conditional relative frequencies.

UNPACKING EXAMPLE 8.SP.1.4

Soojinn determined the gender of the driver for each of the 55 vehicles parked in the school parking lot during the day.

	Male	Female	Total
Cars	8	25	33
Trucks	15	7	22
Total	23	32	55

What is the conditional relative frequency that a driver is female given that the vehicle is a car?

$$\frac{\text{female car drivers}}{\text{total cars}} = \frac{25}{33} \approx 0.758, \text{ or about 76\%}$$

There is a 76% likelihood that a driver is female given that the vehicle is a car.

Two-Way Frequency Tables



Understand that patterns ... can be seen in bivariate categorical data by displaying frequencies in a two-way table. Construct and interpret a two-way table Use relative frequencies ... to describe possible association

ESSENTIAL QUESTION

How can you construct and interpret two-way frequency tables?

EXPLORE ACTIVITY



Making a Two-Way Table

The **frequency** is the number of times an event occurs. A **two-way table** shows the frequencies of data that is categorized two ways. The rows indicate one categorization and the columns indicate another.

A poll of 120 town residents found that 40% own a bike. Of those who own a bike, 75% shop at the farmer's market. Of those who do not own a bike, 25% shop at the farmer's market.

Real

	Farmer's Market	No Farmer's Market	TOTAL
Bike			
No Bike			
TOTAL			



- A Start in the bottom right cell of the table. Enter the total number of people polled.
- **B** Fill in the right column. 40% of 120 people polled own a bike.

The remaining people polled do not own a bike.

C Fill in the top row. 75% of those who own a bike also shop at the market.

The remaining bike owners do not shop at the market.

D Fill in the second row. 25% of those who do not own a bike shop at the market.

The remaining people without bikes do not shop at the market.

E Fill in the last row. In each column, add the numbers in the first two rows to find the total number of people who shop at the farmer's market and who do not shop at the farmer's market.

Alamy Image:

Reflect

1. How can you check that your table is completed correctly?





B Data from 200 flights were collected. The flights were categorized as domestic or international and late or not late. Is there an association between international flights and a flight being late?

	Late	Not Late	TOTAL
Domestic	30	120	150
International	10	40	50
TOTAL	40	160	200



STEP 1

Find the relative frequency of a flight being late.

Total flights that are late \rightarrow $\frac{40}{200} = 0.20 = 20\%$ Total number of flights \rightarrow $\frac{40}{200} = 0.20 = 20\%$

STEP 2

Find the relative frequency of a flight being late among international flights.

Number of international flights that are late \rightarrow $\frac{10}{50} = 0.20 = 20\%$ Total number of international flights \rightarrow $\frac{10}{50} = 0.20 = 20\%$

STEP 3 Compare the relative frequencies. International flights are no more likely to be late than flights in general. There is no association. The relative frequencies show that international flights are just as likely to be late as any other flight.

YOUR TURN

ò

2. Data from 200 middle school and high school students were collected. Students were asked whether or not they had visited at least one national park. Is there an association between being a high school student and visiting a national park? Explain.

	Have Visited a National Park	Have NOT Visited a National Park	TOTAL
Middle School	25	55	80
High School	80	40	120
TOTAL	105	95	200



Guided Practice

 In a survey of 50 students, 60% said that they have a cat. Of the students who have a cat, 70% also have a dog. Of the students who do not have a cat, 75% have a dog. Complete the two-way table. (Explore Activity)

	Dog	No Dog	TOTAL
Cat			
No Cat			
TOTAL			

- **a.** Enter the total number of students surveyed in the bottom right cell of the table.
- **b.** Fill in right column.
- **c.** Fill in top row.
- **d.** Fill in second row.
- e. Fill in last row.
- 2. The results of a survey at a school are shown. Is there an association between being a boy and being left-handed? Explain. (Example 1)

	Left-handed	Right-handed	TOTAL
Boys	14	126	140
Girls	10	90	100
TOTAL	24	216	240

ESSENTIAL QUESTION CHECK-IN

3. Voters were polled to see whether they supported Smith or Jones. Can you construct a two-way table of the results? Why or why not?

15.1 Independent Practice

FL 8.SP.1.4



Date.

4. Represent Real-World Problems One hundred forty students were asked about their language classes. Out of 111 who take French, only 31 do not take Spanish. Twelve take neither French nor Spanish. Use this information to make a two-way table.

Class.

	Take French	Do NOT Take French	TOTAL
Take Spanish			
Do NOT Take Spanish			
TOTAL			

- 5. Represent Real-World Problems Seventh- and eighth-grade students were asked whether they preferred science or math.
 - **a.** Complete the two-way table.

	Prefer Science	Prefer Math	TOTAL
Seventh Grade		72	96
Eighth Grade	32		
TOTAL			176

b. Is there an association between being in eighth grade and preferring math? Explain.

- 6. Persevere in Problem Solving The table gives partial information on the number of men and women who play in the four sections of the Metro Orchestra.
 - a. Complete the table.

	Strings	Brass	Woodwinds	Percussion	TOTAL
Men	13	7		5	33
Women			10		
TOTAL	55			9	98

FOCUS ON HIGHER ORDER THINKING

- Multi-Step The two-way table below shows the results of a survey of Florida teenagers who were asked whether they preferred surfing or snorkeling.
 - **a.** To the right of the number in each cell, write the relative frequency of the number compared to the total for the *row* the number is in. Round to the nearest percent.
 - Prefer Surfing
 Prefer Snorkeling
 TOTAL

 Ages 13–15
 52____;
 78___;
 130
 100%

 Ages 16–18
 52___;
 28___;
 80____

 TOTAL
 104___;
 106__;
 210____
 - **b.** Explain the meaning of the relative frequency you wrote beside 28.

c. To the right of each number you wrote in part a, write the relative frequency of each number compared to the total for the *column* the number is in. Are the relative frequencies the same? Why or why not?

d. Explain the meaning of the relative frequency you wrote beside 28.



Work Area

15.2 Two-Way Relative Frequency Tables



Understand that patterns ... can be seen in bivariate categorical data by displaying frequencies in a two-way table. Construct and interpret a two-way table Use relative frequencies calculated for rows or columns to describe possible association

ESSENTIAL QUESTION

How can categorical data be organized and analyzed?

EXPLORE ACTIVITY 1



Creating a Relative Frequency Table

Real World

The frequency table below shows the results of a survey that Maria took at her school. She asked 50 randomly selected students whether they preferred dogs, cats, or other pets. Convert this table to a *relative frequency* table that uses decimals as well as one that uses percents.



Preferred Pet	Dog	Cat	Other	TOTAL
Frequency	22	15	13	50

A Divide the numbers in the frequency table by the total to obtain relative frequencies as decimals. Record the results in the table below.

Preferred Pet	Dog	Cat	Other	TOTAL
Relative Frequency	$\frac{22}{50} = 0.44$			

B Write the decimals as percents in the table below.

Preferred Pet	Dog	Cat	Other	TOTAL
Relative Frequency	44%			

Reflect

- **1.** How can you check that you have correctly converted frequencies to relative frequencies?
- **2.** Explain why the number in the Total column of a relative frequency table is always 1 or 100%.

EXPLORE ACTIVITY 2 Real



FL 8.SP.1.4

In the previous Explore Activity, the categorical variable was pet preference, and the variable had three possible data values: dog, cat, and other. The frequency table listed the frequency for each value of that single variable. If you have two categorical variables whose values have been paired, you list the frequencies of the paired values in a **two-way frequency table**.

For her survey, Maria also recorded the gender of each student. The results are shown in the two-way frequency table below. Each entry is the frequency of students who prefer a certain pet *and* are a certain gender. For instance, 10 girls prefer dogs as pets. Complete the table.

Preferred Pet Gender	Dog	Cat	Other	TOTAL
Girl	10	9	3	
Воу	12	6	10	
TOTAL				



- A Find the total for each gender by adding the frequencies in each row.
- **B** Find the total for each pet by adding the frequencies in each column.
- **C** Find the grand total, which is the sum of the row totals as well as the sum of the column totals. Write this in the lower-right corner.

Reflect

- 3. Where have you seen the numbers in the Total row before?
- 4. In terms of Maria's survey, what does the grand total represent?



Creating a Two-Way Relative Frequency Table

You can obtain *relative* frequencies from a two-way frequency table:

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- A **joint relative frequency** is found by dividing a frequency that is not in the Total row or the Total column by the grand total.
- A marginal relative frequency is found by dividing a row total or a column total by the grand total.

A **two-way relative frequency table** displays both joint relative frequencies and marginal relative frequencies.



A Divide each number in the two-way frequency table from the previous Explore Activity by the grand total. Write the quotients as decimals.



Preferred Pet Gender	Dog	Cat	Other	TOTAL
Girl	$\frac{10}{50} = 0.2$			
Воу				
TOTAL	$\frac{22}{50} = 0.44$			$\frac{50}{50} = 1$

B Check by adding the joint relative frequencies in a row or column to see if the sum equals that row's or column's marginal relative frequency.

Girl row:	0.2 + + =
Boy row:	+=
Dog column:	0.2 + =
Cat column:	+=
Other column:	+=

Reflect

- **5.** A joint relative frequency in a two-way relative frequency table tells you what portion of the entire data set falls into the intersection of a particular value of one variable and a particular value of the other variable. What is the joint relative frequency of students surveyed who are boys and prefer cats as pets?
- **6.** A marginal relative frequency in a two-way relative frequency table tells you what portion of the entire data set represents a particular value of just one of the variables. What is the marginal relative frequency of students surveyed who are boys?



My Notes

Calculating Conditional Relative Frequencies

One other type of relative frequency that you can obtain from a two-way frequency table is a *conditional relative frequency*. A **conditional relative frequency** is found by dividing a frequency that is not in the Total row or the Total column by the frequency's row total or column total.

EXAMPLE 1 (Red)

FL 8.SP.1.4

From Maria's two-way frequency table you know that 22 students are girls and 15 students prefer cats. You also know that 9 students are girls who prefer cats. Use this to find each conditional relative frequency.

A Find the conditional relative frequency that a student surveyed prefers cats as pets, given that the student is a girl.

Divide the number of girls who prefer cats by the number of girls. Express your answer as a decimal and as a percent.

 $\frac{9}{22}$ = 0.409, or 40.9%

B Find the conditional relative frequency that a student surveyed is a girl, given that the student prefers cats as pets.

Divide the number of girls who prefer cats by the number of students who prefer cats. Express your answer as a decimal and as a percent.

$$\frac{9}{15} = 0.6$$
, or 60%

Reflect

7. When calculating a conditional relative frequency, why do you divide by a row total or a column total and not by the grand total?





8. You can obtain conditional relative frequencies from a two-way relative frequency table. Find the conditional relative frequency that a student prefers cats as pets, given that the student is a girl.

Finding Possible Associations Between Variables

You can use conditional relative frequency to see if there is an association between two variables.

EXAMPLE 2

STEP 2

Maria conducted her survey because she was interested in the question "Does gender influence what type of pet people prefer?" If there is no influence, then the distribution of gender within each subgroup of pet preference should roughly equal the distribution of gender within the whole group. Use the results of Maria's survey to investigate possible influences of gender on pet preference.

STEP 1 Identify the percent of all students surveyed who are girls: 44%

Determine each conditional relative frequency.

Of the 22 students who prefer dogs as pets, 10 are girls. Percent who are girls, given a preference for dogs as pets: 45%

Of the 15 students who prefer cats as pets, 9 are girls. Percent who are girls, given a preference for cats as pets: 60%

Of the 13 students who prefer other pets. 3 are girls. Percent who are girls, given a preference for other pets: 23%

STEP 3 Interpret the results by comparing each conditional relative frequency to the percent of all students surveyed who are girls.

The percent of girls among students who prefer dogs is close to 44%, so gender does not appear to influence preference for dogs.

The percent of girls among students who prefer cats is much greater than 44%, so girls are more likely than boys to prefer cats.

The percent of girls among students who prefer other pets is much less than 44%, so girls are less likely than boys to prefer other pets.



ò

9. Suppose you analyzed the data by focusing on boys rather than girls. How would the percent in Step 1 change? How would the percents in Step 2 change? How would the conclusions in Step 3 change?



My Notes

8.SP.1.4



Guided Practice

 In a class survey, students were asked to choose their favorite vacation destination. The results are displayed by gender in the two-way frequency table. (Explore Activities 1–3)

Preferred Pet Gender	Seashore	Mountains	Other	TOTAL
Girl	7	3	2	
Воу	5	2	6	
TOTAL				

- **a.** Find the total for each gender by adding the frequencies in each row. Write the row totals in the Total column.
- **b.** Find the total for each preferred vacation spot by adding the frequencies in each column. Write the column totals in the Total row.
- **c.** Write the grand total (the sum of the row totals and the column totals) in the lower-right corner of the table.
- **d.** Create a two-way relative frequency table by dividing each number in the above table by the grand total. Write the quotients as decimals.

Preferred Pet Gender	Seashore	Mountains	Other	TOTAL
Girl				
Воу				
TOTAL				

- **e.** Use the table to find the joint relative frequency of students surveyed who are boys and who prefer vacationing in the mountains.
- **f.** Use the table to find the marginal relative frequency of students surveyed who prefer vacationing at the seashore.
- **g.** Find the conditional relative frequency that a student surveyed prefers vacationing in the mountains, given that the student is a girl. Interpret this result. (Examples 1–2)

ESSENTIAL QUESTION CHECK-IN

2. How can you use a two-way frequency table to learn more about its data?

15.2 Independent Practice



Stefan surveyed 75 of his classmates about their participation in school activities as well as whether they have a part-time job. The results are shown in the two-way frequency table. Use the table for Exercises 3-6.

Class_

Activity Job	Clubs Only	Sports Only	Both	Neither	TOTAL
Yes		12		9	51
No	5		10		
TOTAL	15	18			75



- **3. a.** Complete the table.
 - **b.** Explain how you found the correct data to enter in the table.

4. Create a two-way relative frequency table using decimals. Round to the nearest hundredth.

Activity Job	Clubs Only	Sports Only	Both	Neither	TOTAL
Yes					
No					
TOTAL					

- **5.** Give each relative frequency as a percent.
 - a. the joint relative frequency of students surveyed who participate in

school clubs only and have part-time jobs _____

b. the marginal frequency of students surveyed who do not have a

part-time job _____

c. the conditional relative frequency that a student surveyed participates in both school clubs and sports, given that the student

has a part-time job _____

- - **8.** Analyze Relationships What is the difference between relative frequency and conditional relative frequency?

this data, the head of quality control divided each number in each row by the row total. Is this correct? Explain.

b. Draw Conclusions Are any of the data the head of guality control

entered into the two-way relative frequency table correct? If so, which

TOTAL 385 215 600 **a.** Critique Reasoning To create a two-way relative frequency table for





Work Area

6. Discuss possible influences of having a part-time job on participation in school activities. Support your response with an analysis of the data.

HOT FOCUS ON HIGHER ORDER THINKING

Accept/ Reject

is and which isn't? Explain.

Wood

White Oak

Redwood

Accepted

245

140

MODULE QUIZ

Ready to Go On?

15.1 Two-Way Frequency Tables

Martin collected data from students about whether they played a musical instrument. The table shows his results. Use the table for Exercises 1–4.

	Instrument	No Instrument	TOTAL
Boys	42	70	112
Girls	48		88
Total	90	110	200

- 1. Of the students surveyed, how many played an instrument?
- 2. How many girls surveyed did NOT play an instrument?
- **3.** What is the relative frequency of a student playing an instrument? Write the answer as a percent.
- **4.** What is the relative frequency of a boy playing an instrument? Write the answer as a decimal.

15.2 Two-Way Relative Frequency Tables

Students were asked how they traveled to school. The two-way relative frequency table shows the results. Use the table for Exercises 5–7. Write answers as decimals rounded to the nearest hundredth.

5. What is the joint relative frequency of high school students who ride the bus?

	Method				
School	Car Bus Other TOTAL				
Middle School	0.18	0.14	0.10	0.42	
High School	0.38	0.12	0.08	0.58	
TOTAL	0.56	0.26	0.18	1.00	

- **6.** What is the marginal relative frequency of students surveyed who are in middle school?
- **7.** What is the conditional relative frequency that a student rides the bus, given that the student is in middle school?

ESSENTIAL QUESTION

8. How can you use two-way tables to solve real-world problems?



MODULE 15 MIXED REVIEW Assessment Readiness



Selected Response

The table gives data on the length of time that teachers at Tenth Avenue School have taught. Use the table for Exercises 1–5.

	Fewer than 10 years	10 or more years	TOTAL
Male	9	6	15
Female	?	4	25
TOTAL	30	10	40

- 1. How many female teachers have taught for fewer than 10 years?
 - (A) 4(C) 21(B) 9(D) 30
- 2. What is the relative frequency of teachers who have taught for 10 or more years?
 - (A) 10% (C) 30%
 - **B** 25% **D** 60%
- **3.** What is the relative frequency of male teachers who have taught for fewer than 10 years?
 - (A) 0.09 (C) 0.6
 - **(B)** 0.225 **(D)** 1.50
- **4.** What is the joint relative frequency of female teachers who have taught for more than 10 years?
 - (A) 4%(C) 16%(C) 16%
 - (B) 10%
 (D) 25%
- **5.** What is the marginal relative frequency of teachers who are female?
 - (A) 0.16 (C) 0.4
 - **B** 0.25 **D** 0.625

- **6.** A triangle has an exterior angle of *x*°. Which of the following represents the measure of the interior angle next to it?
 - (A) $(180 x)^{\circ}$ (C) $(90 x)^{\circ}$
 - **(B)** $(x 180)^{\circ}$ **(D)** $(x 90)^{\circ}$
- 7. What is the volume of a cone that has a diameter of 12 cm and a height of 4 cm? Use 3.14 for π and round to the nearest tenth.

(A) 25.12 cm ³	(C) 150.72 cm ³
0	<u> </u>

(B) 602.88 cm^3 **(D)** $1,808.64 \text{ cm}^3$

Mini-Task

8. The table gives data on books read by members of the Summer Reading Club.

	Fewer than 25 books	25 or more books	TOTAL
Boys	7	21	28
Girls	9	27	36
TOTAL	16	48	64

- **a.** Find the relative frequency of a club member reading fewer than 25 books.
- **b.** Find the relative frequency of a girl club member reading fewer than 25 books.
- Is there an association between being a girl and reading fewer than 25 books? Explain.

UNIT 6 Study Guide Review



MODULE 14 Scatter Plots

ESSENTIAL QUESTION

How can you use scatter plots to solve real-world problems?

EXAMPLE 1

As part of a research project, a researcher made a table of test scores and the number of hours of sleep a person got the night before the test. Make a scatter plot of the data. Does the data show a positive association, negative association, or no association?

Sleep (hours)	Test score
4	30
5	40
6	50
6	70
8	100
9	90
10	100



The data show a positive association. Generally, as the number of hours of sleep increases, so do the test scores.

EXAMPLE 2

Write an equation for a trend line of the data shown on the graph.



 $m = \frac{30-15}{9-4} = 3$ 15 = 3(4) + bb = 3

$$y=3x+3$$

Find the slope.

Find the y-intercept.

Use the slope and y-intercept to write the equation.

Key Vocabulary

cluster (agrupación) outlier (valor extremo) scatter plot (diagrama de dispersión) trend line (línea de tendencia)

EXERCISES

1. The table shows the income of 8 households, in thousands of dollars, and the number of televisions in each household. (Lesson 14.1)

Income (\$1000)	20	20	30	30	40	60	70	90
Number of televisions	4	0	1	2	2	3	3	4

- **a.** Make a scatter plot of the data.
- **b.** Describe the association between income and number of televisions. Are any of the values outliers?



- **2.** The scatter plot shows the relationship between the price of a product and the number of potential buyers. (Lesson 14.2)
 - **a.** Draw a trend line for the scatter plot.
 - **b.** Write an equation for your trend line.
 - **c.** When the price of the product is \$3.50, the number of potential buyers will be

about _____

d. When the price of the product is \$5.50, the number of potential buyers will be

about _____



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MODULE 15 Two-Way Tables

ESSENTIAL QUESTION

How can you use two-way tables to solve real-world problems?

EXAMPLE

A movie theater kept a record of patrons who bought tickets for a particular movie for two different times. The results are shown in the two-way frequency table. Create a two-way relative frequency table of these data.

	5:00 р.м. Showing	8:00 р.м. Showing	Total
Adults	22	39	61
Children	40	25	65
Total	62	64	126

Step 1: Divide each entry by the total number of patrons. Round to the nearest hundredth.

	5:00 р.м. Showing	8:00 р.м. Showing	Total
Adults	$\frac{22}{126}$ \approx 0.17	$\frac{39}{126}$ \approx 0.31	$\frac{61}{126}$ \approx 0.48
Children	$\frac{40}{126}\approx 0.32$	$\frac{25}{126}$ \approx 0.20	$rac{65}{126} \approx 0.52$
Total	$\frac{62}{126}$ \approx 0.49	$rac{64}{126}$ $pprox$ 0.51	$\frac{126}{126} \approx 1.00$

Step 2: Convert decimals to percents.

	5:00 р.м. Showing	8:00 р.м. Showing	Total
Adults	17%	31%	48%
Children	32%	20%	52%
Total	49%	51%	100%

EXERCISES

Use the tables in the Example to answer each question.

- 1. What is the joint relative frequency of patrons who are adults and attended the 8:00 P.M. showing?
- 2. What is the marginal relative frequency of patrons who went to the 8:00 P.M. showing?
- **3.** What is the conditional relative frequency that a patron is an adult, given that the patron attends the 8:00 P.M. showing?

Key Vocabulary

conditional relative frequency (frecuencia relativa condicional)

frequency (*frecuencia*) joint relative frequency (frecuencia relativa común)

marginal relative frequency (frecuencia relativa *marginal*)

relative frequency (frecuencia relativa)

two-way frequency table (tabla de frecuencia de doble entrada)

two-way relative frequency table (tabla de frecuencia relativa de doble entrada)

two-way table (tabla de doble entrada)

Unit 6 Performance Tasks

- 1. CAREERS IN MATH Psychologist A psychologist gave a test to 15 women of different ages to measure their short-term memory. The test score scale goes from 0 to 24, and a higher score means that the participant has a better short-term memory. The scatter plot shows the results of this study.
 - **a.** Describe the pattern in the data. Is there a positive or negative correlation?



- **b.** Draw a line of best fit on the scatter plot and estimate its slope. Interpret the slope in the context of the problem.
- **c.** In another test, a 70-year-old woman scored 8. Does your line of best fit predict a higher or lower score? What may have happened?
- 2. Kalila has developed two different varieties of tomatoes, called Big Red and Sweet Summer, which she grows in her garden. When she harvests the tomatoes, she measures the diameters of each variety. The results are shown in the table.

	$\begin{array}{l} {\rm Diameter} \\ \leq {\rm 2~in.} \end{array}$	Diameter > 2 in.	
Big Red	36	45	
Sweet Summer	28	39	

a. Use the data to create a two-way frequency table.

	Diameter \leq 2 in.	Diameter $>$ 2 in.	Total
Big Red			
Sweet Summer			
Total			

- **b.** What is the relative frequency of a tomato having a diameter that is greater than two inches? Round to the nearest percent.
- **c.** What is the relative frequency of a Big Red tomato having a diameter that is greater than two inches? Round to the nearest percent.
- **d.** Is there an association between Big Red tomatoes and diameters that are greater than two inches? Explain.



Assessment Readiness



Selected Response

 A local election conducted an exit poll of the age and gender of its voters. The results are shown in the two-way frequency table.

	18–62 years old	63 years and older	Total
Female	142	22	164
Male	126	15	141
Total	268	37	305

What percent of the voters were female and 18 to 62 years old?

- **(A)** 6.5% **(C)** 53%
- **B** 47% **D** 87%
- 2. What type of association is there between the speed of a car and the distance the car travels in a given time at that speed?
 - (A) cluster
 - (B) negative association
 - © no association
 - D positive association
- **3.** Using 3.14 for π , what is the volume of the sphere to the nearest tenth?





Read graphs and diagrams carefully. Look at the labels for important information.

4. Which scatter plot could have a trend line given by the equation y = -4x + 70?



Unit 6 471

 A group of middle school students were asked whether they prefer communicating with their friends by text message or email. The results are shown in the two-way frequency table.

	Text Message	Email	Total
Female	28	16	44
Male	31	18	49
Total	59	34	93

What is the conditional relative frequency that a student prefers email, given that the student is female?

- A 17%
 C 47%
- **(B)** 36% **(D)** 64%
- **6.** The vertices of a triangle are (11, 9), (7, 4), and (1, 11). What are the vertices after the triangle has been reflected over the *y*-axis?
 - (A) (9, 11), (4, 7), (11, 1)
 - **(B)** (11, −9), (7, −4), (1, −11)
 - **((**9, 11), (4, 7), (11, 1)
 - () (-11, 9), (-7, 4), (-1, 11)
- **7.** Which of the following is **not** shown on the scatter plot below?



- (A) cluster
- (B) negative association
- © outlier
- **D** positive association

Mini-Task

8. A scatter plot and trend line of the weight of a Chihuahua puppy versus age is shown.



a. The trend line for these data is given by y = 4x + 3. What does the 3 represent in this context?

b. If you use the trend line to predict the weight of the puppy after 60 months, the result is 243 ounces, or about 15 pounds. Is this a reasonable weight for the Chihuahua at 5 years old? Explain.