LESSON 11.3

Angle-Angle Similarity

FL 8.G.1.5

Use informal arguments to establish facts about . . . the angle-angle criterion for similarity of triangles. *Also 8.EE.2.6, 8.EE.3.7*

ESSENTIAL QUESTION

How can you determine when two triangles are similar?

EXPLORE ACTIVITY 1 FL 8.G.1.5

Discovering Angle-Angle Similarity

Similar figures have the same shape but may have different sizes. Two triangles are **similar** if their corresponding angles are congruent and the lengths of their corresponding sides are proportional.

A Use your protractor and a straightedge to draw a triangle. Make one angle measure 45° and another angle measure 60°.

B Compare your triangle to those drawn by your classmates. How are the triangles the same?

____

How are they different?

____

C Use the Triangle Sum Theorem to find the measure of the third angle of your triangle.

____

Reflect

1. If two angles in one triangle are congruent to two angles in another triangle, what do you know about the third pair of angles?

____

2. **Make a Conjecture** Are two pairs of congruent angles enough information to conclude that two triangles are similar? Explain.

____

____

____

Using the AA Similarity Postulate

Math On the Spot
my.hrw.com

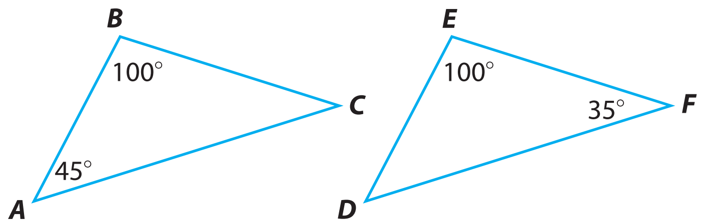
Angle-Angle (AA) Similarity Postulate

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

EXAMPLE 1

FL 8.G.1.5

Explain whether the triangles are similar.



The figure shows only one pair of congruent angles. Find the measure of the third angle in each triangle.

$$45^\circ + 100^\circ + m\angle C = 180^\circ$$
$$145^\circ + m\angle C = 180^\circ$$
$$145^\circ + m\angle C - 145^\circ = 180^\circ - 145^\circ$$
$$m\angle C = 35^\circ$$

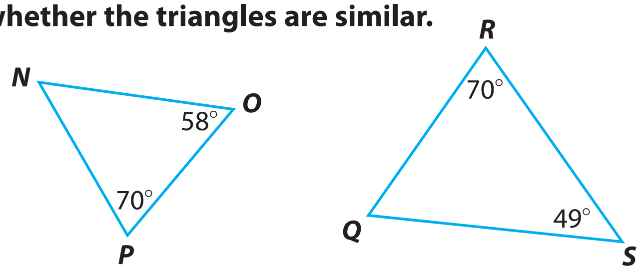
$$100^\circ + 35^\circ + m\angle D = 180^\circ$$
$$135^\circ + m\angle D = 180^\circ$$
$$135^\circ + m\angle D - 135^\circ = 180^\circ - 135^\circ$$
$$m\angle D = 45^\circ$$

Because two angles in one triangle are congruent to two angles in the other triangle, the triangles are similar.

Math Talk
Mathematical Practices
Are all right triangles similar? Why or why not?

YOUR TURN

3. Explain whether the triangles are similar.



Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Finding Missing Measures in Similar Triangles

Because corresponding angles are congruent and corresponding sides are proportional in similar triangles, you can use similar triangles to solve real-world problems.

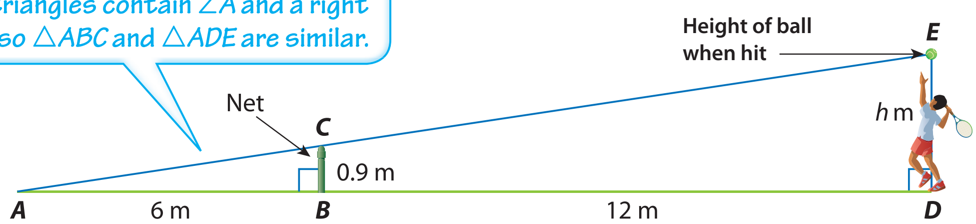
Math On the Spot
my.hrw.com

EXAMPLE 2 Real World

FL 8.EE.3.7

While playing tennis, Matt is 12 meters from the net, which is 0.9 meter high. He needs to hit the ball so that it just clears the net and lands 6 meters beyond the base of the net. At what height should Matt hit the tennis ball?

Both triangles contain $\angle A$ and a right angle, so $\triangle ABC$ and $\triangle ADE$ are similar.



In similar triangles, corresponding side lengths are proportional.

$$\frac{AD}{AB} = \frac{DE}{BC} \longrightarrow \frac{6 + 12}{6} = \frac{h}{0.9}$$ Substitute the lengths from the figure.

$$0.9 \times \frac{18}{6} = \frac{h}{0.9} \times 0.9$$ Use properties of equality to get h by itself.

$$0.9 \times 3 = h$$ Simplify.

$$2.7 = h$$ Multiply.

Matt should hit the ball at a height of 2.7 meters.

Reflect

4. **What If?** Suppose you set up a proportion so that each ratio compares parts of one triangle, as shown below.

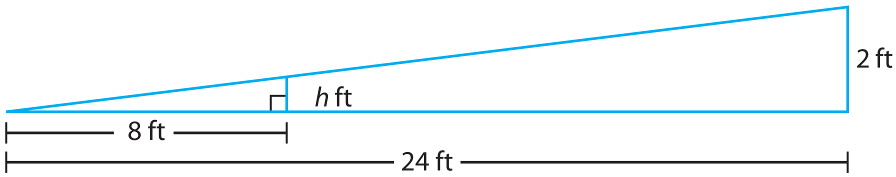
$$\begin{array}{r} \text{height of } \triangle ABC \longrightarrow \\ \text{base of } \triangle ABC \longrightarrow \end{array} \frac{BC}{AB} = \frac{DE}{AD} \begin{array}{l} \longleftarrow \text{height of } \triangle ADE \\ \longleftarrow \text{base of } \triangle ADE \end{array}$$

Show that this proportion leads to the same value for h as in Example 2.

Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

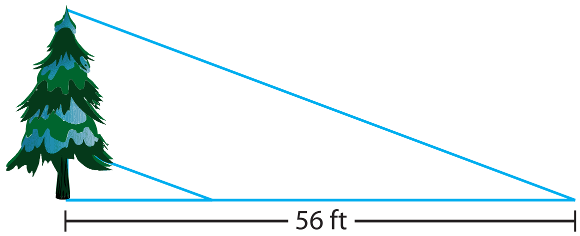
YOUR TURN

5. Rosie is building a wheelchair ramp that is 24 feet long and 2 feet high. She needs to install a vertical support piece 8 feet from the end of the ramp. What is the length of the support piece in inches?



____

6. The lower cable meets the tree at a height of 6 feet and extends out 16 feet from the base of the tree. If the triangles are similar, how tall is the tree?



____

EXPLORE ACTIVITY 2

FL 8.EE.2.6

Using Similar Triangles to Explain Slope

You can use similar triangles to show that the slope of a line is constant.

A Draw a line ℓ that is not a horizontal line. Label four points on the line as *A*, *B*, *C*, and *D*.

You need to show that the slope between points *A* and *B* is the same as the slope between points *C* and *D*.

B Draw the rise and run for the slope between points *A* and *B*. Label the intersection as point *E*. Draw the rise and run for the slope between points *C* and *D*. Label the intersection as point *F*.

C Write expressions for the slope between *A* and *B* and between *C* and *D*.

Slope between *A* and *B*: $\frac{BE}{\square}$ Slope between *C* and *D*: $\frac{\square}{CF}$

D Extend $\overleftrightarrow{AE}$ and $\overleftrightarrow{CF}$ across your drawing. $\overleftrightarrow{AE}$ and $\overleftrightarrow{CF}$ are both horizontal lines, so they are parallel.

Line ℓ is a ______________ that intersects parallel lines.

E Complete the following statements:

$\angle BAE$ and ________ are corresponding angles and are ______________.

$\angle BEA$ and ________ are right angles and are ______________.

F By Angle–Angle Similarity, $\triangle ABE$ and ________ are similar triangles.

G Use the fact that the lengths of corresponding sides of similar triangles are proportional to complete the following ratios: $\frac{BE}{DF} = \frac{\square}{CF}$

H Recall that you can also write the proportion so that the ratios compare parts of the same triangle: $\frac{\square}{AE} = \frac{DF}{\square}$.

I The proportion you wrote in step **H** shows that the ratios you wrote in **C** are equal. So, the slope of line ℓ is constant.

Reflect

7. **What If?** Suppose that you label two other points on line ℓ as *G* and *H*. Would the slope between these two points be different than the slope you found in the Explore Activity? Explain.

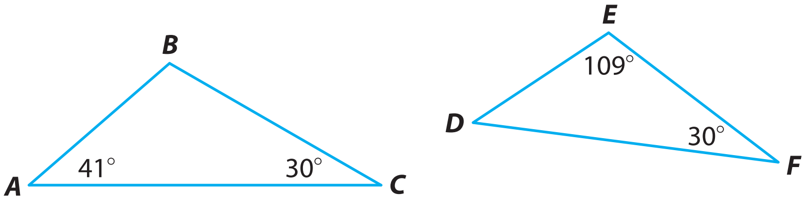
____

____

____

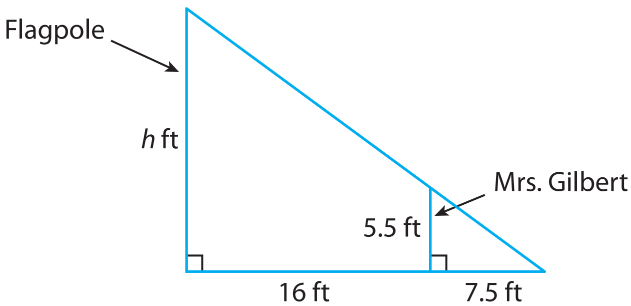
Guided Practice

1. Explain whether the triangles are similar. Label the angle measures in the figure. (Explore Activity 1 and Example 1)



$\triangle ABC$ has angle measures ____________________ and $\triangle DEF$ has angle measures ____________________. Because ________________ in one triangle are congruent to ________________ in the other triangle, the triangles are ________________.

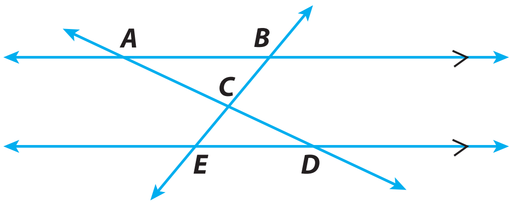
2. A flagpole casts a shadow 23.5 feet long. At the same time of day, Mrs. Gilbert, who is 5.5 feet tall, casts a shadow that is 7.5 feet long. How tall in feet is the flagpole? Round your answer to the nearest tenth. (Example 2)



$$\frac{5.5}{\square} = \frac{h}{\square}$$

$h =$ ______ feet

3. Two transversals intersect two parallel lines as shown. Explain whether $\triangle ABC$ and $\triangle DEC$ are similar. (Example 1)



$\angle BAC$ and $\angle EDC$ are ____________ since they are ______________________________.

$\angle ABC$ and $\angle DEC$ are ____________ since they are ______________________________.

By _______________, $\triangle ABC$ and $\triangle DEC$ are ____________.

ESSENTIAL QUESTION CHECK-IN

4. How can you determine when two triangles are similar?

____

____

____

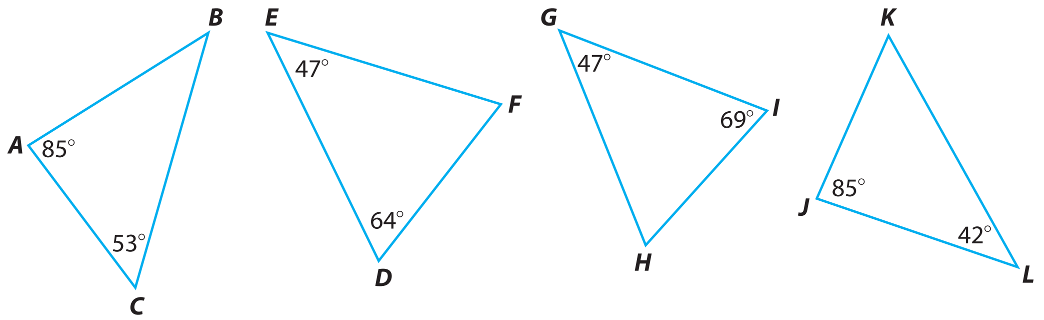
Name ______________________ Class ______________ Date ____________

11.3 Independent Practice

FL 8.EE.2.6, 8.EE.3.7, 8.G.1.5

Personal Math Trainer
my.hrw.com
Online Assessment and Intervention

Use the diagrams for Exercises 5–7.



5. Find the missing angle measures in the triangles.

____

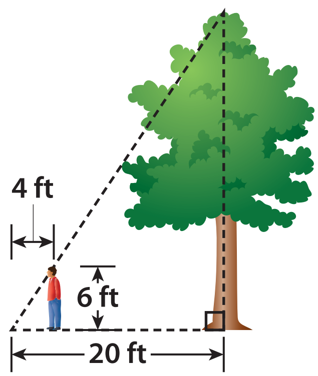
6. Which triangles are similar?

____

7. **Analyze Relationships** Determine which angles are congruent to the angles in $\triangle ABC$.

____

8. **Multistep** A tree casts a shadow that is 20 feet long. Frank is 6 feet tall, and while standing next to the tree he casts a shadow that is 4 feet long.



a. How tall is the tree? ______________

b. How much taller is the tree than Frank? ______________

9. **Represent Real-World Problems** Sheila is climbing on a ladder that is attached against the side of a jungle gym wall. She is 5 feet off the ground and 3 feet from the base of the ladder, which is 15 feet from the wall. Draw a diagram to help you solve the problem. How high up the wall is the top of the ladder?

____

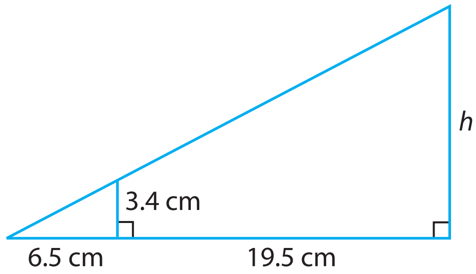
10. **Justify Reasoning** Are two equilateral triangles always similar? Explain.

____

____

____

11. **Critique Reasoning** Ryan calculated the missing measure in the diagram shown. What was his mistake?



$$\frac{3.4}{6.5} = \frac{h}{19.5}$$

$$19.5 \times \frac{3.4}{6.5} = \frac{h}{19.5} \times 19.5$$

$$\frac{66.3}{6.5} = h$$

$$10.2 \text{ cm} = h$$

____

____

____

H.O.T. FOCUS ON HIGHER ORDER THINKING

Work Area

12. **Communicate Mathematical Ideas** For a pair of triangular earrings, how can you tell if they are similar? How can you tell if they are congruent?

____

____

____

____

13. **Critical Thinking** When does it make sense to use similar triangles to measure the height and length of objects in real life?

____

____

____

14. **Justify Reasoning** Two right triangles on a coordinate plane are similar but not congruent. Each of the legs of both triangles are extended by 1 unit, creating two new right triangles. Are the resulting triangles similar? Explain using an example.

____

____

____

____

____

____