LESSON
11.2

Angle Theorems for Triangles

FL 8.G.1.5
Use informal arguments to establish facts about the angle sum and exterior angle of triangles.... *Also 8.EE.3.7, 8.EE.3.7b*

? ESSENTIAL QUESTION

What can you conclude about the measures of the angles of a triangle?

EXPLORE ACTIVITY 1 FL 8.G.1.5

Sum of the Angle Measures in a Triangle

There is a special relationship between the measures of the interior angles of a triangle.

A Draw a triangle and cut it out. Label the angles *A*, *B*, and *C*.

B Tear off each "corner" of the triangle. Each corner includes the vertex of one angle of the triangle.

C Arrange the vertices of the triangle around a point so that none of your corners overlap and there are no gaps between them.

D What do you notice about how the angles fit together around a point?

_____
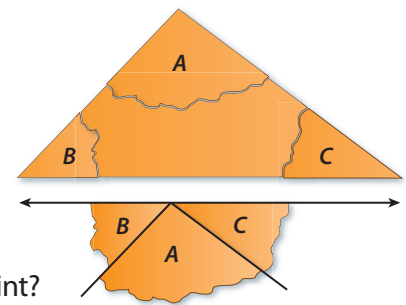
E What is the measure of a straight angle? _____

F Describe the relationship among the measures of the angles of △*ABC*.

_____

The Triangle Sum Theorem states that for △*ABC*, m∠*A* + m∠*B* + m∠*C* = _____.

Reflect

1. Justify Reasoning Can a triangle have two right angles? Explain.

_____

_____

_____

2. Analyze Relationships Describe the relationship between the two acute angles in a right triangle. Explain your reasoning.

_____

_____

_____

EXPLORE ACTIVITY 2   FL 8.G.1.5

Justifying the Triangle Sum Theorem

You can use your knowledge of parallel lines intersected by a transversal to informally justify the Triangle Sum Theorem.

Follow the steps to informally prove the Triangle Sum Theorem. You should draw each step on your own paper. The figures below are provided for you to check your work.

A Draw a triangle and label the angles as ∠1, ∠2, and ∠3 as shown.

B Draw line *a* through the base of the triangle.

C The Parallel Postulate states that through a point not on a line ℓ, there is exactly one line parallel to line ℓ. Draw line *b* parallel to line *a*, through the vertex opposite the base of the triangle.

D Extend each of the non-base sides of the triangle to form transversal *s* and transversal *t*. Transversals *s* and *t* intersect parallel lines *a* and *b*.

E Label the angles formed by line *b* and the transversals as ∠4 and ∠5.

F Because ∠4 and _____ are alternate interior

angles, they are _____.

Label ∠4 with the number of the angle to which it is congruent.

G Because ∠5 and _____ are alternate interior angles,

they are _____.

Label ∠5 with the number of the angle to which it is congruent.

H The three angles that lie along line *b* at the vertex of the triangle are ∠1, ∠4, and ∠5. Notice that these three angles lie along a line.

So, m∠1 + m∠2 + m∠5 = _____.

Because angles 2 and 4 are congruent and angles 3 and 5 are congruent, you can substitute m∠2 for m∠4 and m∠3 for m∠5 in the equation above.

So, m∠1 + m∠2 + m∠3 = _____.

This shows that the sum of the angle measures in a triangle is

always _____.
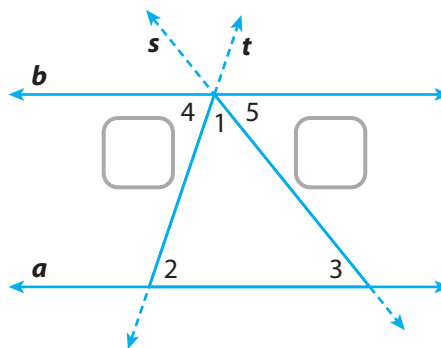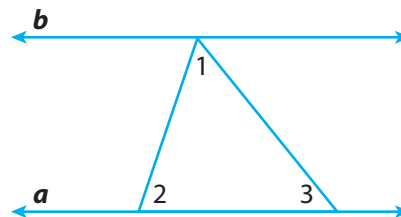
Reflect

3. Analyze Relationships How can you use the fact that $m\angle 4 + m\angle 1 + m\angle 5 = 180°$ to show that $m\angle 2 + m\angle 1 + m\angle 3 = 180°$?

_____

_____

_____

Finding Missing Angle Measures in Triangles

If you know the measures of two angles in a triangle, you can use the Triangle Sum Theorem to find the measure of the third angle.

EXAMPLE 1

FL   8.EE.3.7

Find the missing angle measure.

STEP 1 Write the Triangle Sum Theorem for this triangle.

$$m\angle D + m\angle E + m\angle F = 180°$$

STEP 2 Substitute the given angle measures.

$$55° + m\angle E + 100° = 180°$$

STEP 3 Solve the equation for $m\angle E$.

$$55° + m\angle E + 100° = 180°$$

$$\begin{array}{ll} 155° + m\angle E = 180° & \text{Simplify.}\\ \underline{-155°} \qquad\quad \underline{-155°} & \text{Subtract } 155° \text{ from both sides.}\\ m\angle E = \quad 25° \end{array}$$

So, $m\angle E = 25°$.
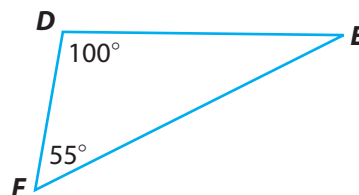
YOUR TURN

Find the missing angle measure.

4.
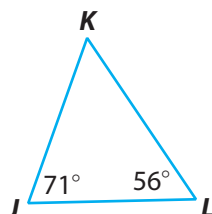
$m\angle K = $ _____

5.

$m\angle R = $ _____

© Houghton Mifflin Harcourt Publishing Company
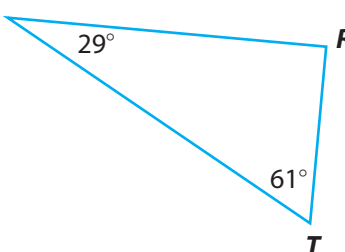
Exterior Angles and Remote Interior Angles

An **interior angle** of a triangle is formed by two sides of the triangle. An **exterior angle** is formed by one side of the triangle and the extension of an adjacent side. Each exterior angle has two remote interior angles. A **remote interior angle** is an interior angle that is not adjacent to the exterior angle.
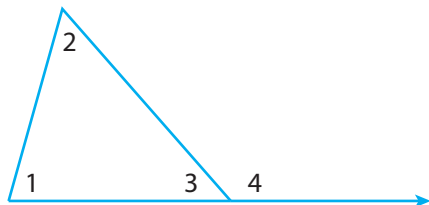
- $\angle 1$, $\angle 2$, and $\angle 3$ are interior angles.
- $\angle 4$ is an exterior angle.
- $\angle 1$ and $\angle 2$ are remote interior angles to $\angle 4$.

There is a special relationship between the measure of an exterior angle and the measures of its remote interior angles.

A Extend the base of the triangle and label the exterior angle as $\angle 4$.

B The Triangle Sum Theorem states:

$m\angle 1 + m\angle 2 + m\angle 3 =$ _____.

C $\angle 3$ and $\angle 4$ form a _____,

so $m\angle 3 + m\angle 4 =$ _____.

D Use the equations in **B** and **C** to complete the following equation:

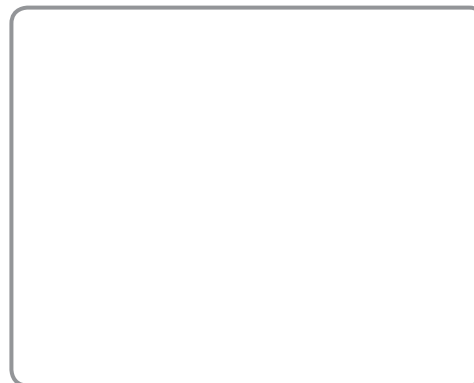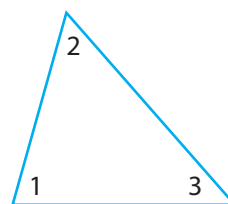$m\angle 1 + m\angle 2 +$ _____ $=$ _____ $+ m\angle 4$

E Use properties of equality to simplify the equation in **D**:

_____

The Exterior Angle Theorem states that the measure of an _____ angle

is equal to the sum of its _____ angles.

Reflect

6. Sketch a triangle and draw all of its exterior angles. How many exterior angles does a triangle have at each vertex?

_____

7. How many total exterior angles does a triangle have?

_____

Using the Exterior Angle Theorem

You can use the Exterior Angle Theorem to find the measures of the interior angles of a triangle.
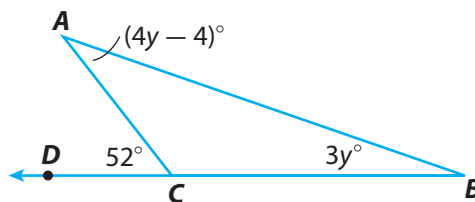
EXAMPLE 2

FL 8.EE.3.7b

Find m∠A and m∠B.

STEP 1 Write the Exterior Angle Theorem as it applies to this triangle.

$$m\angle A + m\angle B = m\angle ACD$$

STEP 2 Substitute the given angle measures.

$$(4y - 4)° + 3y° = 52°$$

STEP 3 Solve the equation for y.

$$(4y - 4)° + 3y° = 52°$$

$4y° - 4° + 3y° = 52°$	Remove parentheses.
$7y° - 4° = 52°$	Simplify.
$\underline{+4° \qquad +4°}$	Add 4° to both sides.
$7y° = 56°$	Simplify.
$\dfrac{7y°}{7} = \dfrac{56°}{7}$	Divide both sides by 7.
$y = 8$	Simplify.

Math Talk
Mathematical Practices

Describe two ways to find m∠ACB.

STEP 4 Use the value of y to find m∠A and m∠B.

m∠A = 4y − 4	m∠B = 3y
= 4(8) − 4	= 3(8)
= 32 − 4	= 24
= 28	

So, m∠A = 28° and m∠B = 24°.

YOUR TURN

8. Find m∠M and m∠N.

m∠M = _____

m∠N = _____

© Houghton Mifflin Harcourt Publishing Company

Find each missing angle measure. (Explore Activity 1 and Example 1)
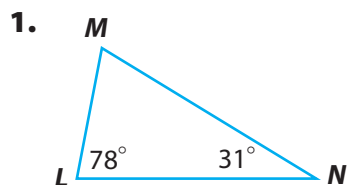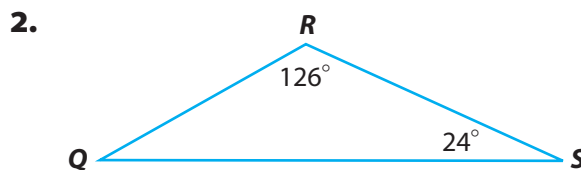
1.
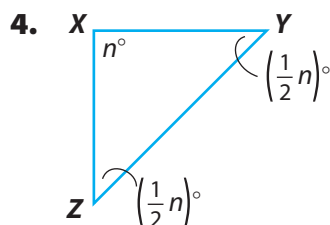

$m\angle M = $ _____

2.

$m\angle Q = $ _____

Use the Triangle Sum Theorem to find the measure of each angle in degrees. (Explore Activity 2 and Example 1)
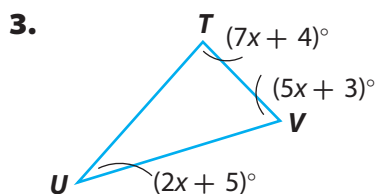
3.

$m\angle T = $ _____, $m\angle U = $ _____,

$m\angle V = $ _____

4.

$m\angle X = $ _____, $m\angle Y = $ _____,

$m\angle Z = $ _____

Use the Exterior Angle Theorem to find the measure of each angle in degrees. (Explore Activity 3 and Example 2)

5.

$m\angle C = $ _____, $m\angle D = $ _____,

$m\angle DEC = $ _____

6.

$m\angle L = $ _____, $m\angle M = $ _____,
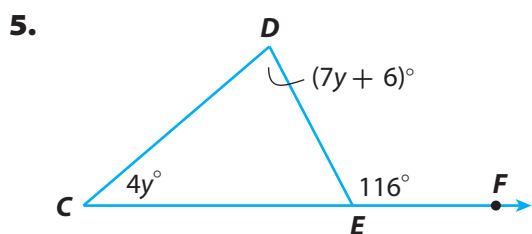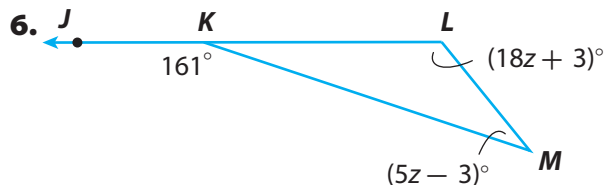
$m\angle LKM = $ _____

? ESSENTIAL QUESTION CHECK-IN

7. Describe the relationships among the measures of the angles of a triangle.

_____

_____

_____

11.2 Independent Practice

Personal Math Trainer

Online Assessment and Intervention

my.hrw.com

Find the measure of each angle.

8.

m∠E = _____
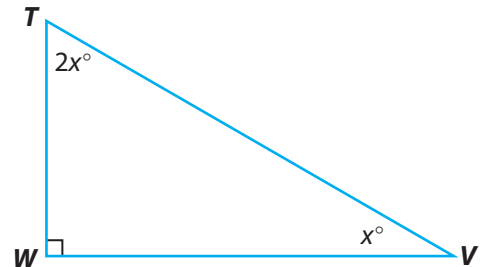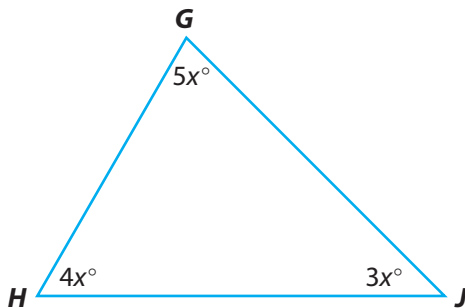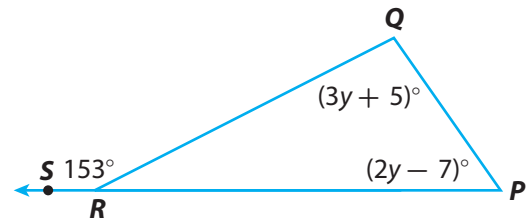
m∠F = _____

9.

m∠T = _____

m∠V = _____

10.

m∠G = _____

m∠H = _____

m∠J = _____

11.

m∠Q = _____

m∠P = _____

m∠QRP = _____

12.

m∠ACB = _____

m∠BCD = _____

m∠DCE = _____

13.

m∠K = _____

m∠L = _____
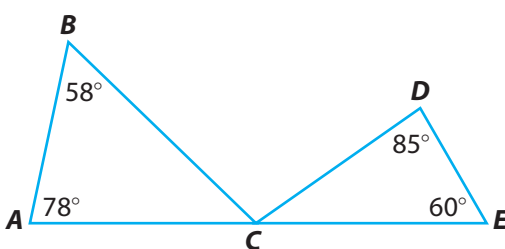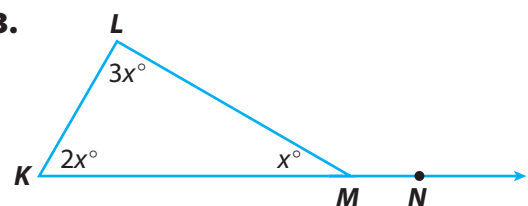
m∠KML = _____

m∠LMN = _____

14. Multistep The second angle in a triangle is five times as large as the first. The third angle is two-thirds as large as the first. Find the angle measures. _____

15. Analyze Relationships Can a triangle have two obtuse angles? Explain.

_____

_____

_____

_____

16. Critical Thinking Explain how you can use the Triangle Sum Theorem to find the measures of the angles of an equilateral triangle.

_____

_____

_____

_____

17. a. Draw Conclusions Find the sum of the measures of the angles in quadrilateral *ABCD*. (Hint: Draw diagonal $\overline{AC}$. How can you use the figures you have formed to find the sum?)
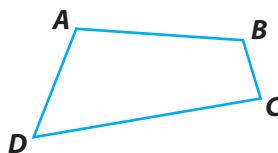
Sum = _____

b. Make a Conjecture Write a "Quadrilateral Sum Theorem." Explain why you think it is true.

_____

_____

_____

_____

_____

_____

18. Communicate Mathematical Ideas Describe two ways that an exterior angle of a triangle is related to one or more of the interior angles.

_____

_____

_____

_____